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## Characterization of Contra Continuous Functions

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### ABSTRACT

In this paper, weaker forms of continuous functions called contra  $\alpha g^*s$ -continuous functions are introduced. Also, a new form of contra continuity called almost contra  $\alpha g^*s$ -continuous functions which are weaker than contra  $\alpha g^*s$ -continuity is introduced and studied their basic properties relating to them.

**Keywords and Phrases:**  $\alpha g^*s$ -closed sets,  $\alpha g^*s$ -continuous functions, contra  $\alpha g^*s$ -continuous functions.

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### 1. Introduction

Dontchev [4] introduced a new class of functions called contra-continuous functions in topological spaces. New weaker forms of functions called contra-semi continuous functions were introduced and investigated by Dontchev and Noiri [5]. Jafari and Noiri [8], [9] introduced other new weaker forms of this class of functions called contra- $\alpha$ -continuous and contra-pre continuous functions. Also, Contra super continuity is a continuation of research done by Dontchev [4] and Jafari and Noiri [7].

In this paper, we introduce and investigate the properties of new class of functions called contra  $\alpha g^*s$ -continuous functions as a generalization of contra continuity. Also, almost contra  $\alpha g^*s$ -continuous functions are introduced and obtained some of their properties in topological spaces.

## 2. Preliminaries

Throughout this paper  $(X, \tau)$ ,  $(Y, \mu)$  and  $(Z, \sigma)$  (or simply  $X$ ,  $Y$  and  $Z$ ) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. For a subset  $A$  of a space  $X$ , the closure and interior of  $A$  with respect to  $\tau$  are denoted by  $\text{cl}(A)$  and  $\text{int}(A)$  respectively.

**Definition 2.1.** [10] A subset  $A$  of  $X$  is  $\alpha g^*s$ -closed if  $\alpha \text{cl}(A) \subset U$  whenever  $A \subset U$  and  $U$  is  $g$ s-open in  $X$ .

**Definition 2.2.** [10] The intersection of all  $\alpha g^*s$ -closed sets containing a set  $A$  is called  $\alpha g^*s$ -closure of  $A$  and is denoted by  $\alpha g^*s\text{-cl}(A)$ .

A set  $A$  is  $\alpha g^*s$ -closed set if and only if  $\alpha g^*s\text{-cl}(A) = A$ .

**Definition 2.3.** [10] The union of all  $\alpha g^*s$ -open sets contained in  $A$  is called  $\alpha g^*s$ -interior of  $A$  and is denoted by  $\alpha g^*s\text{-int}(A)$ .

A set  $A$  is  $\alpha g^*s$ -open if and only if  $\alpha g^*s\text{-int}(A) = A$ .

**Definition 2.4.** In a space  $X$ ,

(i) If every  $\alpha g^*s$ -closed set is closed in  $X$ , then  $X$  is called  $T_{\alpha g^*s}$  space [10].

**Definition 2.5.** A function  $f : X \rightarrow Y$  is called a

- (i)  $\alpha g^*s$ -continuous [10], if the inverse image of every closed set in  $Y$  is  $\alpha g^*s$ -closed in  $X$ .
- (ii) pre  $\alpha g^*s$ -continuous [10], the image of every  $\alpha g^*s$ -closed set in  $X$  is  $\alpha g^*s$ -closed in  $Y$ .
- (iii)  $\alpha g^*s$ -irresolute [10], if the inverse image of every  $\alpha g^*s$ -closed set in  $Y$  is  $\alpha g^*s$ -closed in  $X$ .
- (iv) contra continuous [4], for every open set  $V$  in  $Y$ ,  $f^{-1}(V)$  is closed in  $X$ .

## 3. Contra $\alpha g^*s$ -continuous functions

In this section, a new class of functions called contra  $\alpha g^*s$ -continuous functions is introduced and obtains some of their properties and relationships with some other related functions are discussed.

**Definition 3.1.** A function  $f : X \rightarrow Y$  is said to be contra  $\alpha g^*s$ -continuous if

$f^{-1}(V) \in \text{sg}\omega\alpha\text{-C}(X)$  for every  $V \in \mathcal{O}(Y)$ .

**Remark 3.1.** We can observe that contra  $\alpha g^*s$ -continuous and  $\alpha g^*s$ -continuous are independent of each other.

**Example 3.1.** Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$  and  $\mu = \{Y, \phi, \{a, b\}\}$  be topologies on  $X$  and  $Y$  respectively. Let  $f : X \rightarrow Y$  be the identity function. Then  $f$  is  $\alpha g^*s$ -continuous function but not contra  $\alpha g^*s$ -continuous, since for the open set  $\{a, b\}$  in  $Y$ ,  $f^{-1}(\{a, b\}) = \{a, b\}$  is not  $\alpha g^*s$ -closed in  $X$ .

**Example 3.2.** Let  $X = Y = \{a, b, c\}$ ,  $\tau = \{X, \phi, \{c\}\}$  and  $\mu = \{Y, \phi, \{a, b\}\}$  be topologies on  $X$  and  $Y$  respectively. Define the identity function  $f : X \rightarrow Y$ . Then  $f$  is contra  $\alpha g^*s$ -continuous but not  $\alpha g^*s$ -continuous, for the open set  $\{a, b\}$  in  $Y$ ,  $f^{-1}(\{a, b\}) = \{a, b\}$  is not  $\alpha g^*s$ -open in  $X$ .

**Remark 3.2.** Every contra continuous is contra  $\alpha g^*s$ -continuous.

**Theorem 3.1.** If a function  $f : X \rightarrow Y$  is  $\alpha g^*s$ -irresolute,  $g : Y \rightarrow Z$  is contra continuous then  $g \circ f : X \rightarrow Z$  is contra  $\alpha g^*s$ -continuous.

**Proof:** Let  $G \in O(Z)$ . Since  $g$  is contra continuous,  $g^{-1}(G) \in C(Y)$ . Thus  $g^{-1}(G) \in \alpha g^*s-C(Y)$ . Since  $f$  is  $\alpha g^*s$ -irresolute,  $f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G) \in \alpha g^*s-C(X)$ . Thus  $g \circ f$  is contra  $\alpha g^*s$ -continuous.

**Theorem 3.2.** Composition of contra  $\alpha g^*s$ -continuous and continuous functions is again contra  $\alpha g^*s$ -continuous.

**Proof.** Let  $V \in O(Z)$ . As  $g$  is continuous,  $g^{-1}(V) \in O(Y)$ . Then by contra  $\alpha g^*s$ -continuity,  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V) \in \alpha g^*s-C(X)$ . Thus  $g \circ f$  is contra  $\alpha g^*s$ -continuous.

**Lemma 3.1.** [7] The following properties holds for  $A, B \subset X$ :

- (i)  $x \in \ker(A)$  if and only if  $A \cap F \neq \phi$  for any  $F \in C(X, x)$
- (ii)  $A \subset \ker(A)$  and  $A = \ker(A)$  if  $A \in O(X)$ .
- (iii) if  $A \subset B$ , then  $\ker(A) \subset \ker(B)$

**Theorem 3.3.** The followings conditions are equivalent for a function  $f: X \rightarrow Y$ :

- (i)  $f$  is contra  $\alpha g^*s$ -continuous.
- (ii) for every  $F \in C(Y)$ ,  $f^{-1}(F) \in \alpha g^*s-O(X)$ .
- (iii) for each  $x \in X$  and each  $F \in C(Y, f(x))$ , there exists  $U \in \alpha g^*s-O(X, x)$  such that  $f(U) \subset F$ .
- (iv) for each  $x \in X$  and  $V \in O(Y)$  not containing  $f(x)$ , there exists  $K \in \alpha g^*s-C(X)$  not containing  $x$  such that  $f^{-1}(V) \subset K$ .
- (v)  $f(\alpha g^*s-cl(A)) \subset \ker(f(A))$  holds for  $A \subset X$
- (vi)  $\alpha g^*s-cl(f^{-1}(B)) \subset f^{-1}(\ker(B))$  holds for  $B \subset Y$ .

**Proof.** (i)  $\rightarrow$  (ii) Let  $F \in C(Y)$  and so  $Y-F \in O(Y)$ . From (i),  $f^{-1}(Y-F) = X-f^{-1}(F) \in \alpha g^*s-C(X)$ . Thus  $f^{-1}(F) \in \alpha g^*s-O(X)$ . Hence (ii) holds.

(ii)  $\rightarrow$  (i): Let  $G \in O(Y)$  and so  $Y-G \in C(Y)$ . From (ii),  $f^{-1}(Y-G) = X-f^{-1}(G) \in \alpha g^*s-O(X)$ , which shows that  $f^{-1}(G) \in \alpha g^*s-C(X)$ . Thus (i) hold.

(iii)  $\rightarrow$  (ii) Let  $F \in C(Y, f(x))$ . Then  $x \in f^{-1}(F)$ . By (iii),  $f^{-1}(F) \in \alpha g^*s-O(X, x)$ . Let  $U = f^{-1}(F)$ . Then  $f(U) = f(f^{-1}(F)) \subseteq F$ . Therefore (iii) holds.

(ii)  $\rightarrow$  (iii) Let  $F \in C(Y, f(x))$ . Then  $x \in f^{-1}(F)$ . From (iii),  $U_x \in \alpha g^*s-O(X, x)$  with  $f(U_x) \subseteq F$ , that is  $U_x \in f^{-1}(F)$ . Thus  $f^{-1}(F) = \cup\{U_x : x \in f^{-1}(F)\}$ , which is the union of  $\alpha g^*s$ -open sets. Thus  $f^{-1}(F) \in \alpha g^*s-O(X)$ .

(iv)  $\rightarrow$  (iii) Let  $V \in O(Y)$  not containing  $f(x)$ . Then  $Y-V \in C(Y, f(x))$ . By (iii),  $U \in \alpha g^*s-O(X, x)$  such that  $f(U) \subseteq Y-V$  and so  $U - f^{-1}(Y-V) = X-f^{-1}(V)$ . Hence  $f^{-1}(V) \subseteq X - U$ . Put  $K = X - U$  and so  $K \in \alpha g^*s-C(Y)$  not containing  $x$  such that  $f^{-1}(V) \subseteq K$ .

(iii)  $\rightarrow$  (iv) Let  $F \in C(Y, f(x))$  and so  $Y-F \in O(Y)$  not containing  $f(x)$ . From (iv),  $K \in \alpha g^*s-C(X)$  not containing  $x$  with  $f^{-1}(Y-F) \subseteq K$  and so  $X-f^{-1}(F) \in K$ . Thus  $X-K \in f^{-1}(F)$ , that is  $f(X-K) \subseteq F$ . Put  $U = X - K$ , then  $U \in \alpha g^*s-O(X, x)$  with  $f(U) \subseteq F$ .

(ii)  $\rightarrow$  (v) Let  $A \subseteq X$ . Assume that  $y \notin \ker(f(A))$ . From lemma 3.1,  $F \in C(Y, y)$  with  $f(A) \cap F = \phi$ . Thus,  $A - f^{-1}(F) = \phi$  and so  $A - X \subseteq f^{-1}(F)$ . But from (ii),  $f^{-1}(F) \in \alpha g^*s\text{-}O(X)$  and hence  $X - f^{-1}(F) \in \alpha g^*s\text{-}C(X)$ . Thus  $\alpha g^*s\text{-}cl(X - f^{-1}(F)) = X - f^{-1}(F)$ . Now  $A - X \subseteq f^{-1}(F)$ , that is  $\alpha g^*s\text{-}cl(A) \in \alpha g^*s\text{-}cl(X - f^{-1}(F)) = X - f^{-1}(F)$ . Therefore  $\alpha g^*s\text{-}cl(A) \in f^{-1}(F) = \phi$ , which implies  $f(\alpha g^*s\text{-}cl(A)) \cap F = \phi$  and hence  $y \notin \alpha g^*s\text{-}cl(A)$ . Thus  $f(\alpha g^*s\text{-}cl(A)) \in \ker(f(A))$  holds for every  $A \subseteq X$ .

(v)  $\rightarrow$  (vi) Let  $B \subset Y$ . Then  $f^{-1}(B) \subset X$ . From (iv) and by Lemma 3.1,  $f(\alpha g^*s\text{-}cl(f^{-1}(B))) \subset \ker(f(f^{-1}(B))) \subset \ker(B)$ . Thus, for every  $B \subseteq Y$ , we have

$$\alpha g^*s\text{-}cl(f^{-1}(B)) \subseteq f^{-1}(\ker(B)).$$

(vi)  $\rightarrow$  (i) Let  $V \in O(Y)$ . Then from (vi) and Lemma 3.1,  $\alpha g^*s\text{-}cl(f^{-1}(V)) \subseteq f^{-1}(\ker(V)) = f^{-1}(V)$  and  $\alpha g^*s\text{-}cl(f^{-1}(V)) = f^{-1}(V)$ . Thus  $f^{-1}(V) \subseteq \alpha g^*s\text{-}C(X)$ . Hence  $f$  is contra  $\alpha g^*s$ -continuous.

**Theorem 3.4.** Let  $f: X \rightarrow Y$  be contra  $\alpha g^*s$ -continuous with  $Y$  is regular space. Then  $f$  is  $\alpha g^*s$ -continuous.

**Proof.** Let  $x \in X$  and  $V \in O(Y, f(x))$ . Then by regularity of  $Y$ ,  $W \in O(Y, f(x))$  with  $cl(W) \subseteq V$ . As  $f$  is contra  $\alpha g^*s$ -continuous and from **Theorem 3.3** (iii),  $U \in \alpha g^*s\text{-}O(X, x)$  such that  $f(U) \subset cl(W)$ . Then  $f(U) \subset cl(W) \subseteq V$ . Thus  $f$  is  $\alpha g^*s$ -continuous.

**Definition 3.2.** A space  $X$  is called locally  $\alpha g^*s$ -indiscrete if  $V \in \alpha g^*s\text{-}O(X)$  then  $V \in C(X)$ .

**Theorem 3.5.** If a function  $f: X \rightarrow Y$  is contra  $\alpha g^*s$ -continuous with  $X$  is locally  $\alpha g^*s$ -indiscrete then  $f$  is continuous.

**Proof.** Let  $U \in O(Y)$ . As  $f$  is contra  $\alpha g^*s$ -continuous and  $X$  is locally  $\alpha g^*s$ -indiscrete space,  $f^{-1}(U) \in O(X)$ . Thus  $f$  is continuous.

**Definition 3.3.** A function  $f$  is said to be weakly  $\alpha g^*s$ -continuous, if for each  $x \in X$

and  $V \in O(Y, f(x))$ , there exists  $U \in \alpha g^*s\text{-}O(X, x)$  with  $f(U) \subset cl(V)$ .

**Theorem 3.6.** Every contra  $\alpha g^*s$ -continuous function is weakly  $\alpha g^*s$ -continuous function.

**Proof.** Let  $V \in O(Y)$ . Since  $cl(V) \in C(X)$  and from **Theorem 3.3**(ii),  $f^{-1}(cl(V)) \in \alpha g^*s\text{-}O(X)$ . Let  $U = f^{-1}(cl(V))$ , then  $f(U) \subset f(f^{-1}(cl(V))) \subset cl(V)$ . Thus  $f$  is almost weakly  $\alpha g^*s$ -continuous.

**Definition 3.4.** For any  $A \subset X$ ,  $\alpha g^*s$ -frontier of  $A$  defined as  $\alpha g^*s\text{-}cl(A) - \alpha g^*s\text{-}int(A)$  and is denoted by  $\alpha g^*s\text{-}Fr(A)$ .

**Theorem 3.7.** The set of all points  $x$  of  $X$  at which the function  $f: X \rightarrow Y$  is not contra  $\alpha g^*s$ -continuous is identical with the union of  $\alpha g^*s$ -frontier of the inverse images of closed sets of  $Y$  containing  $f(x)$ .

**Proof.** Suppose  $f$  is not contra  $\alpha g^*s$ -continuous at  $x \in X$ . From **Theorem 3.3** (iii), for every  $U \in \alpha g^*s\text{-}O(X, x)$ ,  $F \in C(Y, f(x))$  such that  $f(U) \cap (Y - F) = \phi$ , that is  $U \cap f^{-1}(Y - F) = \phi$ . Therefore,  $x \in \alpha g^*s\text{-}cl(f^{-1}(Y - F)) = \alpha g^*s\text{-}cl(X - f^{-1}(F))$ . Also,  $x \in f^{-1}(F) \in \alpha g^*s\text{-}cl(f^{-1}(F))$ . Thus,  $x \in \alpha g^*s\text{-}cl(f^{-1}(F)) - \alpha g^*s\text{-}cl(X - f^{-1}(F))$ , that is  $x \in \alpha g^*s\text{-}cl(f^{-1}(F)) - \alpha g^*s\text{-}int(f^{-1}(F))$ . Hence  $x \in \alpha g^*s\text{-}Fr(f^{-1}(F))$ .

On the other hand, for some  $F \in C(Y, f(x))$ ,  $x \in \alpha g^*s\text{-}Fr(f^{-1}(F))$ . As  $f$  is contra  $\alpha g^*s$ -continuous,  $U \in \alpha g^*s\text{-}O(X, x)$  such that  $f(U) \subset F$ . Therefore,  $x \in U \subseteq f^{-1}(F)$  and hence  $x \in \alpha g^*s\text{-}int(f^{-1}(F)) \subseteq X - \alpha g^*s\text{-}Fr(f^{-1}(F))$ , which is contradiction to the fact that  $x \in \alpha g^*s\text{-}Fr(f^{-1}(F))$ . Hence  $f$  is not contra  $\alpha g^*s$ -continuous.

**Definition 3.5.** If  $X$  cannot be expressed as the union of two disjoint nonempty  $\alpha g^*s$ -open sets, then  $X$  is said to be  $\alpha g^*s$ -connected.

**Theorem 3.8.** Let  $f: X \rightarrow Y$  be contra  $\alpha g^*s$ -continuous from a  $\alpha g^*s$ -connected space  $X$  onto any space  $Y$ . Then  $Y$  is not a discrete space.

**Proof.** Let  $f$  be contra  $\alpha g^*s$ -continuous with  $X$  is  $\alpha g^*s$ -connected. On the contrary, let  $Y$  be a discrete space. Let  $A \in O(Y)$ ,  $C(Y)$  and so  $f^{-1}(A) \in \alpha g^*s-O(X)$ ,  $\alpha g^*s-C(X)$ , which shows that  $X$  is  $\alpha g^*s$ -connected. Thus  $Y$  is not a discrete space.

**Theorem 3.9.** Every contra  $\alpha g^*s$ -continuous surjective space with  $\alpha g^*s$ -connected space is connected.

**Proof.** Let  $f$  be contra  $\alpha g^*s$ -continuous with  $X$  is  $\alpha g^*s$ -connected. Suppose  $Y$  is not connected. Then  $U, V \in O(Y)$  with  $U \cap V = \emptyset$  such that  $Y = U \cup V$ . Thus  $U$  and  $V$  are clopen sets in  $Y$ . By contra  $\alpha g^*s$ -continuity of  $f$ ,  $f^{-1}(U), f^{-1}(V) \in \alpha g^*s-O(X)$ . As  $f$  is surjective,  $f^{-1}(U)$  and  $f^{-1}(V)$  are non-empty disjoint and  $X = f^{-1}(U) \cup f^{-1}(V)$ , which shows that  $X$  is  $\alpha g^*s$ -connected. Thus  $Y$  must be connected.

**Theorem 3.10.** Let  $X$  be  $\alpha g^*s$ -connected with  $Y$  is  $T_1$ -space. Then  $f$  is constant, if  $f$  is contra  $\alpha g^*s$ -continuous.

**Proof.** Let  $f: X \rightarrow Y$  is contra  $\alpha g^*s$ -continuous with  $X$  is  $\alpha g^*s$ -connected and  $Y$  is  $T_1$ -space. Then by  $T_1$ -space,  $\nabla = \{f^{-1}(y) : y \in Y\}$  which is the disjoint  $\alpha g^*s$ -open partition of  $X$ . If  $|\nabla| \geq 2$ , there exists a proper  $\alpha g^*s$ -open and  $\alpha g^*s$ -closed set, which is contradiction to the hypothesis. Thus  $\nabla = 1$  and so  $f$  is a constant map.

**Definition 3.8 [10]:** A space  $X$  is said to be  $\alpha g^*s-T_2$ , if for any  $x, y \in X$  with  $x \neq y$ , there exist  $U, V \in \alpha g^*s-O(X)$  with  $U \cap V = \emptyset$  such that  $x \in U$  and  $y \in V$ .

**Definition 3.9. [11]** A space  $X$  is called Ultra Hausdroff space, if for every pair of distinct points  $x$  and  $y$  in  $X$ , there exist disjoint clopen sets  $U$  and  $V$  in  $X$  containing  $x$  and  $y$  respectively.

**Theorem 3.12.** Let  $f: X \rightarrow Y$  be contra  $\alpha g^*s$ -continuous injective function. If  $Y$  is Ultra Hausdroff, then  $X$  is  $\alpha g^*s-T_2$ .

**Proof.** Let  $x, y \in X$  with  $x \neq y$ . Then  $f(x) \neq f(y)$ . By Ultra Hausdroff space there exists disjoint clopen sets  $U$  and  $V$  containing  $f(x)$  and  $f(y)$  respectively. Then  $x \in f^{-1}(U)$  and  $y \in f^{-1}(V)$ , where  $f^{-1}(U), f^{-1}(V) \in \alpha g^*s-O(X)$  with  $f^{-1}(U) \cap f^{-1}(V) = \emptyset$ . Thus  $X$  is  $\alpha g^*s-T_2$ .

**Definition 3.10. [10]** (i) if every  $\alpha g^*s$ -closed cover has a finite subcover, then  $X$  is said to be  $\alpha g^*s$ -closed compact.

(ii) if every countable cover of  $X$  by  $\alpha g^*s$ -closed sets has a finite subcover, then  $X$  is said to be countably  $\alpha g^*s$ -closed compact.

(iii) if every  $\alpha g^*s$ -closed cover has a countable subcover, then  $X$  is said to be  $\alpha g^*s$ -Lindelof.

**Theorem 3.13.** The following properties holds for a contra  $\alpha g^*s$ -continuous surjection function  $f: X \rightarrow Y$ :

(i)  $Y$  is compact, if  $X$  is  $\alpha g^*s$ -closed compact.

(ii)  $Y$  is countably compact, if  $X$  is countably  $\alpha g^*s$ -closed compact.

(iii)  $Y$  is Lindelof, if  $X$  is  $\alpha g^*s$ -Lindelof.

#### 4. Almost Contra $\alpha g^*s$ -Continuous Functions in Topological Spaces

In this section, a new type of continuous functions called an almost contra  $\alpha g^*s$ -continuous functions, which are weaker than contra  $\alpha g^*s$ -continuous functions are introduced and studied some of their properties.

**Definition 4.1.** A function  $f: X \rightarrow Y$  is said to be almost contra  $\alpha g^*s$  continuous if  $f^{-1}(V)$  is  $\alpha g^*s$ -closed in  $X$  for each regular-open set  $V$  in  $Y$ .

**Theorem 4.1:** For the function  $f: X \rightarrow Y$ , following statements are equivalent:

- (i)  $f$  is almost contra  $\alpha g^*s$ -continuous
- (ii) for every  $F \in RO(Y)$ ,  $f^{-1}(F) \in \alpha g^*sO(X)$
- (iii) for each  $x \in X$  and for each  $F \in RC(Y, f(x))$  there exists  $U \in \alpha g^*sO(X, x)$  such that  $f(U) \subseteq F$ .
- (iv) for each  $x \in X$  and for each  $V \in RO(Y)$  such that  $f(x) \notin V$  there exists  $K \in \alpha g^*sC(X)$  not containing  $x$  such that  $f^{-1}(V) \subseteq K$ .

**Proof:** (i)  $\Rightarrow$  (ii) Let  $F \in RO(Y)$  then  $Y - F \in RC(Y)$ . From (i),  $f^{-1}(Y - F) = X - f^{-1}(F) \in \alpha g^*sC(X)$  and hence  $f^{-1}(F) \in \alpha g^*sO(X)$ .

(ii)  $\Rightarrow$  (i) Let  $G \in RC(Y)$  then  $Y - G \in RO(Y)$ . By (ii),  $f^{-1}(Y - G) = X - f^{-1}(G) \in \alpha g^*sO(X)$ , this implies that  $f^{-1}(G) \in \alpha g^*sC(X)$ .

(ii)  $\Rightarrow$  (iii) Let  $F \in RC(Y, f(x))$  such that  $x \in f^{-1}(F)$ . By (ii),  $U \in \alpha g^*sO(X, x)$  such that  $f(U) \subseteq F$ , so  $Ux \subseteq f^{-1}(F)$ . Thus  $f^{-1}(F) = \cup\{Ux : x \in f^{-1}(F)\}$ .

(iii)  $\Rightarrow$  (ii) Let  $F \in RC(Y, f(x))$  and  $x \in f^{-1}(F)$ . By (iii), there exists  $U \subseteq \alpha g^*sO(X, x)$  such that  $f(U) \subseteq F$ , that implies  $Ux \subseteq f^{-1}(F)$ . Thus,  $f^{-1}(F) = \cup\{Ux : x \in f^{-1}(F)\}$ . Hence  $f^{-1}(F) \in \alpha g^*sO(X)$ .

(iii)  $\Rightarrow$  (iv) Let  $V \in RO(Y)$  such that  $f(x) \notin V$  then  $Y - V \in RC(Y, f(x))$ . By (iii) there exists  $U \in \alpha g^*sO(X, x)$  such that  $f(U) \subseteq Y - V$ , that is  $U \subseteq f^{-1}(Y - V) = X - f^{-1}(V)$  and hence  $f^{-1}(V) \subseteq X - U$ . Take  $K = X - U$  then  $K \in \alpha g^*sC(X)$  such that  $f^{-1}(V) \subseteq K$ .

(iv)  $\Rightarrow$  (iii): Let  $F \in RC(Y, f(x))$  then  $Y - F \in RO(Y)$ . From (iv), there exists  $K \in \alpha g^*sC(X)$  not containing  $x$  such that  $f^{-1}(Y - F) \subseteq K$ , that is  $X - f^{-1}(F) \subseteq K$ . Hence  $X - K \subseteq f^{-1}(F)$  this implies  $f(X - K) \subseteq F$ . Take  $U = X - K$  then  $U \in \alpha g^*sO(X, x)$  such that  $f(U) \subseteq F$ .

**Theorem 4.2.** If  $f: X \rightarrow Y$  is an almost contra  $\alpha g^*s$ -continuous injective and  $Y$  is Weakly Hausdorff then  $X$  is  $\alpha g^*s-T_1$ -space.

**Proof:** Let  $x, y \in X$  with  $x \neq y$ . Then there exist  $V, W \in RC(Y)$  such that  $f(x) \in V$  and  $f(y) \in W$  as  $Y$  Weakly Hausdorff. Since  $f$  is almost contra  $\alpha g^*s$ -continuous then  $f^{-1}(V), f^{-1}(W) \in \alpha g^*sO(X)$  such that  $x \in f^{-1}(V), y \notin f^{-1}(V)$  and  $y \in f^{-1}(W), x \notin f^{-1}(W)$ . Hence  $X$  is  $\alpha g^*s-T_1$ -space.

**Theorem 4.3.** If  $f: X \rightarrow Y$  is almost contra  $\alpha g^*s$ -continuous injective function from a space  $X$  into an Ultra Hausdorff space  $Y$  then  $X$  is  $\alpha g^*s-T_2$ .

**Proof:** Let  $x, y \in X$  with  $x \neq y$  then  $f(x) \neq f(y)$  as  $f$  is injective. As  $Y$  is Ultra Hausdorff there exist  $U \in CO(Y, f(x))$  and  $V \in CO(Y, f(y))$  such that  $U \cap V = \emptyset$ . Since  $f$  is almost

contra  $\alpha g^*s$ -continuous,  $x \in f^{-1}(U)$  and  $y \in f^{-1}(V)$  where  $f^{-1}(U) \in \alpha g^*sO(X, x)$  and  $f^{-1}(V) \in \alpha g^*sO(X, y)$  such that  $f^{-1}(U) \cap f^{-1}(V) = \phi$ . Hence  $X$  is  $\alpha g^*s-T_2$ .

**Theorem 4.4.** Let  $f: X \rightarrow Y$  be almost contra  $\alpha g^*s$ -continuous closed and injection function. If  $Y$  is Ultra Normal then  $X$  is  $\alpha g^*s$ -normal.

**Theorem 4.5.** If a function  $f: X \rightarrow Y$  is almost contra  $\alpha g^*s$ -continuous from a  $\alpha g^*s$ -connected space  $X$  on to a topological space  $Y$ , then  $Y$  is also connected space.

Proof: If  $Y$  is not connected then  $U, V \in O(Y)$  such that  $Y = U \cup V$  and hence  $U, V \in CO(Y)$ . Then  $f^{-1}(U), f^{-1}(V) \in \alpha g^*sO(X)$  since  $f$  is almost contra  $\alpha g^*s$ -continuous function and  $f^{-1}(U) \cap f^{-1}(V) = \phi$  such that  $X = f^{-1}(U) \cup f^{-1}(V)$  which is a contradiction to the fact that  $X$  is  $\alpha g^*s$ -connected. Hence  $Y$  must be connected space.

**Theorem 4.6.** Let  $f: X \rightarrow Y$  and  $g: Y \rightarrow Z$  are any two functions. Then we have the following:

- (i)  $g \circ f$  is almost contra  $\alpha g^*s$ -continuous if  $f$  is almost contra  $\alpha g^*s$ -continuous and  $g$  is an R-map.
- (ii)  $g \circ f$  is  $\alpha g^*s$ -continuous and contra  $\alpha g^*s$ -continuous if  $f$  is almost contra  $\alpha g^*s$ -continuous and  $g$  is perfectly-continuous.

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