Vol.10.Issue.3.2022 (July-Sept)

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

http://www.bomsr.com

Email:editorbomsr@gmail.com

INTERNATIONAL STANDARD SERIAL NUMBER 2348-0580

RESEARCH ARTICLE

Enhanced Estimation of Population Mean Utilizing known Sample Size Information

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DOI:10.33329/bomsr.10.3.4

ABSTRACT

Using the known auxiliary parameters and the sample size information, we propose a new family of estimators for the population mean of the main variable in this study. The proposed class of estimators' sampling characteristics, such as bias and Mean Squared Error (MSE), are deduced up to approximately degree one. By reducing the MSE of the introduced estimators, the optimal values of the scalars of the proposed family of estimators are achieved. For these ideal values of the constants, the MSE of the proposed estimators' minimal value is likewise determined. The proposed estimator is hypothetically compared to the previously described existing population mean estimators. The proposed estimators' efficiency requirements for being more effective than the aforementioned current estimators are also obtained. Utilizing an actual, natural population, these efficiency conditions are confirmed. When compared to other population mean estimators, it has been found that the suggested estimators have lower MSEs.

Keywords: Main Variable, Auxiliary Variable, Auxiliary Parameter, Bias, MSE.

Introduction

Instead of estimating a parameter, it is always preferable to calculate it. However, sampling is always the most effective method for obtaining information on the parameter if the population is sizable, and we estimate it using the sample data. The matching statistic is the best estimator to use when trying to estimate any parameter that is being studied, hence the best estimator to use when trying to estimate the population mean ($\overline{Y}$) of the primary variable (Y) is the sample mean ($\overline{y}$).

Despite the fact that $\bar{y}$ is an unbiased estimate of $\bar{Y}$ of Y, it has a sizable sampling variance, thus we even look for biased estimators with a smaller MSE. The purpose of searching an improved estimator of $\bar{Y}$ is fulfilled by the use of auxiliary variable X, having a high positive or negative correlation with Y. The usage of X, which has a strong association with Y, serves the objective of finding a better estimator of $\bar{Y}$.

One of the most popular and straightforward estimating techniques is the ratio approach. The usual ratio estimator was developed by Cochran (1940) using positive correlated auxiliary data. Following Cochran (1940), a number of researchers, including Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999), Singh *et al.* (2004), Al-Omari (2009), Yan and Tian (2010), Subramani and Kumarpandiyan (2012), Jeelani *et al.* (2013), and Yadav *et al.* (2019), revised the classical ratio estimator utilizing known X, including Coefficient of Variation. Ratio and product estimators of the exponential kind were advised by Bahl and Tuteja (1991). Jerajuddin and Kishun (2016) used sample size along with auxiliary parameters to enhance the efficiency of the standard ratio estimator. To improve estimation, Singh and Tailor (2003) made use of data on the correlation coefficient of Y and X that was already known. A transformed X was utilized by Upadhyaya and Singh (1999).

Gupta and Shabbir (2008), Koyuncu and Kadilar (2009), and Al-Omari *et al.* (2009) suggested innovative efficient ratio type estimators utilizing X parameters under simple random sampling (SRS) and rank set sampling (RSS) processes. Shabbir and Gupta (2011) and Singh and Solanki (2012) provided better ratio type estimators of $\bar{Y}$ under SRS and stratified random sampling approaches employing auxiliary information in quantitative and qualitative formats. In contrast, Yadav and Mishra (2015), Yadav *et al.* (2016), and Abid *et al.* (2016) proposed elevated ratio estimators of $\bar{Y}$ using known median of Y and a few customary and unusual supplementary parameters. Yadav and Kadilar (2013a, 2013b) and Sharma and Singh (2013) proposed improved ratio and product type estimators of $\bar{Y}$ using known parameters of X.

Different auxiliary information-based enhanced estimators were proposed by Yadav *et al.* (2017) and Yadav and Pandey (2017), respectively. Using well-known conventional and unconventional location parameters, Ijaz and Ali (2018), Yadav *et al.* (2018), and Zatezalo *et al.* (2018) developed improved ratio and ratio-cum-regression type estimators of $\bar{Y}$. Yadav *et al.* (2019) and Zaman (2019) used information on the usual and non-usual features of X to improve the estimation of $\bar{Y}$. While Yadav *et al.* (2021) worked on a new class of $\bar{Y}$ estimators utilising regression-cum-ratio exponential estimators, Baghel and Yadav (2020) proposed a novel estimator for enhanced $\bar{Y}$ estimation using known X parameters. With the help of data on X, Yadav *et al.* (2022) proposed an enhanced estimator for calculating average peppermint oil yields.

The goal of this study is to suggest some new estimators with higher efficiencies in comparison to other competing estimators that are being taken into consideration. We investigate the proposed estimator's large sample characteristics for a degree one approximation. The entire paper has been organised into several sections, including a review of existing estimators, a proposal for an estimator, a comparison of their efficacy, an empirical investigation, results and discussion, and a conclusion. The paper also includes a list of references at the end.

Review of Existing Estimators

For an approximation of order one, we have shown many $\bar{Y}$ estimators in this section, along with their MSEs. Let the finite population U is made up of N different and recognizable units $U_1, U_2, ..., U_N$ and the 'Simple Random Sampling Without Replacement' (SRSWOR) method is used to collect a sample of size n units from this population, assuming that Y and X has a strong correlation between them. Let (Y_i, X_i) be the observation on the i[th] unit of the population, $i = 1, 2, ..., N$. The manuscript contains the notations shown below.

N - Population Size

n - Sample Size

Y - Study variable

X - Auxiliary variable

$\bar{Y}, \bar{X}$ - Population means

$\bar{y}, \bar{x}$ - Sample means

S_y, S_x - Population Standard Deviations

S_{yx} - Population Covariance between Y and X

C_y, C_x - Coefficients of Variations

M_x - Median of X

ρ - Correlation coefficient between Y and X

β_1 - Coefficient of Skewness of X

β_2 - Coefficient of Kurtosis of X

where,

$$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i \,,\; \bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i \,,\; C_y = \frac{S_y}{\bar{Y}}, S_y^2 = \frac{1}{N-1}\sum_{i-1}^{N}(Y_i - \bar{Y})^2 \,, C_x = \frac{S_x}{\bar{X}},$$

$$S_x^2 = \frac{1}{N-1}\sum_{i-1}^{N}(X_i - \bar{X})^2 \,,\; \rho_{yx} = \frac{Cov(x,y)}{S_x S_y},\; C_{yx} = \rho_{yx} C_y C_x,\; \lambda = \frac{1}{n} - \frac{1}{N},$$

$$Cov(x,y) = \frac{1}{N-1}\sum_{i-1}^{N}(Y_i - \bar{Y})(X_i - \bar{X}),\; \beta_1 = \frac{N\sum_{i-1}^{N}(X_i - \bar{X})^3}{(N-1)(N-2)\,S_x^3},$$

$$\beta_2 = \frac{N(N+1)\sum_{i-1}^{N}(X_i - \bar{X})^4}{(N-1)(N-2)(N-3)\,S_x^4} - \frac{3(N-1)^2}{(N-2)(N-3)}$$

The associated statistic $\bar{y}$ is the most appropriate estimator for $\bar{Y}$, given by,

$$t_0 = \bar{y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$$

It is unbiased for $\bar{Y}$, and given an approximation of order one, its sampling variance is,

$$V(t_0) = \lambda \bar{Y}^2 C_y^2 \qquad (1)$$

Cochran (1940) suggested the usual ratio estimator of $\bar{Y}$, utilizing the known $\bar{X}$ as,

$$t_r = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)$$

Where, $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$ and $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} X_i$

It is a biased estimator and the MSE for the first degree approximation is,

$$MSE(t_r) = \lambda \bar{Y}^2 [C_y^2 + C_x^2 - 2C_{yx}] \qquad (2)$$

Sisodia and Dwivedi (1981) utilized the known C_x and given an estimator of $\bar{Y}$ as,

$$t_1 = \bar{y}\left(\frac{\bar{X} + C_x}{\bar{x} + C_x}\right)$$

The MSE of t_1 for an approximation of degree one is,

$$MSE(t_1) = \lambda \bar{Y}^2 [C_y^2 + \theta_1^2 C_x^2 - 2\theta_1 C_{yx}] \qquad (3)$$

Where, $\theta_1 = \frac{\bar{X}}{\bar{X} + C_x}$

Upadhyaya and Singh (1999) suggested the following estimator of $\bar{Y}$ by using the known β_2 as,

$$t_2 = \bar{y}\left(\frac{\bar{X}C_x + \beta_2}{\bar{x}C_x + \beta_2}\right)$$

The MSE of t_2 for an approximation of order one is,

$$MSE(t_2) = \lambda \bar{Y}^2 [C_y^2 + \theta_2^2 C_x^2 - 2\theta_2 C_{yx}] \qquad (4)$$

Where, $\theta_2 = \frac{\bar{X}C_x}{\bar{X}C_x + \beta_2}$

Singh and Tailor (2003) worked on improved estimation of $\bar{Y}$ using known ρ between Y and X and introduced an estimator of $\bar{Y}$ as,

$$t_3 = \bar{y}\left(\frac{\bar{X} + \rho}{\bar{x} + \rho}\right)$$

The MSE of t_3 for the first order approximation is,

$$MSE(t_3) = \lambda \bar{Y}^2 [C_y^2 + \theta_3^2 C_x^2 - 2\theta_3 C_{yx}] \tag{5}$$

Where, $\theta_3 = \frac{\bar{X}}{\bar{X} + \rho}$

Singh *et al.* (2004) utilized the known information on β_2 and proposed an enhanced estimator of $\bar{Y}$ as,

$$t_4 = \bar{y}\left(\frac{\bar{X} + \beta_2}{\bar{x} + \beta_2}\right)$$

The MSE of t_4 for an approximation of order one is,

$$MSE(t_4) = \lambda \bar{Y}^2 [C_y^2 + \theta_4^2 C_x^2 - 2\theta_4 C_{yx}] \tag{6}$$

Where, $\theta_4 = \frac{\bar{X}}{\bar{X} + \beta_2}$

Yan and Tian (2010) suggested an estimator of $\bar{Y}$ by using the known β_1 as,

$$t_5 = \bar{y}\left(\frac{\bar{X} + \beta_1}{\bar{x} + \beta_1}\right)$$

The MSE of t_5 for an approximation of order one is,

$$MSE(t_5) = \lambda \bar{Y}^2 [C_y^2 + \theta_5^2 C_x^2 - 2\theta_5 C_{yx}] \tag{7}$$

Where, $\theta_5 = \frac{\bar{X}}{\bar{X} + \beta_1}$

Subramani and Kumarpandiyan (2013) used the known M_x and given an estimator of $\bar{Y}$ as,

$$t_6 = \bar{y}\left(\frac{\bar{X} + M_x}{\bar{x} + M_x}\right)$$

The MSE of t_6 for an approximation of order one is,

$$MSE(t_6) = \lambda \bar{Y}^2 [C_y^2 + \theta_6^2 C_x^2 - 2\theta_6 C_{yx}] \tag{8}$$

Where, $\theta_6 = \frac{\bar{X}}{\bar{X} + M_x}$

Jerajuddin and Kishun (2016) utilized the known information of n, the sample size and suggested the following estimator of $\bar{Y}$ as,

$$t_7 = \bar{y}\left(\frac{\bar{X} + n}{\bar{x} + n}\right)$$

The MSE of t_7 for an approximation of order one is,

$$MSE(t_7) = \lambda \bar{Y}^2 [C_y^2 + \theta_7^2 C_x^2 - 2\theta_7 C_{yx}] \quad (9)$$

Where, $\theta_7 = \dfrac{\bar{X}}{\bar{X} + n}$

Suleiman and Adewara (2021) introduced a class of estimators of $\bar{Y}$ and given the seven members as,

$$t_{8(i)} = \bar{y}\left[\alpha_i + (1-\alpha_i)\left(\frac{a\bar{X} + bn}{a\bar{x} + bn}\right)\right],\ i = 1, 2, ..., 7$$

where, α_i are the characterizing constants to be obtained such that $MSE(t_{8(i)})$ is least and a and b are either constants or auxiliary parameters. The values of (a, b) are (1, C_x), (C_x, β_2), (1, ρ), (1, β_2), (1, β_1), (1, M_x) and (M_x, 1) respectively.

The optimum values of the constants α_i is given by,

$$\alpha_{i(opt)} = \frac{C_{yx}}{\theta_{8(i)} C_x^2}$$

where, $\theta_{8(i)} = \dfrac{a\bar{X}}{a\bar{X} + bn}$

The least MSE of $t_{8(i)}$ for the optimal value of α_i is,

$$MSE_{\min}[t_{8(i)}] = \lambda \bar{Y}^2 C_y^2 (1 - \rho^2) \quad (10)$$

Proposed Estimators

Motivated by Suleiman and Adewara (2021), we suggest the following modified family of estimators of $\bar{Y}$ as,

$$t_{p(i)} = \bar{y}\left[\alpha_1 + \alpha_2\left(\frac{a\bar{X} + bn}{a\bar{x} + bn}\right)\right],\ i = 1, 2, ..., 7$$

where, α_1 and α_2 are the characterizing scalars such that $\alpha_1 + \alpha_2 \neq 1$ and a and b are either constants or auxiliary parameters as in Suleiman and Adewara (2021). It is to be worth mentioning that, if $\alpha_1 + \alpha_2 = 1$, the introduced family reduces to Suleiman and Adewara (2021) family of estimators. Thus the Suleiman and Adewara (2021) family of estimators of $\bar{Y}$ is the special case of the introduced family.

We employ the following common approximations to examine the sample characteristics, such as bias and MSE, of the suggested estimators:

$\bar{y} = \bar{Y}(1 + e_0)$ and $\bar{x} = \bar{X}(1 + e_1)$ with $E(e_0) = E(e_1) = 0$ and $E(e_0^2) = \lambda C_y^2$ $E(e_1^2) = \lambda C_x^2$, $E(e_0 e_1) = \lambda C_{yx}$

Expressing $t_{p(i)}$ in terms of $e_i's\,(i=0,1)$, we have

$$t_{p(i)}=\bar{Y}(1+e_0)\left[\alpha_1+\alpha_2\left(\frac{a\bar{X}+bn}{a\bar{X}(1+e_1)+bn}\right)\right]$$

$$=\bar{Y}(1+e_0)\left[\alpha_1+\alpha_2\left(\frac{a\bar{X}+bn}{a\bar{X}+bn+a\bar{X}e_1}\right)\right]$$

$$=\bar{Y}(1+e_0)\left[\alpha_1+\alpha_2\left(\frac{1}{1+\frac{a\bar{X}}{a\bar{X}+bn}e_1}\right)\right]$$

$$=\bar{Y}(1+e_0)\left[\alpha_1+\alpha_2\left(\frac{1}{1+\theta_{8(i)}e_1}\right)\right],\text{ where, }\theta_{8(i)}=\frac{a\bar{X}}{a\bar{X}+bn}$$

$$=\bar{Y}(1+e_0)[\alpha_1+\alpha_2(1+\theta_{8(i)}e_1)^{-1}]$$

$$=\bar{Y}(1+e_0)[\alpha_1+\alpha_2(1-\theta_{8(i)}e_1+\theta_{8(i)}^2e_1^2)]$$

$$=\bar{Y}[\alpha_1(1+e_0)+\alpha_2(1+e_0-\theta_{8(i)}e_1-\theta_{8(i)}e_0e_1+\theta_{8(i)}^2e_1^2)]$$

When $\bar{Y}$ is subtracted from both sides of the equation above, we get,

$$t_{p(i)}-\bar{Y}=\bar{Y}[\alpha_1(1+e_0)+\alpha_2(1+e_0-\theta_{8(i)}e_1-\theta_{8(i)}e_0e_1+\theta_{8(i)}^2e_1^2)-1] \tag{11}$$

Considering both sides of the expectation in (11), we obtain the bias of $t_{p(i)}$ as,

$$B(t_{p(i)})=\bar{Y}[\alpha_1+\alpha_2(1-\theta_{8(i)}\lambda C_{yx}+\theta_{8(i)}^2\lambda C_x^2)-1] \tag{12}$$

Squaring on both sides of (11) and taking expectation, we get the MSE of $t_{p(i)}$ as,

$$MSE(t_{p(i)})=\bar{Y}^2E[\alpha_1(1+e_0)+\alpha_2(1+e_0-\theta_{8(i)}e_1-\theta_{8(i)}e_0e_1+\theta_{8(i)}^2e_1^2)-1]^2$$

$$MSE(t_{p(i)})=\bar{Y}^2E\begin{bmatrix}1+\alpha_1^2(1+2e_0+e_0^2)+\alpha_2^2(1+e_0^2+3\theta_{8(i)}^2e_1^2-4\theta_{8(i)}e_0e_1)\\-2\alpha_1(1+e_0)-2\alpha_2(1+e_0-\theta_{8(i)}e_1-\theta_{8(i)}e_0e_1+\theta_{8(i)}^2e_1^2)\\+2\alpha_1\alpha_2(1+2e_0+e_0^2-\theta_{8(i)}e_1-2\theta_{8(i)}e_0e_1+\theta_{8(i)}^2e_1^2)\end{bmatrix} \tag{13}$$

Taking into account various expectations, we have,

$$MSE(t_{p(i)})=\bar{Y}^2\begin{bmatrix}1+\alpha_1^2\lambda C_y^2+\alpha_2^2(1+\lambda C_y^2+3\theta_{8(i)}^2\lambda C_x^2-4\theta_{8(i)}\lambda C_{yx})\\-2\alpha_1-2\alpha_2(1-\theta_{8(i)}\lambda C_{yx}+\theta_{8(i)}^2\lambda C_x^2)\\+2\alpha_1\alpha_2(1+\lambda C_y^2-2\theta_{8(i)}\lambda C_{yx}+\theta_{8(i)}^2\lambda C_x^2)\end{bmatrix}$$

$$MSE(t_{p(i)})=\bar{Y}^2[1+\alpha_1^2A+\alpha_2^2B-2\alpha_1-2\alpha_2C+2\alpha_1\alpha_2D] \tag{14}$$

Where,

$A=\lambda C_y^2$

$B=(1+\lambda C_y^2+3\theta_{8(i)}^2\lambda C_x^2-4\theta_{8(i)}\lambda C_{yx})$

$C=(1-\theta_{8(i)}\lambda C_{yx}+\theta_{8(i)}^2\lambda C_x^2)$

$D=(1+\lambda C_y^2-2\theta_{8(i)}\lambda C_{yx}+\theta_{8(i)}^2\lambda C_x^2)$

The optimum values of α_1 and α_2, which minimizes the MSE of $t_{p(i)}$ are respectively given as,

$\alpha_{1(opt)}=\dfrac{DC-B}{D^2-AB}$ and $\alpha_{2(opt)}=\dfrac{D-AC}{D^2-AB}$

The least value of $MSE(t_{p(i)})$ for the optimal values of α_1 and α_2 is,

$$MSE(t_{p(i)})=\bar{Y}^2\left[1-\frac{\begin{Bmatrix}C(D-AC)(D^2-AB)+2(DC-B)(D^2-AB)\\-2(DC-B)(D-AC)-A(DC-B)^2-B(D-AC)^2\end{Bmatrix}}{(D^2-AB)^2}\right]$$

$$MSE(t_{p(i)})=\bar{Y}^2\left[1-\frac{P}{Q^2}\right] \tag{15}$$

Where,

$$P=\begin{Bmatrix}C(D-AC)(D^2-AB)+2(DC-B)(D^2-AB)\\-2(DC-B)(D-AC)-A(DC-B)^2-B(D-AC)^2\end{Bmatrix}$$

$Q=(D^2-AB)$

Theoretical Efficiency Comparison

In this section, we have made a theoretical comparison between the efficiency of the recommended estimator and the previously described existing $\bar{Y}$ estimators. We have also identified the conditions in which the suggested estimator outperforms the completion estimate.

When the following conditions are met, the suggested estimator $t_{p(i)}$ is more effective than the estimator t_0.

$V(t_0)-MSE_{\min}(t_{p(i)})>0$, or

$\lambda C_y^2+\dfrac{P}{Q^2}>1$

The introduced estimator $t_{p(i)}$ performs better than the estimator t_r of Cochran (1940) for the following condition if,

$MSE(t_r)-MSE_{\min}(t_{p(i)})>0$, or

$$\lambda[C_y^2 + C_x^2 - 2C_{yx}] + \frac{P}{Q^2} > 1$$

The suggested estimator $t_{p(i)}$ has lesser MSE than the estimator t_1 of Sisodia and Dwivedi (1981) for the following condition if,

$MSE(t_1) - MSE_{\min}(t_{p(i)}) > 0$, or

$$\lambda[C_y^2 + \theta_1^2\, C_x^2 - 2\theta_1\, C_{yx}] + \frac{P}{Q^2} > 1$$

The estimator $t_{p(i)}$ outperforms the estimator t_2 developed by Upadhyaya and Singh (1999) under the condition if,

$MSE(t_2) - MSE_{\min}(t_{p(i)}) > 0$, or

$$\lambda[C_y^2 + \theta_2^2\, C_x^2 - 2\theta_2\, C_{yx}] + \frac{P}{Q^2} > 1$$

The introduced estimator $t_{p(i)}$ is more effective in comparison to t_3 of Singh and Tailor (2003) if the following criteria are met,

$MSE(t_3) - MSE_{\min}(t_{p(i)}) > 0$, or

$$\lambda[C_y^2 + \theta_3^2\, C_x^2 - 2\theta_3\, C_{yx}] + \frac{P}{Q^2} > 1$$

The estimator $t_{p(i)}$ is more efficient than t_4 of Singh *et al.* (2004) for the criteria if,

$MSE(t_4) - MSE_{\min}(t_{p(i)}) > 0$, or

$$\lambda[C_y^2 + \theta_4^2\, C_x^2 - 2\theta_4\, C_{yx}] + \frac{P}{Q^2} > 1$$

When compared to Yan and Tian (2010) estimator t_5, the recommended estimator $t_{p(i)}$ is more effective for the condition if,

$MSE(t_5) - MSE_{\min}(t_{p(i)}) > 0$, or

$$\lambda[C_y^2 + \theta_5^2\, C_x^2 - 2\theta_5\, C_{yx}] + \frac{P}{Q^2} > 1$$

The introduced estimator $t_{p(i)}$ has lesser MSE than t_6 of Subramani and Kumarpandiyan (2013) under the condition if,

$MSE(t_6) - MSE_{\min}(t_{p(i)}) > 0$, or

$$\lambda[C_y^2 + \theta_6^2 C_x^2 - 2\theta_6 C_{yx}] + \frac{P}{Q^2} > 1$$

The suggested estimator $t_{p(i)}$ has lesser MSE than the estimator t_7 of Jerajuddin and Kishun (2016) under the following condition if,

$MSE(t_7) - MSE_{\min}(t_{p(i)}) > 0$, or

$$\lambda[C_y^2 + \theta_7^2 C_x^2 - 2\theta_7 C_{yx}] + \frac{P}{Q^2} > 1$$

The estimator $t_{p(i)}$ is better than the family of estimators t_7 of Suleiman and Adewara (2021) under the condition if,

$MSE_{\min}(t_{8(i)}) - MSE_{\min}(t_{p(i)}) > 0$, or

$$\lambda C_y^2(1-\rho^2) + \frac{P}{Q^2} > 1$$

Numerical Study

We have taken into consideration a real natural population from Murthy in order to assess the performances of the suggested and competing estimates of $\bar{Y}$ and to confirm the efficiency criteria of the provided estimator over the indicated existing estimators (1967). Following are the main and auxiliary variables of the considered population under consideration:

Y : Output for 80 factories in a region

X : Number of workers

The parameters of the population under consideration are presented in Table-1.

Table-1: Parameters of the considered population

Parameter	Value	Parameter	Value
N	80	C_y	0.3542
n	20	C_x	0.7505
$\bar{Y}$	51.8264	β_1	1.0500
$\bar{X}$	11.2646	β_2	-0.0634
ρ	0.9413	M_x	7.5750

Table 2 shows the MSEs of $t_{p(i)}$, the mentioned estimators stated above, and the percentage relative efficiency (PRE) of various estimators with respect to t_0.

Table-2: MSE of different estimators and PRE with respect to t_0

Estimator	MSE	PRE	Estimator	MSE	PRE
t_0	12.63661	100.0000	$t_{8(i)}$	1.43999	**877.5485**
t_r	18.97931	66.58098	$t_{p(1)}$	**1.38571**	**911.9231**
t_1	15.25812	82.81892	$t_{p(2)}$	**1.38584**	**911.8376**
t_2	19.45925	64.93883	$t_{p(3)}$	**1.38556**	**912.0219**
t_3	14.45027	87.44895	$t_{p(4)}$	**1.38867**	**909.9793**
t_4	19.33831	65.34496	$t_{p(5)}$	**1.38763**	**910.6613**
t_5	14.01128	90.18883	$t_{p(6)}$	**1.38882**	**909.8811**
t_6	2.782544	454.1387	$t_{p(7)}$	**1.38904**	**909.7369**
t_7	1.838908	687.1801			

The results in Table-2 are also presented in the form of graph in Figure-1.

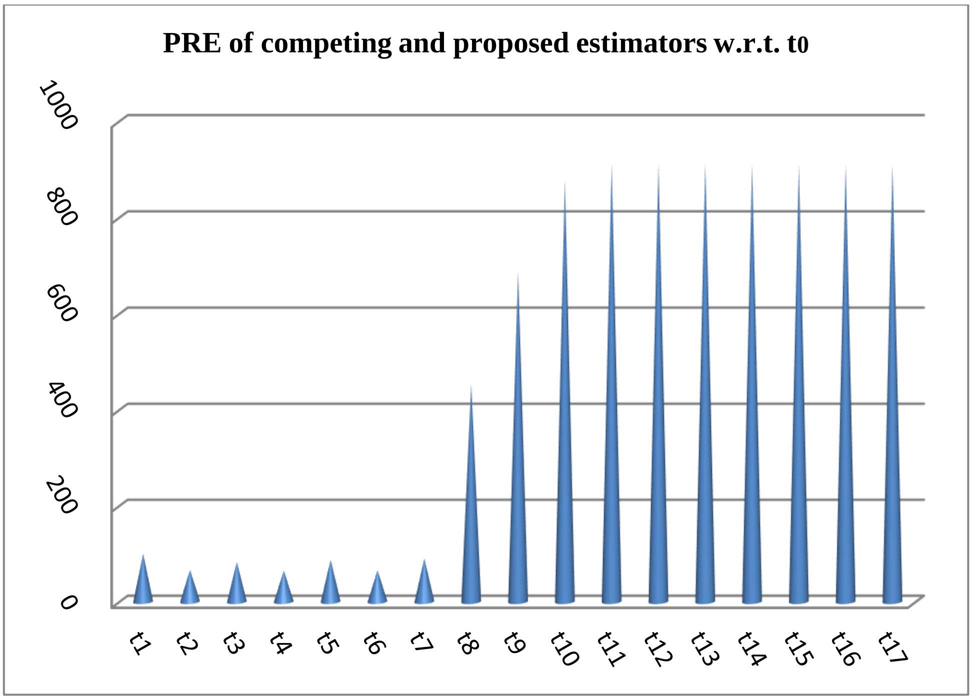

Figure-1: PRE of competing and suggested estimators with respect to t_0

Result and Conclusion

Using the known auxiliary parameters and the sample size information, we proposed a novel family of estimators in this study for improved $\bar{Y}$ estimation. Up to approximation of order one, the bias and MSE of the introduced estimators are investigated. The efficiency criteria of the proposed estimator over the rival estimators are determined by theoretical comparison of the recommended estimator with the previously described existing $\overline{Y}$ estimators. An actual natural population from Murthy (1967) is used to verify these efficiency conditions. It is clear from Table 2 that the competing estimators MSEs fall within the range [1.43999, 12.63661] while the MSEs of the introduced class of estimators ranges in [**1.38556, 1.38904**] and the PREs of the estimators in competition with respect to t_0 lie in the interval [64.93883, 877.5485] while the PREs of introduced estimators with respect to t_0 lie in the interval [**909.7369, 912.0219**]. Thus it is clear that $t_{p(i)}$ estimators of $\overline{Y}$ is the most efficient class of estimators as it has the lesser MSE and the highest PRE in comparison to the estimators of $\overline{Y}$ in competition. Therefore the introduced family of estimators is recommended for the use in different areas of applications.

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