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RESEARCH ARTICLE

WEAKLY FAINTLY CONTINUOUS FUNCTIONS

Sarika M. Patil
Department of Mathematics,
Government First Grade College, Rajnagar, Hubli-580 032
Karnataka State, India.
Email: sarupatil@rediffmail.com
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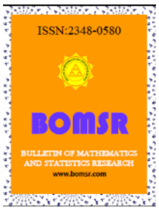

ABSTRACT

In this paper a new class of continuous functions called faintly αg^*s-continuous functions and relationship among faintly αg^*s-continuous functions and αg^*s-connected spaces, αg^*s-compact spaces and αg^*s-regular spaces have been investigated and their properties are obtained.

Keywords and Phrases: αg^*s-closed sets, αg^*s-connected spaces, αg^*s-compact spaces, αg^*s-continuous spaces.

AMS Subject Classification: 54C08, 54C10.

1. Introduction

Generalized open sets play a very important role in General Topology and they arenow the research topics of many topologists worldwide. The weaker forms of continuous functions called faintly continuous functions using θ-open sets were introduced byLong and Herrington [4]. They defined number of properties concerning such functionsand among them; they have showed that every weakly continuous function is faintly continuous. Later Noiri and Popa[6] investigated the properties of weaker form of faintcontinuity called faint semi-continuity, faint pre-continuity and faint β-continuity.

The purpose of this paper is to introduce and investigate the properties of faintly αg^*s-continuous functions. Some new characterizations and several fundamental properties of these functions along with their relationships with certain other types of functions are investigated.

2. Preliminary

Throughout this paper, X and Y always means topological spaces (X, τ) and (Y, σ) and f:(X, τ) → (Y, σ) (simply f:X→Y) denotes a function f of a space(X, τ) into a space (Y, σ).

Definition 2.1: A subset A of a topological space (X, τ) is called

(i) α-open [5] if A ⊆ int(cl(int(A))).

(ii) semi-open [2] if A ⊆ cl(int(A)).

Definition 2.2: A subset A of a topological space (X, τ) is called
(i) g-closed [3] cl(A) ⊆ U whenever A ⊆ U and U is open in X.

(ii) αg*s-closed [7] αcl(A) ⊆ U whenever A ⊆ U and U is gs-open in X.

Definition 2.3: A function f: X→Y is said to be αg*s-continuous [7] if for each open set V in Y, $f^{-1}(V)$ is αg*s-open in X.

Definition 2.4: A function f:X → Y is said to be faintly continuous [6] (resp. faintly semi-continuous [6], faintly pre-continuous [6], faintly α-continuous [1], faintly β- continuous [6], quasi θ-continuous [5]) if for each point x ∈ X and each θ-open set V containing f(x), there exists an open set(resp. semi-open, pre-open, α-open, β-open) set U containing x such that f(U) ⊂ V.

Definition 2.5 [8]: A function f: X → Y is said to be slightly αg*s-continuous if for each clopen set V in Y containing f(x), there exist U ∈ αg*sO(X, x) such that f(U) ⊂ V.

3. Faintly αg*s-Continuous Functions

Definition 3.1: A function f: X → Y is called faintly αg*s-continuous at a point x ∈ X if for each V ∈ θ-O(Y, f(x)), there exists U∈αg*s-O(X, x) such that f (U) ⊆V.

If f has the above property at each point of X, then f is said to be faintly αg*s- continuous.

Theorem 3.2: The following statements are equivalent for a function f: X → Y:

(i) f is faintly αg*s-continuous.
(ii) for each V ∈ θ-O(Y), $f^{-1}(V)$ ∈ αg*s-O(X).
(iii) for each F ∈ θ-C(Y), $f^{-1}(F)$ ∈ αg*s-C(X).
(iv) f is αg*s-continuous.
(v) for every B ⊆ Y, αg*s-cl(f^{-1}(B)) ⊆ f^{-1}(cl_θ(B)).
(vi) for every G ⊆ Y , f^{-1}(int_θ(G)) ⊆ αg*s-int(f^{-1}(G)).

Proof: (i) → (ii) Let f be faintly αg*s-continuous and V ∈ θ-O(Y) such that x ∈ f −1(V). Then there exists U ∈ αg*s-O(X, x) with f (U) ⊆V, that is x ∈ U ⊆ $f^{-1}(V)$. Thus $f^{-1}(V)$ ∈ αg*s-O(X).

(ii) → (i) Let x ∈ X and V ∈ θ-O(Y, f(x)). From (ii), $f^{-1}(V)$ ∈αg*s-O(X, x). Let U = f −1(V), then f (U) ⊆ V . Hence f is faintly αg*s-continuous.

(ii) → (iii) Let V ∈ θ-C(Y), then Y-V ∈ θ-O(X). From (ii), f^{-1}(Y-V) = X- $f^{-1}(V)$ ∈ αg*s-O(X) and hence $f^{-1}(V)$ ∈ αg*s-C(X).
(iii) → (ii) Let V ∈ θ-O(Y), then Y-V ∈ θ-C(Y). From (iii), f^{-1}(Y-V) = X-$f^{-1}(V)$ ∈ αg*s-C(Y) and hence $f^{-1}(V)$ ∈ αg*s-O(X).

From the definition 3.1, we can prove the other equivalent properties.

Remark 3.3: Every αg^*s-continuous function is faintly αg^*s-continuous. However, the converse is not true in general as shown in the below example.

Example 3.4: Let X = {1, 2, 3} and τ = {X, Φ, 1, 2, 3} and σ = {Y, ϕ, {1}, {2}, {1, 2}, {2, 3}}. Then the identity function f: X $\rightarrow$ Y is faintly αg^*s-continuous but not αg^*s-continuous.

Definition 3.5: A function f: X $\rightarrow$ Y is said to be weakly αg^*s-continuous if for each point x $\in$ X and for each V $\in$ O(Y, f(x)), there exists U $\in$ αg^*s-O(X, x) such that f(U) $\subset$ cl(V).

Theorem 3.6: Every weakly αg^*s-continuous function is faintly αg^*s-continuous.

Proof: Let x $\in$ X and V ε θ-O(Y, f(x)). Then there exists W $\in$ O(Y) such that f (x) $\in$ W $\subset$ V, that is f (x) $\in$ W $\subset$ cl(W) $\subset$ V . By weakly αg^*s-continuous, there exists U $\in$ αg^*s-O(X) such that f(U) $\subset$ c l(W), that is f(U) $\subset$ cl(W) $\subset$ V . Therefore for each V $\in$ θ-O(Y, f(x)), there exists U $\in$ αg^*s-O(X, x) such that f(U) $\subset$ V. Hence f is faintly αg^*s-continuous.

Theorem 3.7: Let f:X $\rightarrow$ Y be faintly αg^*s-continuous and Y is regular space. Thenf is αg^*s-continuous.

Proof: Let V $\in$ O(Y). As Y is regular, V $\in$ θ-O(Y). Since f is faintly αg^*s-continuous and from Theorem 3.6, $f^{-1}(V)$ $\in$ αg^*s-O(X). Therefore, for every V $\in$ O(Y), $f^{-1}(V)$ $\in \alpha g^*s$-O(X). Thus f is αg^*s-continuous.

Theorem 3.8: Every faintly αg^*s-continuous function is slightly αg^*s-continuous.

Proof: Let x $\in$ X and V be clopen set in Y containing f(x). Then V$\in$ θ-O(Y). Since fis faintly αg^*s -continuous, there exists U $\in$ αg^*s-O(X, x) such that f(U) $\subset$ V. Hence for everyV $\in$ θ-O(Y), f(U) $\subset$V. Therefore, f is slightly αg^*s-continuous.

Definition 3.9: Let X be topological space. Since the intersection of two clopen sets of X is clopen, the clopen sets of X may be use as a base for a topology for X. This topology is called the ultra-regularization of τ [5] and is denoted by τ_u.

A topological space X is said to be ultra-regular if $\tau = \tau_u$.

Theorem 3.10: The following statements are equivalent for a function f: X $\rightarrow$ Y, if Yis ultra-regular space:

(i) f is αg^*s-continuous.

(ii) f is faintly αg^*s-continuous.

(iii) f is slightly αg^*s-continuous.

Proof: It follows from the Theorems 3.2, 3.8 and Definition 3.9.

Definition 3.11: A αg^*s-frontier of a subset A of a space X is defined as αg^*s-Fr(A) = αg^*s-cl(A) $\cap$ αg^*s-cl(X-A).

Theorem 3.12: The set of all points x $\in$ X in which a function f: X $\rightarrow$ Y is not faintly αg^*s-continuous is the union of αg^*s-frontier of the inverse images of θ-open set containing f(x).

Proof: Suppose f is not faintly αg^*s-continuous at each point x $\in$ X. Then there exists V $\in$ θ-O(Y, f(x)) such that f(U) is not contained in V and hence x $\in$ θ-cl(X−$f^{-1}(V)$).

On the other hand, let $x \in f^{-1}(V) \subset \alpha g^*s\text{-cl}(f^{-1}(V))$ and hence $x \in \alpha g^*s\text{-cl}(f^{-1}(V))$. Therefore, we can observe that $x \in \alpha g^*s\text{-fr}(f^{-1}(V))$.

Conversely, assume that f is faintly αg^*s-continuous at each point $x \in X$ and $V \in \theta\text{-O}(Y, f(x))$. Then, there exists $U \in \alpha g^*s\text{-O}(X, x)$ such that $U \subset f^{-1}(V)$. Hence $x \in \alpha g^*s\text{-int}(f^{-1}(V))$. Therefore $x \notin \alpha g^*s\text{-fr}(f^{-1}(V))$.

Theorem 3.13: Let f: $X \to Y$ be a function and $g : (X, \tau) \to (X \times Y, \tau \times \sigma)$ the graph of f defined by $g(x) = (x, f(x))$ for every $x \in X$. If g is faintly αg^*s-continuous then f is faintly αg^*s-continuous.

Proof: Let $U \in \theta\text{-O}(Y)$, then $X \times U \in \theta\text{-O}(X \times Y)$. It follows that $f^{-1}(U) = g^{-1}(X \times U) \in \alpha g^*s\text{-O}(X, x)$. Hence f is faintly αg^*s-continuous.

Theorem 3.14: Faintly αg^*s-continuous image of a αg^*s-connected space is connected.

Proof: Assume that Y is not connected. Then there exist two non empty open sets V_1 and V_2 such that $V_1 \cap V_2 = \varphi$ and $V_1 \cup V_2 = Y$. Hence $f^{-1}(V_1) \cap f^{-1}(V_2) = \varphi$ and $f^{-1}(V_1) \cup f^{-1}(V_2) = X$. As f is surjective, $f^{-1}(V_1)$, $f^{-1}(V_2)$ are non empty subsets of X. Then $V_1, V_2 \in \theta\text{-O}(X)$, since V_1 and V_2 are both open and closed. As f is faintly αg^*s-continuous, $f^{-1}(V_1), f^{-1}(V_2) \in \alpha g^*s\text{-O}(X)$ and hence X isnot αg^*s-connected which is contradiction to the assumption. Hence Y is connected.

Theorem 3.15: If f: $X \to Y$ is faintly αg^*s-continuous surjective and X is αg^*s- compact then Y is θ-compact.

Proof: Let f be faintly αg^*s-continuous surjective function. Let $\{G_\alpha : \alpha \in \lambda\}$ be any θ-open cover of Y. Since f is faintly αg^*s-continuous, $f^{-1}(G_\alpha)$ is αg^*s-open cover of X. Then there exists a finite subcover $\{f^{-1}(G_i) : i = 1, 2, 3, ...\}$ in X, that is $\{G_i : i = 1, 2, 3..\}$ is a subfamily which covers the space Y. Thus Y is θ-compact.

Definition 3.16 [8]: A topological space X is said to be

(i) $\alpha g^*s\text{-}T_1$ space if for each pair of distinct points x, y in X, there exist pair of αg^*s-opensets, one containing x but not y and the other containing y but not x.

(ii) $\alpha g^*s\text{-}T_2$ space if for each pair of distinct points x, y of X, there exist disjoint αg^*s-open sets U and V such that $x \in U$ and $y \in V$.

Theorem 3.17: Let f: $X \to Y$ be faintly αg^*s-continuous, injective function. If

(i) Y is $\theta\text{-}T_1$ then X is $\alpha g^*s\text{-}T_1$.

(ii) Y is $\theta\text{-}T_2$ then X is $\alpha g^*s\text{-}T_2$.

Proof: (i) Let Y be $\theta\text{-}T_1$. Then for any $x_1, x_2 \in X$ with $x_1 \cap x_2 = \varphi$, there exists $V_1, V_2 \in \theta\text{-O}(Y)$ such that $f(x_1) \in V_1$, $f(x_2) \notin V_1$ and $f(x_1) \notin V_1$, $f(x_2) \in V_2$. Then $f^{-1}(V_1)$, $f^{-}1(V_2) \in \alpha g^*s\text{-O}(X)$ as f is faintly αg^*s-continuous such that $x_1 \in f^{-1}(V_1)$, $x_1 \notin f^{-1}(V_2)$ and $x_2 \notin f^{-1}(V_1)$, $x_2 \in f^{-1}(V_2)$ implies that X is $\alpha g^*s\text{-}T_1$.

(ii) Let Y be $\theta\text{-}T_2$. Then for any $x_1, x_2 \in X$, there exist $V_1, V_2 \in \theta\text{-O}(Y)$ such that $f(x_1) \in V_1$ and $f(x_2) \in V_2$. Then $f^{-1}(V_1), f^{-1}(V_2) \in \alpha g^*s\text{-O}(X)$ containing x_1 and x_2 respectively such that $f^{-1}(V_1) \cap f^{-1}(V_2) = \varphi$ as $V_1 \cap V_2 = \varphi$. Thus X is $\alpha g^*s\text{-}T_2$.

References

[1] S. Jafari an T. Noiri, On faintly α-continuous functions, Indian Jl. Math., 42,(2000), 203-210.

[2] N. Levine, Semi-open Sets and Semi Continuity in Topological Spaces, Amer. Math.Monthly, 70(1963), 36-41.

[3] N. Levine, Generalized closed sets in Topology, Rend. Circ. Mat. Palerno,,19(1970), 89-96.

[4] P. E. Long and L. L. Herrington, The T_θ-topology and faintly continuous functions, Kyungpook Math. Jl. 22, (1982), 7-14.

[5] O. Njasted, On some classes of nearly open sets, Pacific Jl. Math., 15, (1965),961-970.

[6] T. Noiri and V. Popa, Weak forms of faint continuity, Bull. Math. Soc. Sci. Math.Roumanie, 34 (82), (1990), 263-270.

[7] P. G. Patil and T. D. Rayanagoudar, Generalization of New Closed Sets in Topological Spaces, Int. Jl. of Advances in Science and Technology, Vol. 5 No. 2, (2012), 68-78.

[8] T.D.Rayanagoudar, On Some Recent Topics in Topology, Ph.D Thesis, Karnatak University, Dharwad, (2007).