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**RESEARCH ARTICLE**

# INDUCED QUASI STATIC THERMAL STRESSES BY A CIRCULAR INTERNAL POINT HEAT SOURCE IN A CIRCULAR PLATE

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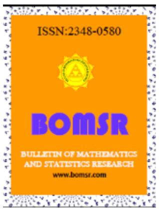


**ABSTRACT**

The present paper deals with the determination of quasi-static thermal stresses due to transient circular internal point heat source situated at certain circle along the radial direction of the circular plate and releasing its heat spontaneously. A circular plate is considered having arbitrary initial temperature and subjected to the boundary surface dissipates heat by convection at $r = a$. Here we compute the effect of transient circular internal point heat source on temperature, displacement and stresses along radial direction and modify Deshmukh *et al* (2011). The governing heat conduction equation has been solved by using integral transform method and the results are obtained in series form in terms of Bessel's functions and the results for temperature change, displacement and stresses have been computed numerically and illustrated graphically.

**Keywords** Thermal stresses, quasi-static, heat conduction equation, circular internal point heat source.

## INTRODUCTION

During the second half of the twentieth century, nonisothermal problems of the theory of elasticity became increasingly important. This is due to their wide application in diverse fields. The high velocities of modern aircraft give rise to aerodynamic heating, which produces intense thermal stresses that reduce the strength of the aircraft structure.

Nowacki (1957) has determined the steady state thermal stresses in circular disk subjected to an axisymmetric temperature distribution on the upper face with zero temperature on the lower face. Roy Choudhary (1972) has determined the quasi-static thermal stresses in a thin circular plate subjected to transient temperature along the circumference of a circle over the upper face with lower face at zero temperature and fixed circular edge thermally insulated. Wankhede (1982) determined the quasi-static thermal stresses in a thin circular disk subjected to arbitrary initial temperature on the upper surface with lower surface at zero temperature and fixed circular edge thermally insulated.

The present paper deals with the determination of quasi-static thermal stresses due to transient circular internal point heat source situated at certain circle along the radial direction of the circular plate and releasing its heat spontaneously. A circular plate is considered having arbitrary initial temperature and subjected to the boundary surface dissipates heat by convection at *r = a*. Here we compute the effect of transient circular internal point heat source on temperature, displacement and stresses along radial direction and modify Deshmukh *et al* (2011). The governing heat conduction equation has been solved by using integral transform method and the results are obtained in series form in terms of Bessel's functions and the results for temperature change, displacement and stresses have been computed numerically and illustrated graphically.

No one previously studied such type of problem. This is new contribution to the field. The results presented here will be useful for engineering problems, particularly in the determination of the state of stress in thin annular cylinders, which constitute essential elements for containers of hot gases or liquids, furnaces and similar facilities.

**FORMULATION OF THE PROBLEM**

Consider a circular plate occupying space D defined by $0 \le r \le a$. Initially the plate is at an arbitrary temperature distribution of $f(r)$ and homogeneous heat convection is maintained at fixed circular edge. Heat generates within the solid at the rate of $\frac{q(r,t)}{k}$. Under these prescribed conditions, the displacement and thermal stresses in the plate with internal heat generation are required to be determined.

The differential equation governing the displacement potential function $\phi(r,t)$ is given by,

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} = (1+\vartheta)a_t T \tag{1}$$

The stress function $\sigma_{rr}$ and $\sigma_{\theta\theta}$ are given by

$$\sigma_{rr} = \frac{-2\mu}{r}\frac{\partial \phi}{\partial r} \tag{2}$$

$$\sigma_{\theta\theta} = -2\mu \frac{\partial^2 \phi}{\partial r^2} \tag{3}$$

while in each case the stress functions $\sigma_{zz}$, $\sigma_{rz}$ and $\sigma_{\theta z}$ zero within the plane state of the stress.

The temperature of the circular plate at time *t* satisfying heat conduction equation as follows,

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{q(r,t)}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}, \tag{4}$$

with boundary condition

$$\frac{\partial T}{\partial r} + H\,T = 0 \quad (r = a, t > 0) \tag{5}$$

and initial condition

$$T = f(r) \quad (t = 0,\ 0 \le r \le a), \tag{6}$$

*where* $k$ is thermal conductivity, $H$ is heat transfer coefficients and $\alpha$ is thermal diffusivity of the material of the plate.

Equations (1)-(6) constitute the mathematical formulation of the problem.

**SOLUTION OF THE HEAT CONDUCTION EQUATION**

To obtain the expression for temperature $T(r,t)$, we introduce the finite Hankel transform over

the variable r and its inverse transform defined by Ozisik (1968) as

$$\bar{T}(\beta_m, t) = \int_{r'=0}^{a} r' K_0(\beta_m, r')\, T(r', t)\, dr' \tag{7}$$

$$T(r,t) = \sum_{m=1}^{\infty} K_0(\beta_m, r)\, \bar{T}(\beta_m, t) \tag{8}$$

$$\text{where } K_0(\beta_m, r) = \frac{\sqrt{2}}{a} \frac{\beta_m}{\sqrt{H^2 + \beta_m{}^2}} \frac{J_0(\beta_m r)}{J_0(\beta_m a)}, \tag{9}$$

$\beta_1, \beta_2, \ldots$ are the positive roots of the transcendental equation

$$\beta_m\ J_0'(\beta_m a) + H J_0(\beta_m a) = 0 \tag{10}$$

where $J_n(x)$ is the Bessel function of the first kind of order n.

On applying the finite Hankel transform defined in Eq. (7) and its inverse transform defined in Eq.(8) to the Eq.(4), one obtains the expression for temperature as

$$T(r,t) = \sum_{m=1}^{\infty} \frac{\sqrt{2}}{a} \frac{\beta_m}{\sqrt{H^2 + \beta_m{}^2}} \frac{J_0(\beta_m r)}{J_0(\beta_m a)}\ e^{-\alpha\, \beta_m{}^2 t} \left\{ \frac{\alpha}{k} \int_0^t e^{\alpha\, \beta_m{}^2 t} \right.$$

$$\left. \times \left[\int_{r'=0}^{a} r'\, K_0(\beta_m, r') q(r', t) dr'\right] dt + \int_{r'=0}^{a} r'\, K_0(\beta_m, r') f(r')\, dr' \right\} \tag{11}$$

**GOODIERS THERMOELASTIC DISPLACEMENT POTENTIAL** $\phi$

Assuming the displacement function $\phi(r,z)$ which satisfies Eq. (1) as

$$\phi(r,t) = -(1+v)a_t \sum_{m=1}^{\infty} \frac{\sqrt{2}}{\beta_m a} \frac{1}{\sqrt{H^2 + \beta_m{}^2}} \left\{ \frac{J_0(\beta_m r)}{J_0(\beta_m a)} e^{-\alpha\, \beta_m{}^2 t}\ \ \frac{\alpha}{k} \int_0^t e^{\alpha\, \beta_m{}^2 t} \right.$$

$$\left. \times \left[\int_{r'=0}^{a} r'\, K_0(\beta_m, r') q(r', t) dr'\right] dt + \int_{r'=0}^{a} r'\, K_0(\beta_m, r') f(r')\, dr' \right\} \tag{12}$$

Now using Eqs. (11) and (12) in Eqs. (2) and (3), one obtains the expressions for stresses respectively as

$$\sigma_{rr} = -\frac{2\sqrt{2}\mu}{ra}(1+\vartheta)a_t \sum_{m=1}^{\infty} \frac{1}{\sqrt{H^2 + \beta_m{}^2}} \frac{J_1(\beta_m r)}{J_0(\beta_m a)}$$

$$\times e^{-\alpha\, \beta_m{}^2 t} \left\{ \frac{\alpha}{k} \int_0^t e^{\alpha\, \beta_m{}^2 t} \left[\int_{r'=0}^{a} r'\, K_0(\beta_m, r') q(r', t) dr'\right] dt \right.$$

$$\left. + \int_{r'=0}^{a} r'\, K_0(\beta_m, r') f(r')\, dr' \right\} \tag{13}$$

$$\sigma_{\theta\theta} = \frac{2\sqrt{2}\mu}{a}(1+\vartheta)a_t \sum_{m=1}^{\infty} \frac{1}{\sqrt{H^2+\beta_m{}^2}} \frac{J_1{}'(\beta_m r)}{J_0(\beta_m a)}$$

$$\times e^{-\alpha\,\beta_m{}^2 t} \left\{ \frac{\alpha}{k}\int_0^t e^{\alpha\,\beta_m{}^2 t}\left[\int_{r'=0}^{a} r'\,K_0(\beta_m, r') q(r',t) dr'\right] dt \right.$$

$$\left. + \int_{r'=0}^{a} r'\,K_0(\beta_m, r') f(r')\, dr' \right\} \quad (14)$$

**SPECIAL CASE AND NUMERICAL CALCULATIONS**

Setting

(1) $\mathrm{f(r)} = \delta(r - r_0)$

$a = 1m$, $\mathrm{r}_0 = 0.5m$, $H = 13$ and $t = 10$sec.

where $\delta(r)$ is well known diract delta function of argument r.

$$F(\beta_m) = \frac{\sqrt{2}}{a} \frac{r_0\beta_m}{\sqrt{H^2+\beta_m{}^2}} \frac{J_0(\beta_m r_0)}{J_0(\beta_m a)}$$

(2) $q = \delta(r - r_0)\, e^{-t}$

$$\bar{q} = \int_{r'=0}^{a} r' \frac{\sqrt{2}}{a} \frac{J_0(\beta_m r')}{J_0(\beta_m a)}$$

$$\times\, \delta(r' - r_0) e^{-t}\, dr'$$

$$= \frac{\sqrt{2}}{a} \frac{r_0\,\beta_m\, e^{-t}}{\sqrt{H^2+\beta_m{}^2}} \frac{J_0(\beta_m r_0)}{J_0(\beta_m a)}$$

**Material Properties**

The numerical calculation has been carried out for copper (pure) circular plate with the material properties defined as

Thermal diffusivity $\alpha$ =112.34$\times$ $10^{-6}\ \mathrm{m^2 s^{-1}}$,

Specific heat $c_\rho = 383$ J/kgK,

Thermal conductivity k = 386 W/Mk,

Poisson ratio $\vartheta = 0.35$,

Coefficient of linear thermal expansion

$$a_t = 16.5 \times 10^{-6}\,\frac{1}{\mathrm{K}},$$

Lame constant $\mu = 26.67$.

**Roots of Transcendental Equation**

The $\beta_1 = 2.2509$, $\beta_2 = 5.1773$, $\beta_3 = 8.1422$, $\beta_4 = 11.1367$, $\beta_5 = 14.1576$, $\beta_6 = 17.2008$.

are the roots of the transcendental equation $\beta_m\ J_0{}'\,(\beta_m a) +\ H\, J_0(\beta_m a) = 0$. The numerical calculation and the graph has been carried out with the help of mathematical software Mat lab.

**DISCUSSION**

In this paper determined the quasi-static thermal stresses due to transient circular internal point heat source situated at certain circle along the radial direction of the circular plate and releasing its heat spontaneously. As a special case mathematical model is constructed for considering copper (pure) circular plate with the material properties specified above.

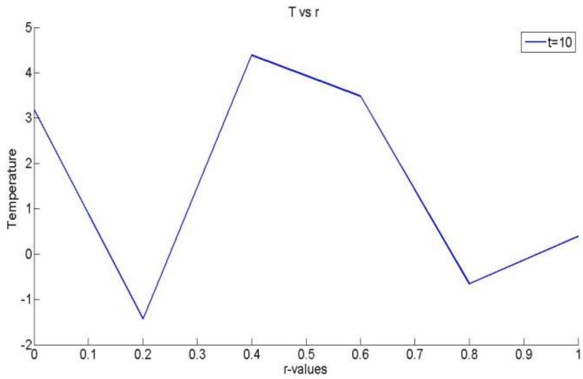


Fig. 1 Temperature distribution $T$

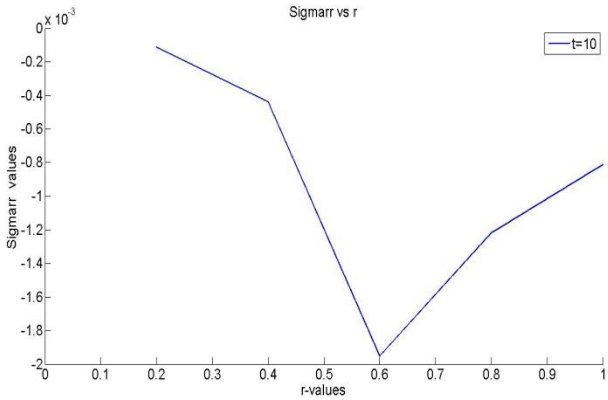


Fig. 2 Displacement $\phi$

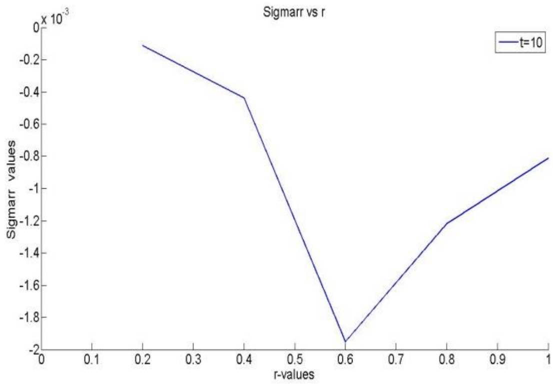


Fig. 3 Radial stress $\sigma_{rr}$

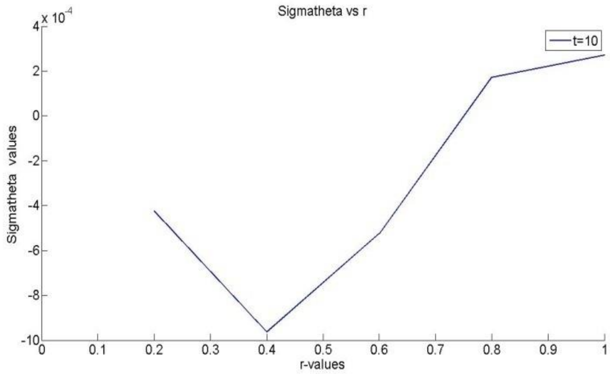


Fig. 4 Angular stress $\sigma_{\theta\theta}$

From Fig. 1, it is observed that temperature is decreasing for $0 \leq r \leq 0.2$ and $0.4 \leq r \leq 0.8$ and increasing for $0.2 \leq r \leq 0.4$, $0.8 \leq r \leq 1$. The overall behavior temperature increases along the radial direction.

From Fig. 2, it is observed that displacement is decreasing for $0.2 \leq r \leq 0.6$ and increasingfor $0.6 \leq r \leq 1$. The overall behavior of the displacement increases along the radial direction.

From Fig. 3, it is observed that the radial stress function $\sigma_{rr}$ is decreasing for $0.2 \leq r \leq 0.6$ and increasing for $0.6 \leq r \leq 1$. The overall behavior of the radial stress develops tensile stress along the radial direction.

From Fig. 4, it is observed that the angular stress function $\sigma_{\theta\theta}$ is decreasing for $0.2 \leq r \leq 0.4$ and increasing for $0.4 \leq r \leq 1$. The overall behavior of the angular stress develops tensile stress along the radial direction.

**Conclusion**

We can conclude that temperature and displacement increases along the radial direction due to transient circular internal point heat source in a circular plate and radial stresses, angular stresses develops tensile stress along the radial direction due to transient circular internal point heat source in a circular plate.

The results obtained here are useful in engineering problems particularly in the determination of state of stress in a circular plate and base of furnace of boiler of a thermal power plant and gas power plant.

**REFERENCES**

[1]. Deshmukh K. C., Quazi Y. I., Warbhe S. D., Kulkarni V. S., *Theoretical & applied Mechanics Letters,* **1**, 031007 (2011).

[2]. Nowacki W., *Bull. Acad. Polon. Sci. Scr. Scl. Tech*. **5**, 227 (1957).

[3]. Ozisik M. N., *Boundary Value Problems of Heat Conduction* (International Text Book Company, Scranton, Pennsylvania,135-148, (1968).

[4]. Roy Choudhary S. K., *Bull. Acad. Polon. Sci, Scr Scl, Tech*. **20**, (1972).

[5]. Wankhede P. C., *Indian Journal of Pure and Applied Mathematics* **13**, 1273 (1982).