Vol.11.Issue.2.2023 (April-June)

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

http://www.bomsr.com

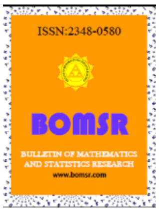
Email:editorbomsr@gmail.com

INTERNATIONAL STANDARD SERIAL NUMBER 2348-0580

RESEARCH ARTICLE

IFsgp* - CONTINUOUS MAPPING AND IRRESOLUTE MAPPING IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

Chandhini. J[1], Uma. N[2]
[1]Research scholar, [2]Asso. Prof. & Head,
PG & Research Department of Mathematics,
Sri Ramakrishna College of Arts and Science, Coimbatore,
Tamil Nadu. (INDIA).
[1]chandhujk@gmail.com [2]uma.n@srcas.ac.in
DOI:10.33329/bomsr.11.2.24



ABSTRACT

In this paper, Continuous Mapping and Irresolute Mapping of Intuitionistic Fuzzy semi generalized pre star are introduced. Here, we have also investigated their relations and properties with other Intuitionistic Fuzzy sets.

Keywords: Intuitionistic fuzzy topological spaces; IFsgp* - closed sets; IFsgp* - open sets; IFsgp* - continuous mapping; IFsgp* - irresolute mapping.

1. INTRODUCTION

The fuzzy concept was introduced by **Zadeh [20]** in 1965. Later **Atanassov [1]** in 1986, presented the Intuitionistic fuzzy sets. In 1997, **Coker D [4]** proposed Intuitionistic fuzzy topological spaces using Intuitionistic fuzzy sets. **Coker** and **Demirci [5]** found the Intuitionistic fuzzy points in 1997. **Gurcay** and **Coker [6]** defined fuzzy continuity in Intuitionistic fuzzy topological spaces in 1997. **Young Bae Jun** and **Seok Zun Song [19]** initiated the Intuitionistic fuzzy semi pre-open sets and the Intuitionistic fuzzy semi-pre-continuous mapping in 2005. **Joen Jun** and **Park [8]** found the Intuitionistic fuzzy alpha continuity and intuitionistic fuzzy pre continuity in 2005. In 2009, **Santhi** and **Jyanthi [11] [12]** defined the concept of Intuitionistic fuzzy generalized semi-continuous mapping. In 2010, **Thakur** and **Jyoti**

pandey Bajpai [17] proposed the Intuitionistic fuzzy w closed sets and Intuitionistic fuzzy w continuity. In 2010 **Shyla Isac Mary** and **Thangavelu [16]** discovered regular pre-semi-closed in topological spaces. **Sakthivel [15]** found the Intuitionistic fuzzy alpha generalized closed set and intuitionistic fuzzy alpha generalized open set in 2012. **Ramesh** and **Thirumalaiswamy [9]** in 2013, instituted the Generalized semi pre continuous and irresolute mapping in Intuitionistic fuzzy topological spaces. In 2016, **Jeyaraman, Ravi** and **Yuvarani [7]** proposed the Intuitionistic fuzzy alpha generalized semi closed set. **Chandhini** and **Uma [3]** instituted an Intuitionistic fuzzy semi-generalized pre-star closed and open set in Intuitionistic fuzzy topological spaces in 2022.

In this study, we have provided new concepts of Intuitionistic fuzzy semi-generalized pre-star continuous mapping and Intuitionistic fuzzy generalized pre-star irresolute mapping in Intuitionistic fuzzy topological spaces with some of their characteristics. Furthermore, we have demonstrated some of IFsgp* of IFTS's features and showed that it is productive.

2. PRELIMINARIES

Definition 2.1 [1]: Let the non-empty fixed set be X. An **Intuitionistic fuzzy set (IFS in short)** A in X is an object having the form $A = \{< x, \mu_A(x), \nu_A(x) > / x \in X\}$ where the functions $\mu_A(x) : X \to [0, 1]$ denote the degree of membership (namely μ_A (x)) and $\nu_A(x) : X \to [0, 1]$ and the degree of non - membership (namely $\nu_A(x)$) of each element $x \in X$ to a set A respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

Definition 2.2 [1]: The IFSs A and B of the form $A = \{< x, \mu_A(x), \nu_A(x) > / x \in X\}$ and $B = \{< x, \mu_B(x), \nu_B(x) > / x \in X\}$. Then

(a) $A \subset B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,

(c) $A^C = \{ < \mu_A(x), \nu_A(x) > / x \in X\}$,

(d) $A \cap B = \{< x, \mu_A(x) \cap \mu_B(x), \nu_A(x) \cup \nu_B(x) > / x \epsilon X\}$,

(e) $A \cup B = \{< x, \mu_A(x) \cup \mu_B(x), \nu_A(x) \cap \nu_B(x) > / x \epsilon X\}$.

We shall use $A = < x, \mu_A, \nu_A >$ instead of $A = \{< x, \mu_A(x), \nu_A(x) > / x \epsilon X\}$ and $A = \{< x, (\mu_A(x)\ \mu_B(x)), (\nu_A(x), \nu_B(x)) >\}$ for $A = \{< x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B)\}$.

And $0_\sim = \{< x, 0, 1 > /\ x \epsilon X\}$ is the empty set and $1_\sim = \{< x, 1, 0 > /\ x \epsilon X\}$ is the whole set of X ,the Intuitionistic fuzzy set.

Definition 2.3 [4]: Let X be a non-empty set; An **Intuitionistic fuzzy topology** (IFT) is a family τ of IFSs in X satisfying the following axioms:

a) $0_{\sim}, 1_{\sim} \in \tau$,

b) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

c) $UG_i \in \tau$ for any arbitrary family $\{G_i | i \in J\} \subseteq \tau$.

The pair (X, τ) is called an **Intuitionistic fuzzy topological space** (IFTS) and any IFS in τ is known as an **Intuitionistic fuzzy open set** (IFOS) in X. The complement A^C of an IFOS A is an IFTS (X, τ) is called an **Intuitionistic fuzzy closed set** (IFCS) in X.

Definition 2.4 [4]: Let (X, τ) be an IFTS and $A = < x, \mu_A, \nu_A >$ be an IFS in X. Then the I**ntuitionistic fuzzy interior** and an **Intuitionistic fuzzy closure** are defined by

(a) $int(A) = \cup \{G/G \; is \; an \; IFOS \; in \; X \; and \; G \subseteq A\}$,

(b) $cl(A) = \cap \{K/K \; is \; an \; IFCS \; in \; X \; and \; A \subseteq K\}$.

Result 2.5 [4]: The two Intuitionistic fuzzy sets A and B of an Intuitionistic fuzzy topological space(X, τ). Then

a) A is an Intuitionistic fuzzy closed set in $X \Leftrightarrow cl(A) = A$,

b) A is an Intuitionistic fuzzy closed set in $X \Leftrightarrow int(A) = A$,

c) $cl(A^C) = (int(A))^C$,

d) $int(A^C) = (cl(A))^C$,

e) $A \subseteq B \Rightarrow int(A) \subseteq int(B)$,

f) $A \subseteq B \Rightarrow cl(A) \subseteq cl(B)$,

g) $cl(A \cup B) = cl(A) \cup cl(B)$,

h) $int(A \cap B) = int(A) \cap int(B)$.

Definition 2.6 [19]: Let (X, τ) be an IFTS and $A = < x, \mu_A, \nu_A >$ be an IFS in X. Then the **semi closure** of A ($scl(A)$) and **semi-interior** of A ($sint(A)$) are defined as

(a) $sint(A) = \cup \{G/G \; is \; an \; IFsos \; in \; X \; and \; G \subseteq A\}$,

(b) $scl(A) = \cap \{K/K \; is \; an \; IFscs \; in \; X \; and \; A \subseteq K\}$.

Result 2.7 [16]: Let A be an IFS in (X, τ), then

(c) $scl(A) = A \cup int(cl(A))$,

(d) $sint(A) = A \cap cl(int(A))$.

Definition 2.8 [15]: Let (X, τ) be an IFTS and $A = < x, \mu_A, \nu_A >$ be an IFS in X. Then the **alpha closure** of A ($\alpha cl(A)$) and **alpha interior** of A ($\alpha int(A)$) are defined as

(a) $\alpha int(A) = \cup \{G/G \; is \; an \; IF\alpha os \; in \; X \; and \; G \subseteq A\}$,

(b) $\alpha cl(A) = \cap \{K/K \; is \; an \; IF\alpha cs \; in \; X \; and \; A \subseteq K\}$.

Result 2.9 [15]: Let A be an IFS in (X, τ), then

(a) $\alpha cl(A) = A \cup cl(int(cl(A)))$,

(b) $\alpha int(A) = A \cap int(cl(int(A)))$.

Definition 2.10 [19]: An IFS A of an IFTS (X, τ) is an

(a) **Intuitionistic fuzzy semi pre-closed set** (IFspcs) if there exists an IFpcs B such that $int(B) \subseteq A \subseteq B$,

(b) **Intuitionistic fuzzy semi pre-open set** (IFspos) if there exists an IFpos B such that $B \subseteq A \subseteq cl(B)$.

Result 2.11 [12]: Let A be an IFS in (X, τ), then

(a) $spint(A) = \cup \{G/G \text{ is an IFspos in } X \text{ and } G \subseteq A\}$,

(b) $spcl(A) = \cap \{K/K \text{ is an IFspcs in } X \text{ and } A \subseteq K\}$.

Note that for any IFS A in (X, τ), we have $spcl(A^C) = (spint(A))^C$ and $spint(A^C) = (spcl(A))^C$.

Definition 2.12: An IFS $A = \{< x, \mu_A(x), \nu_A(x) > / x \in X\}$ in an IFTS (X, τ) is called an

a) An I**ntuitionistic fuzzy pre - open set [4]** if $A \subseteq int(cl(A))$ and an **Intuitionistic fuzzy pre - closed set** if $cl(int(A)) \subseteq A$.

b) An **Intuitionistic fuzzy semi - open set [4]** if $A \subseteq cl(int(A))$ and an **Intuitionistic fuzzy semi - closed set** if $int(cl(A)) \subseteq A$.

c) An **Intuitionistic fuzzy α - open set [4]** if $A \subseteq int(cl(int(A)))$ and an **Intuitionistic fuzzy α - closed set** if $cl(int(cl(A))) \subseteq A$.

d) An **Intuitionistic fuzzy regular - open set [4]** is $int(cl(A)) = A$ and an **Intuitionistic fuzzy regular- closed set** is $cl(int(A)) = A$.

e) An **Intuitionistic fuzzy semi generalized pre - star - closed set [3]** (briefly IFsgp* - closed) if $scl(A) \subseteq U$, whenever $A \subseteq U$ and U is an IFgpos in X.

Definition 2.13: Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an

(a) An **Intuitionistic fuzzy continuous [6]** (IF continuous for short) mapping if $f^{-1} \in IFO(X)$ for every $B \in \sigma$.

(b) An **Intuitionistic fuzzy semi continuous [6]** (IFs continuous for short) mapping if $f^{-1} \in IFSO(X)$ for every $B \in \sigma$.

(c) An **Intuitionistic fuzzy α continuous [8]** (IFα continuous for short) mapping if $f^{-1} \in IF\alpha O(X)$ for every $B \in \sigma$.

(d) An **Intuitionistic fuzzy pre continuous [8]** (IFP continuous for short) mapping if $f^{-1} \in IFPO(X)$ for every $B \in \sigma$.

(e) An **Intuitionistic fuzzy regular continuous [18]** (IFr continuous for short) mapping if $f^{-1} \in IFRO(X)$ for every $B \in \sigma$.

(f) An **Intuitionistic fuzzy weakly continuous [17]** (IFw continuous for short) mapping if $f^{-1} \in IFWO(X)$ for every $B \in \sigma$.

(g) An **Intuitionistic fuzzy generalized semi pre regular continuous [10]** (IFgspr continuous for short) mapping if $f^{-1} \in IFgsprO(X)$ for every $B \in \sigma$.

(h) An **Intuitionistic fuzzy generalized semi pre continuous [13]** (IFgsp continuous for short) mapping if $f^{-1} \in IFgspO(X)$ for every $B \in \sigma$.

(i) An **Intuitionistic fuzzy generalized pre star continuous [2]** (IFgp* continuous for short) mapping if $f^{-1} \in IFgp * O(X)$ for every $B \in \sigma$.

Definition 2.14 [6]: Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be **Intuitionistic fuzzy continuous** (IF continuous in short) if $f^{-1}(A)$ is Intuitionistic fuzzy closed in X for every A of Y.

Definition 2.15 [14]: A function f from a IFTS (X, τ) into an IFts (Y, σ). Then f is said to be **Intuitionistic fuzzy irresolute** (IF irresolute in short) $f^{-1}(A)$ is Intuitionistic fuzzy closed in X for every IFCS B in Y.

Remark 2.16:

1) Every IFr closed set is IFsgp* closed set.
2) Every IFr closed set is IFsgp* closed set.
3) Every IFw closed set is IFsgp* closed set.
4) Every IFα closed set is IFsgp* closed set.
5) Every IFαgs closed set is IFsgp* closed set.
6) Every IFsgp* closed set is IFgspr closed set.
7) Every IFsgp* closed set is IFgsp closed set.

3. IFsgp* - CONTINUOUS MAPPING

Definition 3.1: A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called **IFsgp* continuous mapping** if $f^{-1}(V)$ is IFsgp* closed set in (X, τ) for every IF closed set A of (Y, σ).

Example 3.2: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $T_1 = \{x, (0.4, 0.7), (0.6, 0.3)\}$,

$T_2 = \{y, (0.3, 0.6), (0.7, 0.4)\}$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFsgp* continuous mapping.

Theorem 3.3: *Every IF continuous mapping is an IFsgp* continuous mapping but not conversely*.

Proof: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IF continuous mapping. Let A be an IFcs in Y. Since f is IF continuous mapping, $f^{-1}(A)$ is an IFcs in X. Since every IFcs is an IFsgp*cs, $f^{-1}(A)$ is an IFsgp*cs in X. Hence f is an IFsgp* continuous mapping. Converse need not be true.

Example 3.4: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $T_1 = \{x, (0.2, 0.5), (0.8, 0.5)\}$, $T_2 = \{y, (0.3, 0.5), (0.7, 0.5)\}$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $A = \{y, (0.7, 0.5), (0.3, 0.5)\}$ is an IFcs in Y. Then $f^{-1}(A)$ is not IFcs in X. Thus f is an IFsgp* continuous mapping but not IF continuous mapping.

Theorem 3.5: *Every IFr continuous mapping is IFsgp* continuous mapping but not conversely*.

Proof: Let f : (X, τ) → (Y, σ) be an IFr continuous mapping. Let A be an IFcs in Y. Since f is IFr continuous mapping, $f^{-1}(A)$ is an IFrcs in X. Since every IFrcs is an IFsgp*cs, $f^{-1}(A)$ is an IFsgp*cs in X. Hence f is an IFsgp* continuous mapping. Converse need not be true.

Example 3.6: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $T_1 = \{x, (0.4, 0.6), (0.6, 0.4)\}$, $T_2 = \{y, (0.8, 0.8), (0.2, 0.2)\}$. Then $\tau = \{0_{\sim}, T_1, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, T_2, 1_{\sim}\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $A = \{y, (0.2, 0.2), (0.8, 0.8)\}$ is an IFcs in Y. Then $f^{-1}(A)$ is IFsgp*csin X but not IFrcs in X. Thus f is an IFsgp* continuous mapping but not IFr continuous mapping.

Theorem 3.7: *Every IFα continuous mapping is IFsgp* continuous mapping but not conversely.*

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an IFα continuous mapping. Let A be an IFcs in Y. Since f is IFα continuous mapping, $f^{-1}(A)$ is an IFαcs in X. Since every IFαcs is an IFsgp*cs, $f^{-1}(A)$ is an IFsgp*cs in X. Hence f is an IFsgp* continuous mapping. Converse need not be true.

Example 3.8: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $T_1 = \{x, (0.4, 0.6), (0.6, 0.4)\}$, $T_2 = \{y, (0.8, 0.8), (0.2, 0.2)\}$. Then $\tau = \{0_{\sim}, T_1, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, T_2, 1_{\sim}\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and f$(b) = v$. The IFS $A = \{y, (0.2, 0.2), (0.8, 0.8)\}$ is an IFcs in Y. Then $f^{-1}(A)$ is IFsgp*cs in X but not IFαcs in X. Thus f is an IFsgp* continuous mapping but not IFα continuous mapping.

Theorem 3.9: *Every IFw continuous mapping is an IFsgp* continuous mapping but not conversely*.

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an IFw continuous mapping. Let A be an IFcs in Y. Since f is IFw continuous mapping, $f^{-1}(A)$ is an IFwcs in X. Since every IFwcs is an IFsgp*cs, $f^{-1}(A)$ is an IFsgp*cs in X. Hence f is an IFsgp* continuous mapping. Converse need not be true.

Example 3.10: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $T_1 = \{x, (0.4, 0.7), (0.6, 0.3)\}$, $T_2 = \{y, (0.7, 0.8), (0.3, 0.2)\}$. Then $\tau = \{0_{\sim}, T_1, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, T_2, 1_{\sim}\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $A = \{y, (0.3, 0.2), (0.7, 0.8)\}$ is an IFcs in Y. Then $f^{-1}(A)$ is IFsgp*cs in X but not IFwcs in X. Thus f is an IFsgp* continuous mapping but not IFw continuous mapping.

Theorem 3.11: *Every IFαgs continuous mapping is IFsgp* continuous mapping but not conversely.*

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an IFαgs continuous mapping. Let A be an IFcs in Y. Since f is IFαgs continuous mapping, $f^{-1}(A)$ is an IFαgscs in X. Since every IFαgscs is an IFsgp*cs, $f^{-1}(A)$ is an IFsgp*cs in X. Hence f is an IFsgp* continuous mapping. Converse need not be true.

Example 3.12: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $T_1 = \{x, (0.1, 0.4), (0.9, 0.6)\}$, $T_2 = \{y, (0.3, 0.4), (0.7, 0.6)\}$, $T3 = \{y, (0.5, 0.6), (0.5, 0.4)\}$. Then $\tau = \{0_{\sim}, T_1, T_2, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, T_3, 1_{\sim}\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $A = \{y, (0.5, 0.4), (0.5, 0.6)\}$ is an IFcs in Y. Then $f^{-1}(A)$ is IFsgp*cs in X but not IFαgscs in X. Thus f is an IFsgp* continuous mapping but not IFαgs continuous mapping.

Theorem 3.13: *Every IFsgp* continuous mapping is IFgspr continuous mapping but not conversely.*

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an IFsgp* continuous mapping. Let A be an IFcs in Y. Since f is IFsgp* continuous mapping, $f^{-1}(A)$ is an IFsgp*cs in X. Since every IFsgp*cs is an IFgsprcs, $f^{-1}(A)$ is an IFgsprcs in X. Hence f is an IFgspr continuous mapping. Converse need not be true.

Example 3.14: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $T_1 = \{x, (0.1, 0.5), (0.9, 0.5)\}$,

$T_2 = \{y, (0.9, 0.5), (0.1, 0.5)\}$. Then $\tau = \{0_{\sim}, T_1, 1_{\sim}\}$ and $\sigma = \{0_{\sim}, T_2, 1_{\sim}\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS

$A = \{y, (0.1, 0.5), (0.9, 0.5)\}$ is an IFcs in Y. Then $f^{-1}(A)$ is IFgsprcs in X but not IFsgp*cs in X. Thus f is an IFgspr continuous mapping but not IFsgp* continuous mapping.

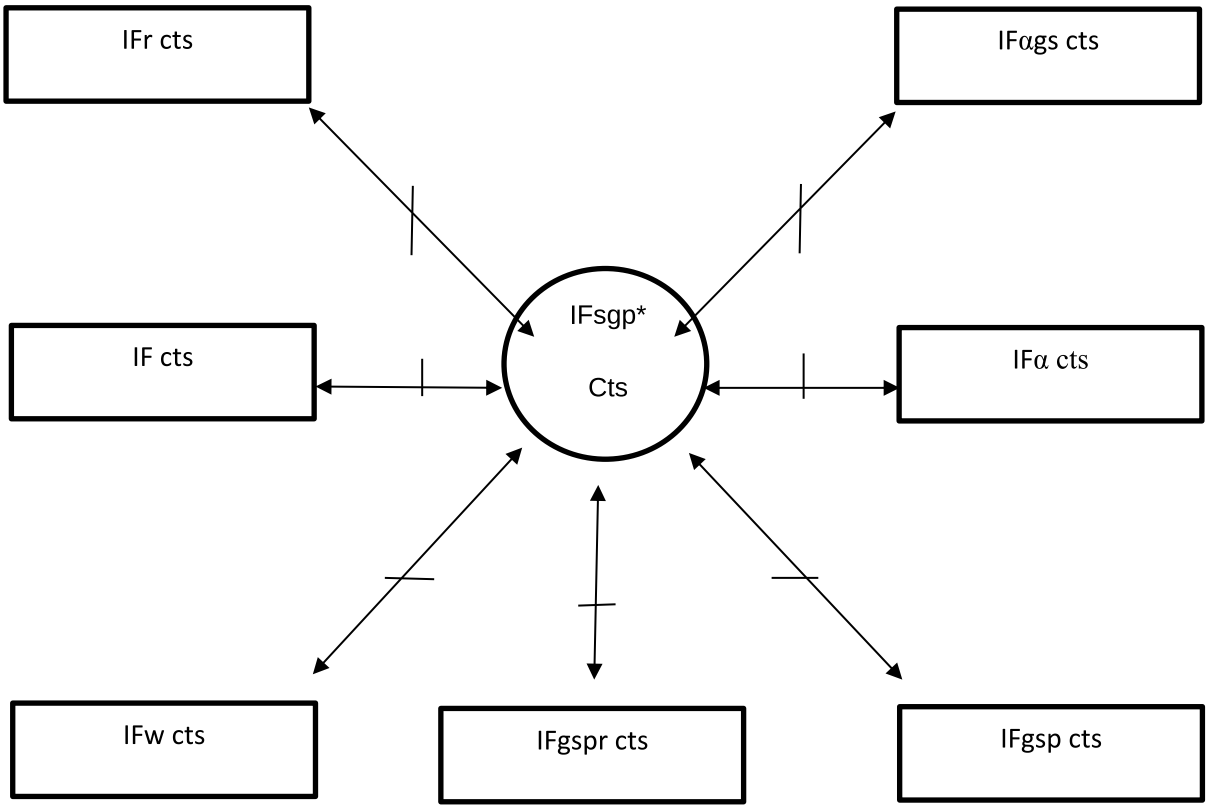
Theorem 3.15: *Every IFsgp* continuous mapping is an IFgsp continuous mapping but not conversely.*

Proof: Let $f : (X, \tau) \to (Y, \sigma)$ be an IFsgp* continuous mapping. Let A be an IFcs in Y. Since f is IFsgp* continuous mapping, $f^{-1}(A)$ is an IFsgp*cs in X. Since every IFsgp*cs is an IFgspcs, $f^{-1}(A)$ is an IFgspcs in X. Hence f is an IFgsp continuous mapping. Converse need not be true.

Example 3.16: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $T_1 = \{x, (0.5, 0.7), (0.5, 0.3)\}$, $T_2 = \{y, (0.6, 0.8), (0.4, 0.2)\}$. Then $\tau = \{0_\sim, T_1, 1_\sim\}$ and $\sigma = \{0_\sim, T_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. The IFS $A = \{y, (0.4, 0.2), (0.6, 0.8)\}$ is an IFcs in Y. Then $f^{-1}(A)$ is IFgspcs in X but not IFsgp*cs in X. Thus f is an IFgsp continuous mapping but not IFsgp* continuous mapping.

The relations between various types of IF continuity is given in the following diagram.

Remark 3.17:



The reverse implications are not true in general. In this diagram 'cts' means continuous.

Theorem 3.18: *A mapping $f : (X, \tau) \to (Y, \sigma)$ is an IFsgp* continuous mapping iff the inverse image of each IFos in Y is an IFsgp*cs in X.*

Proof: Let A be an IFos in Y. This implies AC is an IFcs in Y. Since f is IFsgp* continuous, $f^{-1}(A^C)$ is IFsgp*cs in X. $f^{-1}(A^C) = (f^{-1}(A))^C$, $f^{-1}(A)$ is an IFsgp*os in X.

Theorem 3.19: *Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be a mapping and let $f^{-1}(A)$ is an IFrcs in X for every IFcs A in Y. Then f is an IFsgp* continuous mapping.*

Proof: Let A be an IFcs in Y. Then $f^{-1}(A)$ is an IFrcs in X. Since every IFrcs is IFsgp*cs [by theorem every IFrcs is IFsgp* cs], $f^{-1}(A)$ is an IFsgp*cs in X. Hence f is an IFsgp* continuous mapping.

Theorem 3.20: *Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be an IFsgp* continuous mapping and $g: (Y,\sigma) \rightarrow (Z,\eta)$ is an IF continuous mapping, then $g \circ f : (X,\tau) \rightarrow (Z,\eta)$ is an IFsgp* continuous mapping.*

Proof: Let A be an IFcs in Z. Then $g^{-1}(A)$ is an IFcs in Y, by hypothesis. Since f is an IFsgp* continuous mapping, $f^{-1}(g^{-1}(A))$ is an IFsgp*cs in X. Hence $g \circ f$ is an IFsgp* continuous mapping.

4. IFsgp* - IRRESOLUTE MAPPING

Definition 4.1: A mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ is called **IFsgp* irresolute mapping** if $f^{-1}(V)$ is IFsgp* closed set in (X,τ) for every IFsgp* closed set V of (Y,σ).

Theorem 4.2: *Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be an IFsgp* Irresolute. Then f is an IFsgp* continuous mapping.*

Proof: Let f be IFsgp* irresolute mapping. Let A be any IFcs in Y. Since every IFcs is an IFsgp*cs in Y. By hypothesis $f^{-1}(A)$ is an IFsgp*cs in X. Hence f is an IFsgp* continuous mapping.

Theorem 4.3: *A mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ is an IFsgp* irresolute mapping iff the inverse image of each IFsgp*os in X.*

Proof: The proof is obvious from the definition [IFsgp* irresolute], Since $f^{-1}(A^C) = [f^{-1}(A)]^C$.

Theorem 4.4: *A mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ and $g : (Y,\sigma) \rightarrow (Z,\eta)$ is an IFsgp* irresolute mapping, then $g \circ f : (X,\tau) \rightarrow (Z,\eta)$ is an IFsgp* irresolute mapping.*

Proof: Let A be an IFsgp*cs in Z. Then $g^{-1}(A)$ is an IFsgp*cs in Y. Since f is an IFsgp* irresolute mapping, $f^{-1}(g^{-1}(A))$ is an IFsgp*cs in X. Hence $g \circ f$ is an IFsgp* irresolute mapping.

Theorem 4.5: *A mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ be an IFsgp* irresolute mapping and $g: (Y,\sigma) \rightarrow (Z,\eta)$ is an IFsgp* continuous mapping, then $g \circ f : (X,\tau) \rightarrow (Z,\eta)$ is an IFsgp* continuous mapping.*

Proof: Let A be an IFcs in Z. Then $g^{-1}(A)$ is an IFsgp*cs in Y. Since f is an IFsgp* irresolute mapping, $f^{-1}(g^{-1}(A))$ is an IFsgp*cs in X. Hence $g \circ f$ is an IFsgp* continuous mapping

5. PROPERTIES OF IFsgp* - CONTINUOUS MAPPING AND IRRESOLUTE MAPPING

Definition 5.1 [6]: Let X is a nonempty set and $C \in X$ a fixed element in X. IF $\alpha \in (0,1]$ and $\beta \in [0,1)$ are two real numbers such that $\alpha + \beta \leq 1$ then

(i) $\mathcal{C}(\alpha,\beta) =< x, \mathcal{C}_{\alpha}, \mathcal{C}_{1-\beta} >$ is called an **Intuitionistic fuzzy point** in X, where α denotes the degree of membership of $\mathcal{C}(\alpha,\beta)$ and denotes the degree of nonmembership of $\mathcal{C}(\alpha,\beta)$.

(ii) $\mathcal{C}(\beta) =< x, 0, 1 - \mathcal{C}_{1-\beta} >$ is called a **vanishing Intuitionistic fuzzy point** in X, where β denotes the degree of non-membership of $\mathcal{C}(\beta)$.

Definition 5.2 [6]: Two IFSs are said to be **q-coincident** (AqB in short) iff there exist an element $x \in X$ such that $\mu_A(x) > \nu_A(x)$ or $\nu_B(x) < \mu_B(x)$.

Definition 5.3 [6]: For any two IFSs A and B of X, $\rceil(AqB)$ iff $A \subseteq B^C$.

Theorem 5.4: *Let $f : (X,\tau) \rightarrow (Y,\sigma)$ is an IFsgp* continuous mapping then for each IFP $\mathcal{C}(\alpha,\beta)$ of X and each $V \in \sigma$, $f(\mathcal{C}(\alpha,\beta)) \in V$, there exist an IFsgp*os U of X such that $\mathcal{C}(\alpha,\beta) \in U$ and $f(U) \subseteq V$.*

Proof: Let $\mathcal{C}(\alpha,\beta)$ be an IFP of X and $V \in \sigma$ such that $f(\mathcal{C}(\alpha,\beta)) \in V$. Put $U = f^{-1}(V)$. Then by hypothesis U is an IFsgp*os in X such that $\mathcal{C}(\alpha,\beta) \in U$ and $f(U) = f(f^{-1}(V)) \subseteq V$.

Theorem 5.5: *Let $f : (X,\tau) \rightarrow (Y,\sigma)$ is an IFsgp* continuous mapping then for each IFP $\mathcal{C}(\alpha,\beta)$ of X and each $V \in \sigma$, such that $f(\mathcal{C}(\alpha,\beta))\, q\, V$, there exist an IFsgp*os U of X such that $\mathcal{C}(\alpha,\beta)\, q\, U$ and $f(U) \subseteq V$.*

Proof: Let $\mathcal{C}(\alpha,\beta)$ be an IFP of X and $V \in \sigma$ such that $f(\mathcal{C}(\alpha,\beta))\, q\, V$. Put $U = f^{-1}(V)$. Then by hypothesis U is an IFsgp*os in X such that $\mathcal{C}(\alpha,\beta)\, q\, U$ and $f(U) = f(f^{-1}(V)) \subseteq V$.

Definition: 5.6: Let (X,τ) be an IFTS. The sgp* closure of an IFS A of x denoted by sgp$*$ cl(A) is the intersection of all IFsgp* closed set of X which contains A.

Remark 5.7: it is clear that, $A \subseteq sgp * cl(A) \subseteq cl(A)$ for any IFS A of X.

Theorem 5.8: *Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be an IFsgp* continuous mapping. Then the following statement hold.*

(i) $f(sgp * cl(A)) \subseteq cl(f(A))$, *for every IFS A in X.*

(ii) $sgp * cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$, *for every IFS B in X.*

Proof:

(i) Let $A \subseteq X$. Then $cl(f(A))$ is an IFcs in Y. Since f is an IFsgp* continuous mapping, $f^{-1}(cl(f(A)))$ is an IFsgp*cs in X. ie) $sgp * cl(A) \subseteq f^{-1}(cl(f(A)))$.

Since $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(cl(f(A)))$ and $f^{-1}(cl(f(A)))$ is an IFsgp* closed,

$\Rightarrow sgp * cl(A) \subseteq f^{-1}(cl(f(A)))$. Hence $f[sgp * cl(A)] \subseteq cl(f(A))$.

(ii) Replacing A by $f^{-1}(B)$ in (i), we get $f(sgp * cl(f^{-1}(B)) \subseteq cl(f(f^{-1}(B))) \subseteq cl(B)$.

Hence $sgp * cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$, for every IFS B of Y.

Theorem 5.9: *Let $f : X \to Y$ be a mapping from an IFTS X into an IFTS Y. Then the following statement are equivalent.*

(a) *f is an IFsgp* irresolute mapping.*

(b) *$f^{-1}(B)$ is an IFsgp*os in X for every IFsgp*os B in Y.*

(c) *$sgp * cl(f^{-1}(B)) \subseteq f^{-1}(sgp * (cl(B)))$, for every IFS B in Y.*

(d) *$f^{-1}(sgp * (intB)) \subseteq sgp * int(f^{-1}(B))$, for every IFS B in Y.*

Proof:

(a) ⇒ **(b)** This can be proved by taking the complement of definition 4.1.

(b) ⇒ **(c)** Let B be any IFS in Y. Then $B \subseteq cl(B)$. Also, $f^{-1}(B) \subseteq f^{-1}sgp * (cl(B))$. Since $sgp * (cl(B))$ is an IFsgp*cs in Y, $f^{-1}sgp * (cl(B))$ is an IFsgp*cs in X. Therefore, $sgp * cl(f^{-1}B)) \subseteq f^{-1}sgp * (cl(B))$.

(c) ⇒ **(d)** Let B be any IFS in Y. Then $int(B)$ is an IFos in Y. Then $f^{-1}(int(B))$ is an IFsgp*os in X. Since $sgp * (int(B))$ is an IFsgp*os in X, $f^{-1}(sgp * (int(B)))$ is an IFsgp*os in X. Therefore $sgp * (cl(B))$ is an IFsgp*cs in Y, $f^{-1}(sgp * (cl(B)))$ is an IFsgp*cs in X.

Therefore $sgp * cl(f^{-1}(B)) \subseteq f^{-1}(sgp * (cl(B)))$.

(d) ⇒ **(a)** Let B be any IFsgp*os in Y. Then $sgp * (int(B)) = B$. By our assumption we have

$f^{-1}(B) = f^{-1}(sgp * (int(B))) \subseteq sgp * int(f^{-1}(B))$, So $f^{-1}(B)$ is an IFsgp*os in X. Hence f is an IFsgp* irresolute mapping.

Remark 5.10:

(i) Any union of IFsgp* open set is IFsgp* open.

(ii) Any intersection of IFsgp* closed set is IFsgp* closed.

Theorem 5.11*: For any IFS $A = < x, \mu_A, \nu_A >$ in an IFTS (X, τ), $A \in IF\ sgp * o(X)$ iff ($\forall\, \mathcal{C}(\alpha, \beta) \in A$) there exist $B \in IF\ sgp * o(X)$ such that $\mathcal{C}(\alpha, \beta) \in B \subseteq A$.*

Proof: If $A \in IF\ sgp * o(X)$, then we take $B = A$ so that $\mathcal{C}(\alpha, \beta) \in B \subseteq A$ for every $\mathcal{C}(\alpha, \beta) \in A$. Let A be an IFS in (X, τ) and assume that there exist $B \in IF\ sgp * o(X)$ such that $\mathcal{C}(\alpha, \beta) \in B \subseteq A$. Then

$$A = \bigcup_{\mathcal{C}(\alpha,\beta)\in A} \mathcal{C}(\alpha, \beta) = \bigcup_{\mathcal{C}(\alpha,\beta)\in A} B \subseteq A$$

and so

$$A = \bigcup_{\mathcal{C}(\alpha,\beta)\in A} B$$

which is an IFsgp* open set by Remark 5.10 (i).

Theorem 5.12: *Let $f : X \rightarrow Y$ be a mapping from an IFTS X to an IFTS Y. Then the following assertions are equivalent.*

(i) f *is IFsgp* continuous*

(ii) $f^{-1}(B) \in IFsgp * c(X)$ *for every IFCS B in Y.*

(iii) *For every IFP $C(\alpha,\beta)$ in X and every IFOS B in Y such that $f(C(\alpha,\beta)) \in B$, there exist $A \in IFsgp * o(X)$ such that $C(\alpha,\beta) \in A$ and $f(A) \subseteq B$.*

Proof:

(i) ⇒ (ii) Obvious

(i) ⇒ (iii) Assume that f is IFsgp* continuous. Let $C(\alpha,\beta)$ be an IFP in X and B be an IFOS in Y such that $f(C(\alpha,\beta)) \in B$. Take $A = f^{-1}(B)$. Then $A \in IFsgp * o(X)$ by the definition of IFsgp* continuous and $C(\alpha,\beta) \in f^{-1}(f(C(\alpha,\beta))) \in f^{-1}(B) = A$ and $f(A) = f(f^{-1}(B)) \subseteq B$.

(iii) ⇒ (i) Let B be an IFOS in Y and $C(\alpha,\beta) \in f^{-1}(B)$. Then $f(C(\alpha,\beta)) \in B$. Using (iii), we know that there exist $A \in IFsgp * o(X)$ such that $C(\alpha,\beta) \in A$ and $f(A) \subseteq B$. It follows that $C(\alpha,\beta) \in A \subseteq f^{-1}(B)$ so from above theorem 5.11 that $f^{-1}(B) \in IFsgp * o(X)$. Thus, f is IFsgp* continuous.

6. CONCLUSION

In the present study, we have described the Intuitionistic fuzzy semi generalised pre star continuous mapping and intuitionistic fuzzy semi generalised pre star irresolute mapping. We have also investigated a few of its characteristics and the connections between the various Intuitionistic fuzzy sets. In the future research, we would like to employ intuitionistic fuzzy semi generalised pre star sets to determine the connected and disconnected concepts.

REFERENCE

[1]. K. T. Atanassov, Intuitionistic fuzzy sets, Fuzzy sets and systems, 20(1986), 87 - 96.

[2]. J. Chandhini, N. Uma, IFgp* Neighborhood in Intuitionistic fuzzy topological spaces, Aryabhatta Journal of Mathematics and Informatics, vol 14, Special conference issue, 30 - 31 March 2022.

[3]. J. Chandhini, N. Uma, IFsgp* - Closed Sets, Open sets in Intuitionistic Fuzzy Topological Spaces, International Journal for Research in Applied Science & Engineering Technology (IJRASET), Vol 10 Issue XII, Dec 2022, DOI – https://doi.org/10.22214/ijraset.2022.48169.

[4]. D. Coker, An introduction to intuitionistic fuzzy topological spaces, Fuzzy sets and systems 88, 1997, 81 - 89.

[5]. D. Coker and M. Demirci, On intuitionistic fuzzy points, Notes on intuitionistic fuzzy sets 1(1997), 79 - 84.

[6]. H. Gurcay and D. Coker, On fuzzy continuity in intuitionistic fuzzy topological spaces, J. Fuzzy Math. 5(2) (1997), 365 - 378.

[7]. M. Jeyaraman, O. Ravi and A. Yuvarani, Intuitionistic fuzzy alpha generalized semi closed set, Thai Journal of mathematics volume 14(3), 2016, 757 - 769.

[8]. J. K. Joen, Y. B. Jun, and J. H. Park, Intuitionistiv fuzzy alpha continuity and intuitionistic fuzzy pre continuity, IJMMS, 19(2005), 3091 - 3101.

[9]. K. Ramesh, and Thirumalaiswamy M, Generalized semipre closed set in Intuitionistic fuzzy topological spaces, International journal of computer application technology and research 2(3), 2013, 324 - 328.

[10]. K. Ramesh, and Thirumalaiswamy M, Generalized semipre regular continuous and irresolute mapping in Intuitionistic fuzzy topological spaces, International journal of engineering research and technology, vol 2, Issue 8, 832 - 838, 2013.

[11]. R. Santhi and D. Jyanthi Intuitionistic fuzzy generalized semi continuous mapping, Advances in theoretical and applied mathematics, 5, 2009, 73 - 82.

[12]. R. Santhi and D. Jyanthi Intuitionistic fuzzy generalized semipre closed set, Tripura Math. Soci., 61 - 71, 2009.

[13]. R. Santhi and D. Jyanthi Intuitionistic fuzzy generalized semipre continuous mapping, Int. J. Contemp. Math. Sciences, 5(30) (2010), 1455 - 1469.

[14]. R. Santhi and K. Sakthivel, Intuitionistic fuzzy generalized semicontinuous mapping, Advances in theoretical and applied mathematics, 5(2009), 73-82.

[15]. K. Sakthivel, Intuitionistic fuzzy alpha generalized closed sets and intuitionistic fuzzy alpha generalized open set, The Mathematical Education 4(2012).

[16]. T. Shyla Isac Mary and P. Thangavelu, On Regular Pre semi closed sets in Topological spaces, KBM Journal of Mathematical Sciences and Computer Applications, 1(2010), 9 - 17.

[17]. S. S. Thakur and Jyoti Pandey Bajpai, Intuitionistic fuzzy w - closed sets and Intuitionistic fuzzy w - continuity, International Journal of Contemporary Advanced Mathematics, 1(1), 2010, 1 - 15.

[18]. S. S. Thakur and R. Chaturvedi, Regular generalized closed sets in intuitionistic fuzzy topological spaces, Universitatea Din Bacau, Studii Si Cercetari Seria: Mathematica, 16(2006), 257 - 272.

[19]. Young Bae Jun and Seok Zun Song, Intuitionistic fuzzy semi pre-open sets and Intuitionistic fuzzy semi pre continuous mapping, Jour. of Appl. Math and Computing, 2005, 467 - 474.

[20]. L. A. Zadeh, Fuzzy sets, Information control, 8(1965), 338 – 353.