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RESEARCH ARTICLE

A STUDY ON NEUTRAL FRACTIONAL DIFFERENTIAL SYSTEMS WITH NONLOCAL CONDITION

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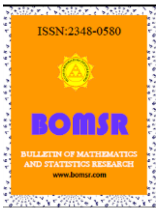

ABSTRACT

This paper is fundamentally involved with the existence of mild solutions for a Neutral Fractional Differential Systems (Abbreviated - NFDS) with Non-Local Conditions (Abbre- viated - NLC) in Banach space $\mathbb{X}$. The main objective of this paper is to investigate the existence theory for a variety of fractional differential equations with applications. We also discuss some definitions, notations about sectorial operators, solution operator, preliminary facts, existence theorems and their results.

Mathematics subject classification : 26A33, 34K40.

Keywords : Fractional calculus, Caputo Derivative, Non-Local Conditions, Banach Contrac- tion Principle.

1 Introduction

Fractional differential equations uses in various fields such as physics, chemistry, mechanics, electricity, biology, economics, control theory, signal and image processing, biophysics, blood flow phenomena, aerodynamics, fitting of experimental data, etc. The fractional differential equation becomes a huge field of mathematics and the possibility of fractional differential equation has been implemented in so many areas of sciences. Now a days the concept of fractional differential equations are powerfully tested in so many various ways. The concept of Fractional derivative appeared for the first time in a famous correspondence between G.A. deL'Hospital and G.W. Leibniz, in 1695. Many mathematicians have further developed this area and we can mention the studies of L. Euler(1730), J.L. Lagrange (1772), P.S. Laplace (1812), J.B.J. Fourier (1822), N.H. Abel (1823), J. Liouville (1832), B.

Riemann (1847), H.L. greer (1859), H. Holmgreen (1865), A.K. Grunwald (1867), A.V. Letnikov (1868), N. Ya. sonin (1869), H. Laurent (1884), P.A. Nekrassov (1888), A.Krug (1890), J. Hadamard (1892), o. Heaviside (1892), S. Pincherle (1902), G.H. Hardy and J.E. Littlewood (1917), H. Weyl (1919), P. Levy (1923), A. marchaud (1927), H.T. Davis (1924), A.Zygmund (1935), E.R. Love (1938), H.Kober (1940), D.V. Widder (1941), M.Riesz (1949) and W. Feller (1952).

In the last decade, fractional calculus has been recognized as one of the best tools to de- scribe long-memory processes. Such models are interesting for engineers and physicists but also for pure mathematicians. The most important among such models are those described by dif- ferential equations containing fractional-order derivatives. Their evolutions behave in a much more complex way than in the classical integer-order case and the study of the corresponding theory is a hugely demanding task. Although some results of qualitative analysis for fractional differential equations can be similarly obtained, many classical methods are hardly applicable directly to fractional differential equations. New theories and methods are thus required to be specifically developed, whose investigation becomes more challenging. Comparing with classical theory of differential equuations, the researches on the theory of fractional differential equations are only on their initial stage of development.

Fractional differential equations received a great attention as these equations as found to be the excellent importance to model the physical concepts and their problems experiencing sudden changes at various instants. The starting stage of this paper is the works in papers.

The starting stage of this paper is the works in papers [5–9]. Some authors in [5] studied the existence of solutions for the problem

$$\frac{d}{dt}[x(t) + F(t, x(t), x(b_1(t)), ..., (b_m(t)))] + Ax(t) = G(t, x(t), x(a_1(t)), ..., x(a_n(t))), \qquad 0 \le t \le a,$$

$$x(0) + g(x) = x_0,$$

by utilizing the fractional powers of operators and Sadovskii fixed point theorem. And in [4], authors also analyzed the subsequent neutral partial partial differential equations of the form

$$\frac{d}{dt}x(t) - F(t, x(t), x(h_1(t))] = -A[x(t) - F(t, x(h_1(t)))] + G(t, x(h_2(t))), t \in j \quad (1.1)$$

$$x(0) + g(x) = x_0 \in X \quad (1.2)$$

by applying the fractional powers of operators and Banach contraction fixed point theorem. Lately, Alka Chadha et al. [3] extends the problem (1.1) - (1.2) into fractional order problem under suitable fixed point theorems. Inspired by the above mentioned works [3–5] the fundamental motivation behind this paper is to demonstrate the existence of mild solutions for the accompanying NFDS in a Banach space $\mathbb{X}$:

$$^{c}\mathcal{D}_t^{\eta}[z(t) - \mathcal{U}(t, z(t), z(t), z(\mu_1(t)), ..., z(\mu_m(t)))] = \mathcal{Q}[z(t) - \mathcal{U}(t, z(t), z(\mu_1(t)),,$$

$$z(\mu_m(t)))] + \mathcal{V}(t, z(t), z(\tilde{\mu_1}(t)), ..., z(\tilde{\mu_n}(t))) + \mathcal{W}(t, z(t), z(\hat{\mu_1}(t)), ..., z(\hat{\mu_q}(t))), t \neq t_k, \quad (1.3)$$

$$t \in J = [0, T] \quad (1.4)$$

$$z(0) = g(z) + z_0 \in \mathbb{X} \quad (1.5)$$

where ${}^{c}\mathcal{D}_t^{\eta}$ is the Caputo fractional derivative of order 0 < η < 1 and $\mathcal{Q} : \mathcal{D}(\mathcal{Q}) \subset \mathbb{X} \to \mathbb{X}$ is a closed linear operator with dense domain $\mathcal{D}(\mathcal{Q})$ in a Banach space $\mathbb{X}$. The functions $\mathcal{U}, \mathcal{V}, \mathcal{W}$, and g are apposite continuous functions to be specified later.

2. Preliminaries

In this section, we discuss some definitions and notations about sectorial operators, solution operator and analytic solution operators required for establishing our results. Throughout this paper, X is a complex banach space equipped with the norm $\|.\|_{\mathbb{X}}$. The symbol C(J; $\mathbb{X}$) stands for the Banach space of all continuous functions from J into X with supremum norm, i.e.,

$$\| y \|_J = \sup_{t \in j} \| y(t) \| .$$

The notation $L(\mathbb{X},Y)$ denotes the Banach spaces of all bounded linear operators from $\mathbb{X}$ into Y with the operator norm denoted by $\|.\|_{L(\mathbb{X},Y)}$ and when $\mathbb{X} = Y$ then we write simply $L(\mathbb{X})$ and $\|.\|_{L(\mathbb{X},Y)}$. In addition, $\mathcal{PC}(J, \mathbb{X})$ represents the Banach space of all the piecewise continuous functions from J into $\mathbb{X}$ with the norm

$$\| u \|_{\mathcal{PC}} = max\{\sup_{t \in j} \| u(t + 0 \|_{\mathbb{X}}, \quad \sup_{t \in j} \| u(t - 0 \|_{\mathbb{X}}\},$$

and $B_r(z, \mathbb{X})$ denotes a closed ball with center z and radius r in $\mathbb{X}$.

Definition 2.1 [10] The Riemann-Liouville fractional integral operator J of order η > 0 is defined by

$$\mathcal{J}_t^{\eta}\mathcal{U}(t) = \frac{1}{\Gamma(\eta)}\int_0^t (t - s)^{\eta-1} \mathcal{U}(s)ds,$$

Where $\mathcal{U} \in L^1\big((0,T); \mathbb{X}\big)$.

Definition 2.2 [10] The Riemann-Liouville fractional derivative is given by,

$\mathcal{D}_t^{\eta}\, \mathcal{U}(t) = \mathcal{D}_t^{m}\mathcal{J}_t^{m-n}\, \mathcal{U}(t),$ $\quad m - 1 < \eta < m, \; m \in \mathbb{N},$

where $\mathcal{D}_t^{m} = \frac{d^m}{dt^m}$, $\mathcal{U} \in L^1((0, T); \mathbb{X})$, $\mathcal{J}_t^{m-n}\, \mathcal{U} \in W^{m,1}((0, T); \mathbb{X})$. Here the notation $W^{m,1}((0, T); \mathbb{X})$ stand for the Sobolev space defined by

$W^{m,1}((0, T); \mathbb{X}) = \{y \in \mathbb{X} : \exists_z \in L^1((0, T); \mathbb{X}) : y(t) \sum_{k=0}^{m-1} d_k \frac{t^k}{k!} + \frac{t^{m-1}}{m-1} * z(t), t \in (0,T)\}$

Note that $z^{(t)} = y^m(t)$, $d_k = y^k(0)$.

Definition 2.3 [10] The Caputo fractional derivative is given by

$${}^{c}\mathcal{D}_t^{\eta}\, \mathcal{U}(t) = \frac{1}{\Gamma(m-n)} \int_0^t (t - s)^{m-\eta-1}\, \mathcal{U}^m(s)ds, m - 1 < \eta < m$$

where $U \in C^{m-1}((0, T); \mathbb{X}) \cap L^1((0, T); \mathbb{X})$ and the following holds,

$$\mathcal{J}_t^{\eta}\left({}^{c}\mathcal{D}_t^{\eta}\,\mathcal{U}(t)\right) = \mathcal{U}(t) - \sum_{k=0}^{m-1} d_k \frac{t^k}{k!}\mathcal{U}^k(0)$$

The laplace transformation of the Caputo derivative of order $\eta > 0$ is given by

$L[{}^{c}\mathcal{D}_t^{\eta}\,\mathcal{U}(t); \lambda] = \lambda^{n}L[u(t)] - \sum_{k=0}^{m-1}\lambda^{\eta-k-1}\mathcal{U}^k(0)$ $\quad m - 1 < \eta < m.$

Definition 2.4 [7] An operator $\mathcal{Q}$, which is closed and linear, is called sectorial operator if there are constants $\omega \in \mathbb{R}, \theta \in [\frac{\pi}{2}, \pi], M > 0$ such that the following two conditions are satisfied:

(1) $\rho(\mathcal{Q}) \supset \Sigma_{(\theta,\omega)} = \{\lambda \in C : \lambda \neq \omega, |\arg(\lambda - \omega)| < \theta\}$

(2) $\| R(\lambda, \mathcal{Q}) \|_{L(\mathbb{X})} \leq \dfrac{M}{|\lambda - \omega|}, \Sigma_{(\theta,\omega)}$

where $\rho(\mathcal{Q})$ be the resolvent set of $\mathcal{Q}$.

For more details we refer to [1]. Consider the following Cauchy problem for the fractional evolution equation

$${}^{c}\mathcal{D}_t^{\eta}\, u(t) = \mathcal{Q}\, u(t), t > 0; u(0) = z, u^k(0) = 0, k = 1, \ldots\ldots, m - 1, \tag{2.1}$$

where $\eta > 0$ and $m = [\eta] + 1$.

Definition 2.5 [1] A solution operator $S_\eta(t)$ of (2.1) is said to be analytic if $S_\eta(t)$ admits an analytic extension to a sector$\Sigma_{\theta_0}$for some $\theta_0 \in (0, \pi/2]$.

An analytic solution operator $S_\eta(t)$ is said to be of analyticity type (θ_0, ω_0) if for each $\theta < \theta_0$, and $\omega > \omega_0$ there exists a positive constant $M = M(\theta, \omega)$ such that $\|S_\eta(t)\| \leq Me^{\omega Ret}$, for $t \in \Sigma_\theta = \{t \in C/\{0\} : arg\ t < \theta\}$. Denote $\mathcal{A}^{\eta}$ $(\theta_0, \omega_0) = \{\mathcal{A} \in \mathcal{C}^{\eta}$; $\mathcal{A}$ generates analytic solution operator $S_\eta(t)$ of type $(\theta_0, \omega_0)\}$

Lemma 2.1 [1] $Let\ \eta \in (0, 2)$. A linear closed densely defined operator $\mathcal{Q} \in \mathcal{A}^{\eta}$ (θ_0, ω_0) if and only if $\lambda^{\eta} \in \rho(\mathcal{Q})$ for each $\lambda \in \Sigma_{\theta_0} + \frac{\pi}{2}(\omega_0)$, and for any $\omega > \omega_0$, $\theta < \theta_0$, there exists a constant $C = C(\theta, \omega)$ such that

$\| \lambda^{\eta-1}R(\lambda_n, Q)\| \leq \frac{C}{|\lambda-\omega|}, \lambda \in \Sigma_{\theta+\frac{\pi}{2}}(\omega)$. ,

Before we define the mild solution for the given problem, initially, we present the following theorem.

Theorem 2.1 Suppose $\mathcal{Q}$ is a sectorial operator and f satisfies the uniform Holder condition with exponent $\beta \in (0, 1]$ then

$$u(t) = S_\eta(t)z_0 + \int_0^t T_\eta(t - s)f(s)ds, \qquad t \in [0, T] \tag{2.2}$$

where

$$S_\eta(t) = \frac{1}{2\pi i}\int_{\Gamma} e^{\lambda t}\lambda^{\eta-1}R(\lambda^{n}, \mathcal{Q})d\lambda, \qquad T_\eta(t) = \frac{1}{2\pi i}\int_{\Gamma} e^{\lambda t}\lambda^{\eta-1}R(\lambda^{n}, \mathcal{Q})d\lambda, \tag{2.3}$$

is the mild solution for fractional Cauchy problem

$$ {}^{c}\mathcal{D}_t^{\eta}\, u(t) = \mathcal{Q}\, u(t) + f(t), \qquad 0 < \eta < 1, \qquad t \in [0, T], \tag{2.4} $$

$$ u(0) = z_0 \in \mathbb{X}. \tag{2.5} $$

where Γ is a suitable path lying on $\Sigma_{\theta\omega}$. For o < η < 1, $T_\eta(t)$ is the η - resolvent family and $S_\eta(t)$ is the solution operator, generated by $\mathcal{Q}$.

If η ∈ (0, 1) and $\mathcal{Q} \in \mathcal{A}^{\eta}$ (θ_0, ω_0), then for any $z \in \mathbb{X}$ and t > 0, we have $S_\eta(t)z \in \mathcal{D}(\mathcal{Q})$ and

$\| S\eta(t) \|_{L(\mathbb{X})} \leq Me^{\omega t}, \| T_\eta(t) \|_{L(\mathbb{X})} \leq Ce^{\omega t}(1 + t^{\eta-1}), \quad t > 0, \quad \omega > \omega_0$

Let

$$ \widetilde{M}_S = \sup_{o \leq t \leq T} \| S\eta(t) \|_{L(\mathbb{X})}, \widetilde{M}_T = \sup_{o \leq t \leq T} Ce^{\omega t}(1 + t^{\eta-1}). $$

Thus we have

$\| S_\eta(t) \|_{L(\mathbb{X})} \leq \widetilde{M}_S$, $T_\eta(t) \|_{L(\mathbb{X})} \leq t^{\eta-1} \widetilde{M}_T$ .ow, we list the following basic assumptions on the functions which will be utilized later to establish main results.

(H1) For 0 < β < 1, the function $\mathcal{Q}^{\beta}\,\mathcal{U} : J \times \mathbb{X}^{m+1} \to \mathbb{X}$ is continuous function satisfy the Lipschitz condition, that is, there exists a constant $L_{\mathcal{U}} > 0$ such that,

$$ \left\|\mathcal{Q}^{\beta}\mathcal{U}(s_1, u_0, u_1, \dots, u_m) - \mathcal{Q}^{\beta}\mathcal{U}\,(s_2, \bar{u}_0, \bar{u}_1, \dots, \bar{u}_m)\right\| \leq L_{\mathcal{U}}(|s_1 - s_2| + \max_{i=0,1,2\dots m} \|\bar{u}_i, \bar{u}_i\|_{\mathbb{X}}) $$

For any 0 ≤ s_1, s_2 ≤ T, u_i, $\bar{u}_i \in \mathbb{X}$, i = 0, 1, ...m. Moreover, there exist constants $\tilde{L}_{\mathcal{U}} > 0$ such that the inequality

$$ \left\|\mathcal{Q}^{\beta}\mathcal{U}(t, u_0, u_1, \dots, u_m)\right\| \leq \tilde{L}_{\mathcal{U}}(\max\{\|u_i\|_{\mathbb{X}} : i = 0, i, \dots, m\} + 1) $$

holds for any $(t, u_0, u_1, \dots, u_m) \in J \times \mathbb{X}^{m+1}$.

(H2) The function $J \times \mathbb{X}^{n+1}$.→ $\mathbb{X}$ is continuous function, and there exist a constant $L_{\mathcal{V}} > 0$ such that the function satisfies the Lipschitz condition:

$$ \|\mathcal{V}\,(s_1, v_0, v_1, \dots, v_n) - \mathcal{V}\,(s_2, \bar{v}_0, \bar{v}_1, \dots, \bar{v}_n)\| \leq L_{\mathcal{V}}\,(|s_1 - s_2| + \max_{i=0,1,2\dots m} \|\bar{v}_i, \bar{v}_i\|_{\mathbb{X}}) $$

for any 0 ≤ s_1, s_2 ≤ T, v_i, $\bar{v}_i \in \mathbb{X}$, i = 0, 1, 2....n. Moreover there exist a constant $\tilde{L}_{\mathcal{V}} > 0$ such that the inequality

$$ \|\mathcal{V}\,(t, v_0, v_1, \dots, v_n)\| \leq \tilde{L}_{\mathcal{V}}(\max\{\|v_i\|_{\mathbb{X}} : i = 0,1, \dots, n\} + 1) $$

holds for any $(t, v_0, v_1, \dots, v_n) \in J \times \mathbb{X}^{n+1}$

(H3) The function $\mathcal{W} : J \times \mathbb{X}^{q+1} \to \mathbb{X}$is continuous function, and there exist a constant $L_{\mathcal{W}} > 0$ such that the function satisfies the Lipschitz condition:

$$ \left\|\mathcal{W}\,(s_1, w_0, w_1, \dots, w_q) - \mathcal{W}(s_2, \bar{w}_0, w_1, \dots, \bar{w}_q)\right\| \leq L_{\mathcal{W}}\,(|s_1 - s_2| + \max_{i=0,1,2\dots m} \|\bar{w}_i, \bar{w}_i\|_{\mathbb{X}}) $$

for any $0 \le s_1, s_2 \le T$, w_i, $\overline{w}_i \in \mathbb{X}$, i = 0, 1, 2....q. Moreover there exist a constant $\tilde{L}_{\mathcal{W}} > 0$ such that the inequality

$$\left\|\mathcal{W}\left(t, w_0, w_1, \ldots, w_q\right)\right\| \le \tilde{L}_{\mathcal{V}}(\max\{\|w_i\|_{\mathbb{X}}: i = 0,1,\ldots,q\} + 1)$$

holds for any $(t, w_0, w_1, \ldots, w_n) \in J \times \mathbb{X}^{n+1}$

(H4) The function g : $\mathbb{X} \to \mathbb{X}$ is continuous and there exist constants $L_g, \tilde{L}_g$ and $L_{g1} > 0$ such that

$$\| g(y) - g(\bar{y}) \| \le L_g \| y - \bar{y} \| \ \mathbb{X}, \qquad y, \bar{y}, \in \mathbb{X}$$

and

$$\| g(y) \| \le \tilde{L}_g \| y \| + L_{g1}, \qquad y \in \mathbb{X}.$$

The second inequality can also written as,

$$\| g(y) - g(0) \| + \| g(0) \| \le L_g \| y \| + L_{g1}, \quad \text{with } \tilde{L}_g = Lg \text{ and } \quad L_{g1} = \| g(0) \|.$$

(H5) $\tilde{\mu}_i, \mu_j, \tilde{\mu}_l \in C(J,J), i = 1,2,\ldots n, j = 1,2,\ldots m, l = 1,2,\ldots q.$ Now we are in a position to define the mild solution for the system (1.3).

Definition 2.6: A function $z \in \mathcal{PC}(J, \mathbb{X})$ is said to be a mild solution of the system (1.3),

if (i) $z(0) = g(z) + z_0$;

$z(t) = S_\eta(t)[z_0 + g(z) - U(0, z(0), z(\mu_1(0)), \ldots, z(\mu_m(0)))] + U(t, z(t), z(\mu_1(t)),\ldots, z(\mu_m(t))) \int_0^t T_\eta(t - s)[\mathcal{V}(s, z(s), z(\tilde{\mu}_1(s)), \ldots . z(\tilde{\mu}_n(s))) + \mathcal{W}(s, z(s), z(\tilde{\mu}_1(s)), \quad \ldots z\left(\tilde{\mu}_q(s)\right))]ds$ is also satisfied.

3 Existence results

In this section, we present and prove our main results. Our result is based on Banach contraction principle.

Theorem 3.1

Let (H1) - (H5) holds and

$$\Lambda = [\widetilde{M_s}\{ Lg + p\, L_1\} + L_{\mathcal{U}} \left\|\mathcal{Q}^{-\beta}\right\| \{1 + \widetilde{M_s}\} + \widetilde{M_T} \frac{T^\eta}{\eta}\{L_{\mathcal{V}} + L_{\mathcal{W}}\}] < 1, \qquad (3.1)$$

Proof

Let $z_0 \in \mathbb{X}$ be fixed. For the sake of brevity, we rewrite that

$(t, z(t), z(\mu_1(t)),\ldots, z(\mu_m(t))) = (t, u(t))$

$(t, z(t), z(\tilde{\mu}_1(t)),\ldots, z(\tilde{\mu}_n(t)))) = (t, v(t))$

and

(t, z(t), z(μ̃1(t)),..., z(μ̃q(t)))) = (t, w(t)).

Define the mapping $\Gamma : \mathcal{PC}(J,\mathbb{X}) \rightarrow \mathcal{PC}(J,\mathbb{X})$ such that

$(\Gamma_z)(t) = S_\eta(t)[z0 + g(z) - \mathcal{U}(0, u(0))] + \mathcal{U}(t, u(t)) + \int_0^t t\, T_\eta(t - s)[\mathcal{V}(s, v(s)) + \mathcal{W}(s, w(s))]ds$ for each $t \in J$. Since $\mathcal{U}$, $\mathcal{V}$ and $\mathcal{W}$ are continuous functions and $S\eta(t)$, $t \geq 0$ and $T\eta(t)$, $t \geq 0$ are compact, therefore it is easy to show that the map Γ is well defined on $\mathcal{PC}(J,\mathbb{X})$.To build the result, it is enough to show that the mapping Γ is a contraction mapping on $\mathcal{PC}(J,\mathbb{X})$. For the better way, we break the proof into the following steps:

(i)

$$\|S_\eta(t)[g(z) - g(\tilde{z})]\| \leq \|S_\eta(t)\|\ \|g(z) - g(\tilde{z})\|$$

$$\leq \widetilde{M}_S\, Lg\, \|z - \tilde{z}\|_{\mathbb{X}}.$$

(ii)

$$\|S_\eta(t)[\mathcal{U}(0,u(z)) - \mathcal{U}(0, \bar{u}(0)]\| \leq \|S_\eta(t)\|\ \left\|\mathcal{Q}^{-\beta}\right\|\left\|\mathcal{Q}^{\beta}\mathcal{U}\big(0,u,(0)\big) - \mathcal{Q}^{\beta}\mathcal{U}\big(0,\bar{u},(0)\big)\right\|$$

$$\leq \widetilde{M}_S\, L_{\mathcal{U}} \left\|\mathcal{Q}^{-\beta}\right\| \sup_{0\leq t\leq T} \|z(s) - \bar{z}(s)\|.$$

$$\leq \widetilde{M}_S\, L_{\mathcal{U}} \left\|\mathcal{Q}^{-\beta}\right\| \|\mathrm{z} - \tilde{z}\|_{\mathbb{X}}.$$

(iii)

$$\left\|\mathcal{U}\big(t,u(t)\big) - \mathcal{U}(t,\bar{u}(t))\right\| = \left\|\mathcal{Q}^{-\beta}\right\|\left\|\mathcal{Q}^{\beta}\mathcal{U}\big(t,u(t)\big) - \mathcal{Q}^{\beta}\mathcal{U}\big(t,\bar{u},(t)\big)\right\|$$

$$\leq L_{\mathcal{U}} \left\|\mathcal{Q}^{-\beta}\right\| \|\mathrm{z} - \tilde{z}\|_{\mathbb{X}}$$

(iv)

$$\left\|\int_0^t T_\eta\,(t-s) - \mathcal{V}(s,\bar{v}(s))]ds\right\| \leq \int_0^t \|T_\eta(t-s)\|\left\|\mathcal{V}\big(s,v(s)\big) - \mathcal{V}(s,\bar{v}(s))\right\| ds$$

$$\leq \widetilde{M}_T L_{\mathcal{V}} \int_0^t (t-s)^{\eta-1} \sup_{o\leq t\leq T} \|(z(s) - \bar{z}(s)\|ds$$

$$\leq \widetilde{M}_T L_{\mathcal{V}} \frac{T^\eta}{\eta} \|\mathrm{z} - \tilde{z}\|_{\mathbb{X}}$$

In similar manner, we have

$$\left\|\int_0^t T_\eta\,(t-s) - \mathcal{W}\big(s,w(s)\big) - \mathcal{W}\big(s,\overline{w}(s)\big)]ds\right\| \leq \int_0^t \|T_\eta(t-s)\|\left\|\mathcal{W}\big(s,w(s)\big) - \mathcal{W}(s,\overline{w}(s))\right\| ds$$

$$\leq \widetilde{M}_T L_{\mathcal{W}} \frac{T^\eta}{\eta} \|\mathrm{z} - \tilde{z}\|_{\mathbb{X}}$$

From the above results, we have

$$\|\Gamma z\,(t) - \Gamma\bar{z}(t)\| \leq [\widetilde{M}_s\{L_g\} + L_u\left\|\mathcal{Q}^{-\beta}\right\|\|\{1 + \widetilde{M}_s\} + \widetilde{M}_T\| \frac{T^\eta}{\eta}\{L_{\mathcal{V}} + L_{\mathcal{W}}\}] \sup_{o\leq t\leq T} \|(z(s) - \bar{z}(s)\|$$

Thus

$$\| (\Gamma z) - (\Gamma \bar{z}) \| \leq \Lambda \| z - \bar{z} \|_{\mathcal{PC}} .$$

Since Λ < 1 by the equation (3.1), it indicates that the map Γ is contraction on : $\mathcal{PC}(J, \mathbb{X})$. Hence, by Banach contraction principle, there exists a unique fixed point z ∈ : $\mathcal{PC}(J, \mathbb{X})$such that Γz(t) = z(t) which is a mild solution of the problem (1.3). Now, the proof of the theorem is completed.

4 Conclusion

In this manuscript, we have analyzed the mild solution of neutral fractional differential systems with nonlocal condition in a Banach space $\mathbb{X}$ with sectorial operator Q.

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