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RESEARCH ARTICLE

A CHAIN PRODUCT-TYPE EXPONENTIAL ESTIMATOR FOR POPULATION MEAN IN DOUBLE SAMPLING

N Sahoo*[1], S K Jhankar[2]

[1,2]School of Statistics, Gangadhar Meher University, Amruta Vihar, Sambalpur, Odisha, India.

*Email: nirustatistics@gmail.com

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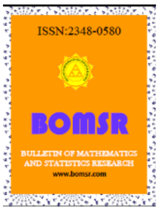


ABSTRACT

This paper deals with the estimation of a finite population mean in two-phase sampling in the presence of two auxiliary variables. An efficient product-type exponential estimator is proposed and the properties of the proposed estimation procedure have been examined up to first order of approximation. The proposed estimator is more reliable as compared to the other consisting existing estimators for different parameter values.

Keywords: Two-phase Sampling, Auxiliary variables, Study variable, Bias, Mean Square Error, Percent Relative Efficiency.

1. Introduction

Consider a finite population $U = (U_1, U_2, \dots\dots\dots\dots\dots.. U_N)$ of N units. Let $\bar{X}, \bar{Y}$ and $\bar{Z}$ denote the population means, C_x, C_y and C_z denote the coefficients of variation, ρ_{yx}, ρ_{yz} and ρ_{xz} denote the correlation coefficients. Let Y be the study variable and X and Z be the auxiliary variables with corresponding values y_i, x_i and z_i $(i = 1,2, \dots\dots\dots N)$. The problem is to estimate $\bar{Y}$ in the presence of two auxiliary variables x and z.

Let $S_y^2 = \sum_{i=1}^{n}(y_i - \bar{Y})^2/(N-1)$ and $S_x^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{X})^2}{(N-1)}$, $S_z^2 = \sum_{i=1}^{n}(z_i - \bar{Z})^2/(N-1)$ and Let $C_y = S_y/\bar{Y}$, $C_x = S_x/\bar{X}$ and $C_z = S_z/\bar{Z}$ be the coefficients of variation of y, x and z respectively. $f_1 = \left(\frac{(1-f)}{n}\right)$, $f_2 = \left(\frac{(1-f')}{n'}\right)$, $f_3 = \left(\frac{(1-f'')}{n}\right)$,

where $f = \frac{n}{N}$, $f' = \frac{n'}{N}$ and $f'' = \frac{n}{n'}$

$$v(\bar{y}) = f_1 \bar{Y}^2 C_y^2$$

2. Traditional Estimators & It's MSE

Suppose the two variables y and x is negatively (high) correlated. For estimating the population mean $\bar{Y}$ of the study variable y, the double sampling product estimator is defined by

$$t_1 = \bar{y}\left(\frac{\bar{x}}{\bar{x}'}\right) \tag{1}$$

The mean square error (MSE) of the double sampling product estimator to the first degree of approximation is given by

$$MSE(t_1) = \left[f_1 S_y^2 + f_3 R S_x^2 (R + 2\beta_{xy})\right] \tag{2}$$

$$t_2 = \bar{y} + b_{yx}(n)(\bar{x}' - \bar{x}) \tag{3}$$

$$MSE(t_2) = \bar{Y}^2 C_y^2 \left[f_1\left(1 - \rho_{yx}^2\right) + f_2 \rho_{yx}^2\right] \tag{4}$$

Singh and Vishwakarma (2007) suggested exponential product type estimator for $\bar{Y}$ as

$$t_3 = \bar{y} exp\left(\frac{\bar{x} - \bar{x}'}{\bar{x} + \bar{x}'}\right) \tag{5}$$

The MSE of the estimator t_3 is

$$MSE(t_3) = \bar{Y}^2\left[f_1 C_y^2 + \frac{f_3}{4}\left(C_x^2 + 4\rho_{yx} C_y C_x\right)\right] \tag{6}$$

The usual chain-type ratio and product estimators of Y under double sampling scheme using two auxiliary variables X and Z are given, respectively, by

$$t_4 = \bar{y}\left(\frac{\bar{x}}{\bar{x}'}\right)\left(\frac{\bar{z}'}{\bar{Z}}\right) \tag{7}$$

$$MSE(t_4) = \bar{Y}^2\left[f_1 C_y^2 + f_3 C_x^2 + f_2 C_z^2 + 2 f_3 \rho_{yx} C_y C_x + 2 f_2 \rho_{yz} C_y C_z\right] \tag{8}$$

Singh and Choudhury suggested the following exponential chain type product estimators of Y under double sampling scheme using two auxiliary variables X and Z:

$$t_5 = \bar{y} exp\left\{\frac{\bar{x} - \left(\frac{\bar{x}'}{\bar{z}'}\right)\bar{Z}}{\bar{x} + \left(\frac{\bar{x}'}{\bar{z}'}\right)\bar{Z}}\right\} \tag{9}$$

$$MSE(t_5) = \bar{Y}^2\left[f_1 C_y^2 + \frac{1}{4}\{f_3 C_x^2 + f_2 C_z^2\} + \{f_3 \rho_{yx} C_y C_x + f_2 \rho_{yz} C_y C_z\}\right] \tag{10}$$

3. Proposed Estimator:

We have proposed a chain product-type exponential estimator for finite population mean of the study variable Y, given as

$$t_{Pe} = \bar{y}\exp\left[\psi_1\left\{\frac{\bar{x} - \bar{x}'}{\bar{x} + \bar{x}'}\right\} + \psi_2\left\{\frac{\bar{Z} - \bar{z}'}{\bar{Z} + \bar{z}'}\right\}\right], \tag{11}$$

where ψ_1 and ψ_2 are unknown constants, whose value is to be determined for optimality conditions.

To derive the expression of Mean square Error of different estimators, let us assume that

$\bar{y} = \bar{Y}(1 + e_0)$, $\bar{x} = \bar{X}(1 + e_1)$, $\bar{x}' = \bar{X}(1 + e_2)$, $\bar{z}' = \bar{Z}(1 + e_3)$

Such that

$E(e_0) = E(e_1) = E(e_2) = E(e_3) = 0$

$E(e_0^2)=\left(\frac{1-f}{n}\right)C_y^2$, $E(e_1^2)=\left(\frac{1-f}{n}\right)C_x^2$

$E(e_2^2)=\left(\frac{1-f'}{n'}\right)C_x^2$, $E(e_3^2)=\left(\frac{1-f'}{n'}\right)C_z^2$

$E(e_0e_1)=\left(\frac{1-f}{n}\right)\rho_{yx}C_yC_x$, $E(e_0e_2)=\left(\frac{1-f'}{n'}\right)\rho_{yx}C_yC_x$

$E(e_0e_3)=\left(\frac{1-f'}{n'}\right)\rho_{yz}C_yC_z$, $E(e_1e_2)=\left(\frac{1-f'}{n'}\right)C_x^2$

$E(e_1e_3)=\left(\frac{1-f'}{n'}\right)\rho_{xz}C_xC_z$, $E(e_2e_3)=\left(\frac{1-f'}{n'}\right)\rho_{xz}C_xC_z$

The mean square error of the proposed estimator t_{Pe} is as follows

$$t_{Pe}=\bar{y}exp\left[\psi_1\left\{(e_1-e_2)(2+e_1+e_2)^{-1}\right\}+\psi_2\{-e_3(2+e_3)^{-1}\}\right]$$

$$=\bar{Y}(1+e_0)\left[1+\psi_1\frac{e_1}{2}-\psi_1\frac{e_2}{2}-\psi_1\frac{e_1^2}{4}+\psi_1\frac{e_2^2}{4}-\psi_2\frac{e_3}{2}+\psi_2\frac{e_3^2}{4}+\psi_1^2\frac{e_1^2}{8}+\psi_1^2\frac{e_2^2}{8}+\psi_2^2\frac{e_3^2}{8}\right]$$

Now

$$B(t_{Pe})=E[t_{Pe}-\bar{Y}]$$

$$=\bar{Y}\left[-\frac{\psi_1}{4}\left(\frac{1-f''}{n}\right)C_x^2+\frac{\psi_2}{4}\left(\frac{1-f'}{n'}\right)C_z^2+\frac{\psi_1^2}{8}\left\{\left(\frac{1-f}{n}\right)+\left(\frac{1-f'}{n'}\right)\right\}C_x^2+\frac{\psi_2^2}{8}\left(\frac{1-f'}{n'}\right)C_z^2+\frac{\psi_1}{2}\left(\frac{1-f''}{n}\right)\rho_{yx}C_yC_x-\frac{\psi_2}{2}\left(\frac{1-f'}{n'}\right)\rho_{yz}C_yC_z\right] \quad (12)$$

$$MSE(t_{Pe})=E[t_{Pe}-\bar{Y}]^2$$

$$=\bar{Y}^2\left[\frac{\psi_1^2}{4}f_3C_x^2+\frac{\psi_2^2}{4}f_3C_z^2+f_1C_y^2+\psi_1f_3\rho_{yx}C_yC_x-\psi_2f_2\rho_{yz}C_yC_z\right] \quad (13)$$

The optimum value of ψ_1 and ψ_2 are found out to be

$$\psi_{1(opt)}=-2\rho_{yx}\left(\frac{C_y}{C_x}\right)$$

$$\psi_{2(opt)}=2\left(\frac{f_2}{f_3}\right)\rho_{yz}\left(\frac{C_y}{C_z}\right)$$

So, the minimum Mean Square Error of the proposed estimator is given as

$$MSE(t_{Pe})_{opt}=\bar{Y}^2C_y^2\left[f_1-\rho_{yx}^2f_3-\frac{f_2^2}{f_3}\rho_{yz}^2\right] \quad (14)$$

4. **Comparative Study**

We get the following efficiency conditions on comparing the MSE of proposed estimator t_{Pe} with the mean square error of the existing predictive estimators $t_i, i=1,2,3,4,5$

The proposed estimator will be more efficient if $MSE(t_i)-MSE(t_{Pe})_{opt}>0$

(i) $\left[f_1S_y^2+f_3RS_x^2(R+2\beta_{xy})\right]>\bar{Y}^2C_y^2\left[f_1-\rho_{yx}^2f_3-\frac{f_2^2}{f_3}\rho_{yz}^2\right]$

(ii) $\left[f_1(1-\rho_{yx}^2)+f_2\rho_{yx}^2\right]>C_y^2\left[f_1-\rho_{yx}^2f_3-\frac{f_2^2}{f_3}\rho_{yz}^2\right]$

(iii) $\left[\frac{f_3}{4}\left(C_x^2+4\rho_{yx}C_yC_x\right)\right]>-C_y^2\left[\rho_{yx}^2f_3+\frac{f_2^2}{f_3}\rho_{yz}^2\right]$

(iv) $f_3C_x^2+f_2C_z^2+2f_3\rho_{yx}C_yC_x+2f_2\rho_{yz}C_yC_z>-C_y^2\left[\rho_{yx}^2f_3+\frac{f_2^2}{f_3}\rho_{yz}^2\right]$

(v) $\frac{1}{4}\{f_3C_x^2+f_2C_z^2\}+\{f_3\rho_{yx}C_yC_x+f_2\rho_{yz}C_yC_z\}>-C_y^2\left[\rho_{yx}^2f_3+\frac{f_2^2}{f_3}\rho_{yz}^2\right]$

5. Numerical Study

Using above conditions, we have performed a numerical study utilizing one population data which is enlisted below.

Population [Source: Singh (1967)]

The variables are

y: Output

x: Number of workers

z: Fixed capital

$N=80,\quad n'=25,\quad n=10,\quad \bar{Y}=5182.638$

$\bar{X}=283.875,\quad \bar{Z}=1126.00,\quad C_y=0.3520,\quad C_x=0.9430$

$C_z=0.7460$, $\rho_{yx}=0.9136$, $\rho_{yz}=0.9413$ and $\rho_{xz}=0.9859$

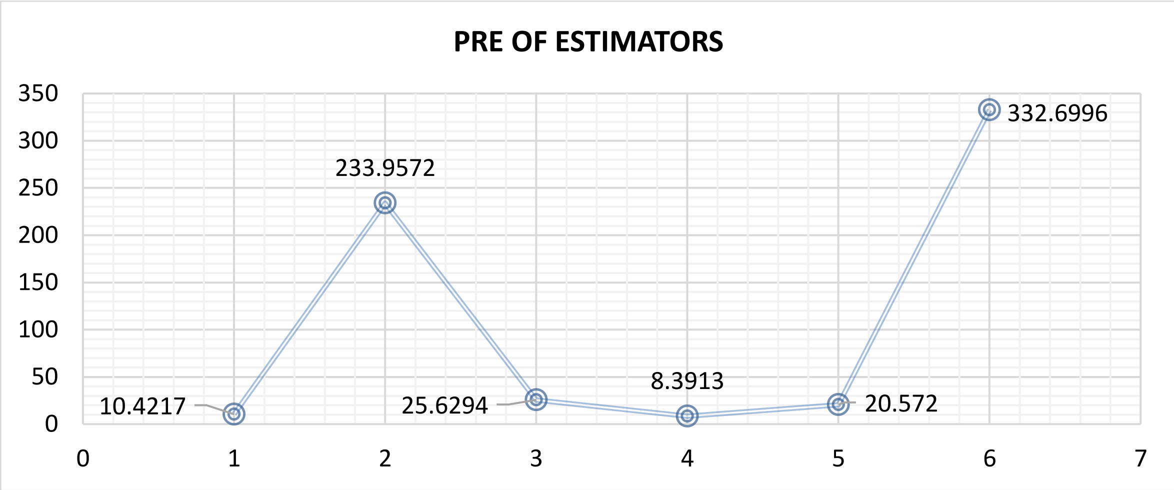
The percent relative efficiency both for proposed estimator as well as existing estimators have been calculated by using the following formula

$$PRE(t_i)=\left[\frac{var(\bar{y})}{MSE(t_i)}\right]\times 100\ ; \qquad (15)$$

TABLE-1

ESTIMATORS	PRE
t_1	10.4217
t_2	233.9572
t_3	25.6294
t_4	8.3913
t_5	20.572
t_{Pe}	**332.6996**

The following line graph represents the percent relative efficiency of proposed estimator as well as traditional estimators.



Conclusion

From the above table-1, it is clear that the proposed estimator has maximum percent relative efficiency that is 332.6996 which is more as compared to the other existing estimators. Based on our theoretical justification, the proposed estimator can be preferred over the other estimators in real practice.

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