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RESEARCH ARTICLE

A REFERENCE CASE FOR MEAN FIELD GAME AND STOCHASTIC DIFFERENTIAL GAME THEORY MODELS

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ABSTRACT

This research paper is going to give a gentle introduction to mean field Games.

It aims to produce a coherent text beginning for simple notions of deterministic control theory progressively to current mean field games theory. The framework gradually extended form single agent stochastic control problems to multi agent stochastic differential mean field games. The concept of Nash equilibrium is introduced to define a solution of the mean field game. To achieve considerable simplifications the numbers of agents goes to infinity and formulate the problem on the basis of Mclean Viasov theory for interacting particles systems. Furthermore, the problem at infinity is being solved by a variation of the stochastic maximum principle. To elaborate more the model in continuous time is presented using MFGS.

Keywords: Stochastic, differential, mean field, games, Nash equilibrium concepts.

INTRODUCTION

Mean field game theory is defined a new branch of game theory mean field game theory is devoted to the analysis of differential games with infinitely many players. For such large population dynamic games, it is unrealistic for a player to collect detailed state information about all other players. There are several approaches to the analysis of differential games with an infinite number of agents. A first one is to look at Nash equilibria in differential games with a large number of players and try to pass to the Nash equilibria in different games with second approach consist in guessing the equations that Nash equilibria of differential games. With infinitely many players should salisty and to show that resolving solutions of their equations allow solving differential games with finitely many players.

To complete the discussion on the master equation, let us finally underline that, beside the MFG system, another possible and natural simplification of this equation is a space discretization, which yields to a more standard transport equation in finite space dimension.

By nature mean field games are related with probability and partial differential equations. Both communities have a different approach of the topic, mostly inspired by the works of caines Huang and Mallam for the probability part and by Lasry and lions for the PDE one.

HAMILTON-JACOBI EQUATIONS:

This sections is an edited and expanded version of Jim Nolens notes. We first recall some basic facts about the solutions of the initial (or) rather, terminal value problem for the Hamilton-jacobi equations of the form

$$u_s + IC\nabla u, y = 0$$

$$u\,(s, y) = u_{oy}$$

In order to explain low such problems come about and to understand why the mean –field games with their coupling of forward and backward equations are natural, we need to recall some basic notions from the control theory.

Dynamic programming principle for stochastic control

For the stochastic control problem there is a dynamic programming principle that is analogous to the LPP for deterministic control. Using the market property of the stochastic process xs, one can easily prove the following.

Theorem:

Let u (y,s) be the value function defined by

U (y, s) = max $\beta \varepsilon B\mathfrak{s}, S\mathfrak{f}[\int_s^S \epsilon(Xt, \beta t, t)dt$

$+\frac{\mathcal{F}(Xs)}{Xs} = y]$ If s < s ≤ s

Then

$u\,(y, s)$= max

$\beta t B s, s\ F\ [\int_s^S \epsilon\ \ (Xt1\ \beta t\ t\ )\ dt$

+ u (Xs, S) / Xs =y]

PROOF

The idea is the same as in the case of deterministic control. Split the integral into two pieces. One over (s, S) and the other over [s, S]. Then condition on E_s and use the marrow property 1 so that the second integral and payoff may be expressed in terms of u (Xs, S). Nash equilibtria in classical differential

GAMES:

Let us return for the sake of simplicity to our first model. It is now time to come back to the contribution hypothesis by considering the game with N players. To simply the account, and because it involves a “toy model”, we look at the same particular can as atom (which is rather technical since the criterion is not regular but is very graphic) in which the meeting begins once proportion θ (we shall assume $\theta = 90\%$ for the sake of simplicity) of the participants have arrivel (but still we fare T to be between times S and S max) in addition let us suppose that all the agents have the same ∞. Various questions then naturally arise:

Does the N-player game have Nash?

Equilibria?

It’s there uniqueness of such equilibria?

Do N player equilibria tend towards the mean field games equilibrium when N→∞?

If need be is the rate of convergence known?

This case is simple, but it allows since we shall answer the above questions is the affirmative (in the symmetrical case) – to pause the way for an approximation of an N-player game by MPG.

This example of approximations of a N-player game through a first order expansion "$Ho + M^{+} H\ 1\ x \ldots\ldots\ldots\ldots\ldots.$" Where (formally Ho is the mean field game and H1 the first order correction coefficient leads to a new type of solution of a N-player Nash equilibrium. The solution of “Ho+$\frac{\pm}{N} Hi$ reflects a strategic world in which agents do not care about other agents individually at least but only about the population dynamics and world in which N1 the number of players. Is only entered to fake into account the granularity of the game and the imperfectness of the continuum hypothesis.

STOCHASTIC DIFFERENTIAL MEAN FIELD GAMES:

This chapter describes in details the n-player stochastic differential games under consideration as well as the associated mean field game solution concepts. We take now an approach opposite to that of the previous chapter I in the sense that we start immediately with the most general solution with common noise, before successively refining and specializing the results. The equilibrium concepts for mean field games developed in this chapter are admittedly cumbersome. But they are central to this thesis so we take him to through explain all of the moving parts.

MFG SOLUTION CONCEPTS:

The basic inputs to the model are the following data. We are given a time horizon S >o, three exponents (q,q,qƐ) with q≥1 a control space, an initial state distribution ∞ƐG(R^d) and the following functions.

$$a, \gamma, \gamma o, h) : (0, s) \times Rd\ X\ Gq\ (Rd)$$

$$\times\ \beta \to Rd \times Rd \times n \times Rd \times n0 \times R1$$

$$\int : Rd \times \emptyset q(R1) \to R.$$

The state idiosyncratic noises and common noise are of dimension d,n, no reply.

Assumptions:

1. The control space B is a closed cover subset of a Euclidean space. (more generally, as in a closed complex Ɛ-compact subset of a breach space would suffice.
2. The exponents satisfy q >q≥1vqƐ and qƐ(0,1) more over quence ðƐ q ^{q1} (Rd).
3. The function a, ʎ, ʎ0, h, and l q (s,y, b, b)are jointly measurable and are contribution in (y, u,b) for each s.

PROPERTIES OF MEAN FIELD SOLUTIONS:

As a first step, this chapter is devoted to the derivation of several useful structure properties of MFG solutions. The first section defines and discuses MFG pre solutions.

- n agents of Players
- each agents chooses an action

From a given set B called the action space

Amume B is a compact matrix space

- A strategy profile is a sector [b, bn] ∈ B^a
- The goal of agent I is to choose biEB so as to maximize the reward ꝺi^n

A REALLY SIMPLE EXAMPLE:

- Simple example not new but gives an idea of the general class of models (older simple exs later on)

  E metrix space, N player (1 ≤Ŝ≤N)

  Choose a position XiƐE according to a criterion Fi (x) where x=(x1, x2……xn) E E^n

GENERAL STRUCTURE:

Particular case dynamical problem horizon T continuous time and space, Brownian noises (both incle and common) no inter temporal preference rate control on drifts (Hamiltonian H), criterion dep only on m

U (x,m,t) (X E R^d, mt P(R^d) or

M_f (R^d), tƐ (0, T) and

H (x, p,m) cortex in PƐR^d

MFG master equation

$$\frac{\propto u}{\propto t} - (u{+}\propto)\Delta \times U + H, \nabla x\, U, M) +$$

$$< \frac{\propto u}{\propto m} - ({+}u{+}\propto)sm + div\,(\frac{\propto{+}{+}}{\propto p} m)\} +$$

$$-\propto \frac{\propto u}{\propto m2}\,(\nabla m, \nabla m) + 2 \propto< \frac{\propto}{\propto m}\,\nabla \times U, \nabla m) = 0$$

And U/t=0 =Uo (X,M) final cost.

U amount of ind. Rand.

Reduced mean field models

In this section we consider reduced mean field models. The models originally studied by raspy and sections. Which consider in a system of Hamilton Jocobi type equation and an associated transport or Fokker planck equation. We present the derivation of such models and discus various methods to prove existence and uniqueness of solutions. Stationary models are then briefly discussed. These are quite interesting in their own right but also under appropriate conditions, encode the long time asymptotic for mean field games, as shown in (GHS 10). We consider also stationary extended models in which the cost for a reference player distribution but also on their actions. Then we look at certain variation structures that some of these problems enjoy, and they connections between mean field models and other row classical problems such as optimal transport and Aubry –matter theory. In this part we discuss only determination control problems. This allows us to using backward statistic differential equations and therefore keeping the presentation elementary. Mean field models with correlations will be considered.

Conclusion

In this paper, we have considered a statistic differential equation whose payoff elements are Nash equilibrium concepts. We have considered a Nash equilibrium solution. We have considered a uniqueness and equistence statements.

The model we presented in this paper stochastic differential game with discontinuous feedback. We are the value of Nash equiliprium feedback in terms of Hamilton Jacobi theory to reduce the existence of a regular solution to a system of non-linear parabolic equations have been studied. Nash equilibrium post for nonzero sum even for N>∞ mar players game analyzed.

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