Vol.11.Issue.2.2023 (April-June)
©KY PUBLICATIONS

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

http://www.bomsr.com

Email:editorbomsr@gmail.com

INTERNATIONAL STANDARD SERIAL NUMBER 2348-0580

RESEARCH ARTICLE

POSSIBLE INNOVATORY PROPAGATION MODELS AND THEIR CORRESPONDING MODELS IN PYTHAGOREAN FUZZY SET VIA MODIFIED VERMA $i.e.$ HYBRID SHANNON ENTROPY

ROHIT KUMAR VERMA
Associate Professor Department of Mathematics
Bharti Vishwavidyalaya, Durg, Chhattisgarh, India
Email:rohitkverma73@rediffmail.com

DOI:10.33329/bomsr.11.2.127

ROHIT KUMAR VERMA

ABSTRACT

We can suggest a few fresh Innovatory Propagation Models (IPM) based on some information measurements. This could be beneficial as model for the transmission of new ideas or the growth of the grain industry.

Key words: Innovatory propagation model, Measures of entropy, Conjugate model, Logistic-type growth model.

1. INTRODUCTION

In contrast to intuitionistic fuzzy sets (IFS), Yager's concept of Pythagorean fuzzy sets (PFSs) offers a fresh technique to express uncertainty and ambiguity with great precision and accuracy. An extension of the intuitionistic fuzzy set is the Pythagorean Fuzzy Set (PFS). The pair of membership grades in IFSs are represented as (μ, ν) when they meet the requirement of $\mu + \nu \leq 1$. The condition $\mu + \nu \leq 1$ was recently extended by Yager and Abbasov [10] and Yager [11] to $\mu^2 + \nu^2 \leq 1$, and they subsequently introduced a class of Pythagorean fuzzy sets (PFSs) whose membership values are ordered pairs (μ, ν) that fulfill the necessary condition of $\mu^2 + \nu^2 \leq 1$ with various aggregation operations and applications in multicriterion decision making. As a result, the PFS decision-making mechanism performs better than the IFS decision-making in real-world scenarios including uncertainty. PFS and Soft Set are combined to create Pythagorean Fuzzy Soft Set (PFSS).

PFSs are a generalized form of IFSs, hence to calculate the distances between PFSs, all three of its components $\mu_{\bar{P}}^2(x), \nu_{\bar{P}}^2(x)$ and $\pi_{\bar{P}}^2(x)$ must be taken into account. Euclidean distance is a well-known measure of separation between PFSs. Consequently, the disparity

$$\left(\mu_{\bar{P}}(x_i) - \frac{1}{\sqrt{3}}\right)^2 + \left(\nu_{\bar{P}}(x_i) - \frac{1}{\sqrt{3}}\right)^2 + \left(\pi_{\bar{P}}(x_i) - \frac{1}{\sqrt{3}}\right)^2 \geq \left(\mu_{\bar{Q}}(x_i) - \frac{1}{\sqrt{3}}\right)^2 + \left(\nu_{\bar{Q}}(x_i) - \frac{1}{\sqrt{3}}\right)^2 + \left(\pi_{\bar{Q}}(x_i) - \frac{1}{\sqrt{3}}\right)^2, \forall x_i$$

This manifests that $\left(\mu_{\bar{Q}}(x), \nu_{\bar{Q}}(x), \pi_{\bar{Q}}(x)\right)$ is located more nearby to $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$ than that of $(\mu_{\bar{P}}(x), \nu_{\bar{P}}(x), \pi_{\bar{P}}(x))$. From a geometrical perspective, the resolution axiom demonstrates that the Euclidean distance between $(\mu_{\bar{P}}(x), \nu_{\bar{P}}(x), \pi_{\bar{P}}(x))$ and $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$ is greater than the distance between $\left(\mu_{\bar{Q}}(x), \nu_{\bar{Q}}(x), \pi_{\bar{Q}}(x)\right)$ and $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$. According to this, $\left(\mu_{\bar{Q}}(x), \nu_{\bar{Q}}(x), \pi_{\bar{Q}}(x)\right)$ are placed closer to $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$ than $(\mu_{\bar{P}}(x), \nu_{\bar{P}}(x), \pi_{\bar{P}}(x))$. From a geometrical perspective, the resolution axiom $i.e.$ $E(\bar{P}) \leq E(\bar{Q})$, if $\bar{P}$ is crisper than $\bar{Q}$, $i.e.$, $\forall x \in X$, $\mu_{\bar{P}}(x) \leq \mu_{\bar{Q}}(x)$ and $\nu_{\bar{P}}(x) \leq \nu_{\bar{Q}}(x)$ for $\max\left(\mu_{\bar{Q}}(x), \nu_{\bar{Q}}(x)\right) \leq \frac{1}{\sqrt{3}}$ and $\mu_{\bar{P}}(x) \geq \mu_{\bar{Q}}(x), \nu_{\bar{P}}(x) \geq \nu_{\bar{Q}}(x)$ for $\min\left(\mu_{\bar{Q}}(x), \nu_{\bar{Q}}(x)\right) \geq \frac{1}{\sqrt{3}}$, is reasonable and logical because the closer PFS to the unique point $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$ with maximum entropy reflects the greater entropy of that PFS.

Then, using a probability type, we create entropies for PFSs. We use the concept of entropy $V_a(P)$ of Verma [7, 8] to a probability mass function $p = p_1, p_2, \dots \dots \dots, p_n$ intending to formulate the probability-type of entropy for PFSs.

$$V_a(P) = \begin{cases} -\sum_{i=1}^{n} \ln\left(\frac{1+ap_i}{p_i}\right) + \sum_{i=1}^{n} \ln(1+a)p_i, a \neq 0 \\ \sum_{i=1}^{n} \ln p_i, a = 0. \end{cases} \tag{1.1}$$

Let $X = x_1, x_2, \dots \dots \dots, x_n$ represent the set of all possible discourses. Therefore, we suggest the following probability type entropy for the PFS $\bar{P}$ with a PFS $\bar{P}$ in X.

$$V_a(P) = \begin{cases} -\frac{1}{n}\sum_{i=1}^{n}\left[\ln\left(1 + a\mu_{\bar{P}}^2(x_i)\right) + \ln\left(1 + a\nu_{\bar{P}}^2(x_i)\right) + \ln\left(1 + a\pi_{\bar{P}}^2(x_i)\right)\right] \\ \quad + \frac{1}{n}\sum_{i=1}^{n}\left[\ln \mu_{\bar{P}}^2(x_i) + \ln \nu_{\bar{P}}^2(x_i) + \ln \pi_{\bar{P}}^2(x_i) + \ln(1+a)\right], a \neq 0 \\ \quad \frac{1}{n}\sum_{i=1}^{n}\left[\ln \mu_{\bar{P}}^2(x_i) + \ln \nu_{\bar{P}}^2(x_i) + \ln \pi_{\bar{P}}^2(x_i) + \ln(1+a)\right], a = 0. \end{cases} \tag{1.2}$$

On the other hand, J. N. Kapur, Uma Kumar, and Vinod Kumar [5] created a few novel, potentially inventive propagation models in 1992. In the novel propagation models, they employed the characteristics of the function $\emptyset(f)$ $i.e.$

$$\frac{1}{c}\frac{df}{dt} = \emptyset(f) \tag{1.3}$$

and the properties used were:

(i) $\emptyset(0) = 0$, (ii) $\emptyset(1) = 0$, (iii) $\emptyset'(f) > 0$ when $f_0 < f < f^*$,

(iv) $\emptyset'(f) < 0$ when $f^* < f < 1$ and (v) $\emptyset'(f) = 0$ when $f = f^*$ (1.4)

where, f_0 is the value of f at $t = 0$. These characteristics demand that $\emptyset(f)$ increase from 0 to a maximum value of $\emptyset(f^*)$, then fall back to 0. These characteristics guarantee that the equation (3.1.1) will produce logistic-type [9] S-shaped growth models with a point of inflection at f^*.

The above properties do not imply that $\emptyset(f)$ has to be necessarily a concave function of f, but if $\emptyset(f)$ is a differentiable concave function, it can satisfy all these properties.

Now, since the general form of the measure of entropy, for any probability distribution $p_1, p_2, \dots \dots \dots p_n$ in the discrete case, is given by

$$\sum_{i=1}^{n} \emptyset(p_i) \tag{1.5}$$

and in the continuous variate case the corresponding measure of entropy with density function $f(x)$ is given by

$$\int_b^d \emptyset\big(f(x)\big)\, dx \tag{1.6}$$

where $\emptyset(.)$ is a concave function.

As a result, the concave functions are useful for producing measurements of entropy and may also be useful for producing growth models of the logistic kind [9].

Later in 2013, Verma [8] employed the parametric entropy measure, $i.e.$

$$V_a(P) = -\sum_{i=1}^{n} \ln\left(\frac{1+ap_i}{p_i}\right) + \sum_{i=1}^{n} \ln(1+a)p_i \tag{1.7}$$

with model $$\frac{1}{c}\frac{df}{dt} = -\ln(1+af) + \ln f + \ln(1+a).f \tag{1.8}$$

and got, $$\frac{1}{c}\frac{df}{dt} = -\ln\frac{1}{f}, \tag{1.9}$$

which is the negation of $\frac{1}{f}$ times the model of Gompertz [3]

Now, when we modify the model (1.6) $i.e.$

$$\frac{1}{c}\frac{df}{dt} = \frac{-\ln(1+af) + a^3 \ln f + \ln(1+a).f}{a^2} \tag{1.10}$$

and got, $$\frac{1}{c}\frac{df}{dt} = -\frac{1}{2}f(1-f), \tag{1.11}$$

which is the negation of Fisher-Pry [2] model of innovation of diffusion or Mckendric-Pai [6] model of logistic.

We give examples of unique innovatory propagation models (IPMs) that were developed in this study utilising the modified Verma [7] parametric measure of entropy and their equivalent models in Pythagorean fuzzy sets, respectively, in the next sections (2.1) and (2.2).

2. OUR WORK

2.1 INNOVATORY PROPAGATION MODEL VIA MODIFIED VERMA ENTROPY

Now since the modified version of Verma [7] parametric measure of entropy is,

$$V_a(P) = \sum_{i=1}^{n} \ln(1+ap_i) - \sum_{i=1}^{n} p_i \ln p_i - \sum_{i=1}^{n} \ln(1+a).p_i, a > 0$$

and in continuous variate case, we get the entropy

$$\int_b^d \big(\ln\big(1+af(x)\big) - f(x)\ln f(x) - \ln(1+a).f(x)\big)dx. \tag{2.1.1}$$

The corresponding IPM is,

$$\frac{1}{c}\frac{df}{dt} = \ln(1+af) - f\ln f - \ln(1+a).f \tag{2.1.2}$$

Now, we consider the corresponding function

$$\emptyset(f) = \ln(1 + af) - f \ln f - \ln(1 + a).f \quad (2.1.3)$$

so that,
$$\emptyset'(f) = \frac{1}{1+af}.a - 1 - \ln f - \ln(1 + a) \quad (2.1.4)$$

and
$$\emptyset''(f) = -\frac{1}{(1+af)^2}.a^2 - \frac{1}{f} = -\frac{a^2f^2+a^2f+2af+1}{f(1+af)^2} < 0 \quad (2.1.5)$$

Thus, $\emptyset(f)$ is a concave function of f, which vanishes only when $f = 0$ and $f = 1$ $i.e.$

$\emptyset(0) = 0$ and $\emptyset(1) = 0$.

Since, the corresponding LTGM is

$$\frac{1}{c}\frac{df}{dt} = \ln(1 + af) - f \ln f - \ln(1 + a).f \quad (2.1.6)$$

Now, let us find the limiting model when $a \to 0, i.e.$

$$\frac{1}{c}\frac{df}{dt} = \lim_{a\to 0}[\ln(1 + af) - f \ln f - \ln(1 + a).f] \quad (2.1.7)$$

$$= f \ln\frac{1}{f} \quad (2.1.8)$$

which is Gompertz's [3] model of logistic.

Without loss of generality, we can modify the model,

$$\frac{1}{c}\frac{df}{dt} = \frac{\ln(1+af)-a^3f\ln f-\ln(1+a)f}{a^2} \quad (2.1.9)$$

For this model

$$\lim_{a\to 0}\left[\frac{\ln(1+af)-a^3f\ln f-\ln(1+a)f}{a^2}\right] = \frac{1}{2}f(1-f) \quad (2.1.10)$$

so that when $a \to 0$, (2.10.9) gives the model

$$\frac{1}{c}\frac{df}{dt} = \frac{1}{2}f(1-f) \quad (2.1.11)$$

Which gives the Fisher-Pry [2] model of innovation of diffusion or Mckendric-Pai [6] model of logistic.

2.2 CORRESPONDING INNOVATORY PROPAGATION MODELS IN PYTHAGOREAN FUZZY SET (PFS)

If the value of $\emptyset_{\bar{P}}(x)$ denotes the level to which element x belongs to set $\bar{P}$. Then, in the Pythagorean fuzzy set (PFS) [7], the equivalent IPM for the modified Verma parametric measure of entropy is given by,

$$\frac{1}{c}\frac{d}{dt}\emptyset_{\bar{P}}(x) = \ln\left(1 + a\mu_{\bar{P}}^2(x_i)\right) + \ln\left(1 + a\nu_{\bar{P}}^2(x_i)\right) + \ln\left(1 + a\pi_{\bar{P}}^2(x_i)\right)$$
$$-\mu_{\bar{P}}^2(x_i)\ln\mu_{\bar{P}}^2(x_i) - \nu_{\bar{P}}^2(x_i)\ln\nu_{\bar{P}}^2(x_i) - \pi_{\bar{P}}^2(x_i)\ln\pi_{\bar{P}}^2(x_i) - \ln(1 + a) \quad (2.2.1)$$

For this model, when $a \to 0$, we get

$$\frac{1}{c}\frac{d}{dt}\emptyset_{\bar{P}}(x) = -\mu_{\bar{P}}^2(x_i)\ln\mu_{\bar{P}}^2(x_i) - \nu_{\bar{P}}^2(x_i)\ln\nu_{\bar{P}}^2(x_i) - \pi_{\bar{P}}^2(x_i)\ln\pi_{\bar{P}}^2(x_i)$$
$$= \mu_{\bar{P}}^2(x_i)\ln\frac{1}{\mu_{\bar{P}}^2(x_i)} + \nu_{\bar{P}}^2(x_i)\ln\frac{1}{\nu_{\bar{P}}^2(x_i)} + \pi_{\bar{P}}^2(x_i)\ln\frac{1}{\pi_{\bar{P}}^2(x_i)} \quad (2.2.2)$$

which is the model of innovatory propagation due to Gompertz [3] in Pythagorean fuzzy set.

Of course, without loss of generality, we can modify the model (2.2.1) into,

$$\frac{1}{c}\frac{d}{dt}\emptyset_{\bar{P}}(x)=\frac{\begin{array}{c}\ln\left(1+a\mu_{\bar{P}}^2(x_i)\right)+\ln\left(1+a\nu_{\bar{P}}^2(x_i)\right)+\ln\left(1+a\pi_{\bar{P}}^2(x_i)\right)\\ -a^3\mu_{\bar{P}}^2(x_i)\ln\mu_{\bar{P}}^2(x_i)-a^3\nu_{\bar{P}}^2(x_i)\ln\nu_{\bar{P}}^2(x_i)-a^3\pi_{\bar{P}}^2(x_i)\ln\pi_{\bar{P}}^2(x_i)\\ \mu_{\bar{P}}^2(x_i)\ln(1+a)-\nu_{\bar{P}}^2(x_i)\ln(1+a)-\pi_{\bar{P}}^2(x_i)\ln(1+a)\end{array}}{a^2} \tag{2.2.3}$$

For this model

$$\lim_{a\to 0}\left[\frac{\begin{array}{c}\ln\left(1+a\mu_{\bar{P}}^2(x_i)\right)+\ln\left(1+a\nu_{\bar{P}}^2(x_i)\right)+\ln\left(1+a\pi_{\bar{P}}^2(x_i)\right)\\ -a^3\mu_{\bar{P}}^2(x_i)\ln\mu_{\bar{P}}^2(x_i)-a^3\nu_{\bar{P}}^2(x_i)\ln\nu_{\bar{P}}^2(x_i)-a^3\pi_{\bar{P}}^2(x_i)\ln\pi_{\bar{P}}^2(x_i)\\ \mu_{\bar{P}}^2(x_i)\ln(1+a)-\nu_{\bar{P}}^2(x_i)\ln(1+a)-\pi_{\bar{P}}^2(x_i)\ln(1+a)\end{array}}{a^2}\right]$$

$$=\frac{1}{2}\left[\mu_{\bar{P}}^2(x_i)+\nu_{\bar{P}}^2(x_i)+\pi_{\bar{P}}^2(x_i)-\left(\mu_{\bar{P}}^4(x_i)+\nu_{\bar{P}}^4(x_i)+\pi_{\bar{P}}^4(x_i)\right)\right] \tag{2.2.4}$$

which is the model of innovatory propagation due to Fisher-Pry [2] or the model of logistic due to Mckendric-Pai [6] in Pythagorean fuzzy set (PFS).

3. CONCLUSION :

This study's objective is to create specific Logistic-Type Growth Models (LTGMs) or Innovatory Propagation Models (IPMs) in classical and Pythagorean Fuzzy Sets (PFS), such as those proposed by Fisher-Pry [2], Mckendric-Pai [6], and Gompertz [3], based on parametric information measures derived from a modified version of Verma [7]. The following applications of these models are possible:

Using statistical data that is projected for the future and then reviewed, we may develop our future strategy.

1. The number of organisms and the amount of food present both affect how quickly fast-growing microorganisms multiply.
2. The initial rate of multiplication allows for a factor of comparison for both media effectiveness and an organism's capacity for reproduction.
3. Vaccines may be made in large amounts on the assumption that a maximum number of organisms is reached; this maximum is based on the nutrient content and is unrelated to the quantity of culture that is injected.

REFERENCES

[1]. Burg, J. P. (1972): The Relationship between Maximum Entropy Spectra and Maximum Likelihood Spectra,in Modern Spectra Analysis ed by D.G.Childers,M.S.A.

[2]. Fisher, J. C. and Pry, P. C. (1971): A Simple Substitution model for technological change, Technological Forecasting and Social Change 3, 75-88.

[3]. Gompertz, B. (1935): On the Science Connected with Human Mortality, Trans. Roy. Soc. 115, 5-11.

[4]. Kapur, J. N. (1997): Measures of Information and Their Applications, Wiley Eastern, New Delhi, pp.101, 473-475.

[5]. Kapur, J. N., Kumar, Uma and Kumar, Vinod (1992): Some Possible Models for Technological Innovation Diffusion;Exploiting Analogus Characteristics of Entropy Measures: CSIR Journal of Scientific and Industrial Research Vol.51, 202-208.

[6]. Mckendric, A. and Pai, M. K. (1911): The Rate of Multiplication of Micro Organisons; A Mathematical Study, Proc. Roy. Soc. Edinburgh, 31,649-653.

[7]. Verma, Rohit Kumar and Verma, Babita (2013): A New Approach in Mathematical Theory of Communication: A New Entropy with its Application), Lambert Academic Publishing pp. 80-82.

[8]. Verma, R. K., Dewangan, C. L. and Jha, P. (2012): An Unorthodox Parametric Measures of Information and Corresponding Measures of Fuzzy Information, Int. Jour. of Pure and Appl. Mathematics, Bulgaria, Vol. 76, No. 4, pp. 599-614.

[9]. Verma, R. K., Dewangan, C. L. and Jha, P. (2012): Some Logistic-Type Growth Models Based on Measures of Information, Int. Jour. of Math. Sci. & Engg. Appls., Vol. 6 No. 1, pp. 217-225.

[10]. Yager, R. R. and Abbasov, A. M. (2013): Pythagorean membership grades, complex numbers, and decision making, International Journal of Intelligent Systems, vol. 28, no. 5, pp. 436–452.

[11]. Yager, R. R. (2013): Pythagorean fuzzy subsets," in Proceedings of the 9th Joint World Congress on Fuzzy Systems and NAFIPS Annual Meeting, IFSA/NAFIPS 2013, pp. 57–61, Edmonton, Canada.

Biography of author: Dr. Rohit Kumar Verma is a well-known author in the field of the journal scope. He obtained his highest degree from RSU, Raipur (C.G.) and worked in engineering institution for over a decade. Currently he is working in a capacity of Associate Professor and HOD, Department of Mathematics, Bharti Vishwavidyalaya, Durg (C.G.). In addition to 30 original research publications in the best journals, he also published two research book by LAP in 2013 and 2023 in the areas of information theory and channel capacity that fascinate him. He is the Chairperson, the Board of Studies Department of Mathematics at Bharti Vishwavidyalaya in Durg, C.G., India. He has published two patents in a variety of fields of research. In addition to numerous other international journals, he reviews for the American Journal of Applied Mathematics (AJAM). In addition to the Indian Mathematical Society, he is a member of the Indian Society for Technical Education (ISTE) (IMS).