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RESEARCH ARTICLE

SIX SIGMA BASED CONTROL CHART FOR SINGLE SERVER, INFINITE CAPACITY MARKOVIAN QUEUEING MODEL USING PROCESS CAPABILITY

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ABSTRACT

Customer satisfaction is an essential requirement in any manufacturing or service industry, and the need to maintain customer satisfaction increases over time. Customer feedback and ratings have a huge impact on the sustainability of the business. With this in mind, this paper analyzes a single-server, infinite-capacity Markovian queuing model that uses six sigma-based control charts and process functions to track the system performance. Numerical examples are also provided.

Keywords: control charts, six sigma, process capability, queuing theory.

AMS: 62P30; 97M40; 60K30; 60K25

INTRODUCTION

Technological explosion in today's world has contributed to the big data resulting from on line transections. In online services, the customer satisfaction plays a major role in the sustainability of the business. This can be achieved by focussing on the minimum response time. When the waiting time reduces, the tendency to give a better rating increases, leading to a better customer satisfaction.

The adoption of six sigma based control charts and the process capability index would be very helpful in monitoring the performance of the system by providing an optimal range of the waiting customers.

CONTROL CHARTS

Shewhart's control charts are used to track process variation over time[11]. This is frequently used in quality control since it gives an upper and a lower limit within which the process is stable. Considerable research has been done in this area over the years, taking into account a variety of factors beginning with the quantity of damaged goods. Current research concentrates on using control charts to analyse queueing systems, particularly customer wait times, no. of customers in the system, customers in the queue etc. which are crucial to consumer happiness.

REVIEW OF LITERATURE:

Haim Shore (2007) developed attribute control chart [3]. M.V .Kharparde and S. D. Dhabe (2010) constructed control chart with random queue length for the queuing model (M/M/1): (∞/FCFS) [5]. T. Poongodi and S. Muthulakshmi (2013) investigated waiting time in the system using control chart for (M/M/1) : (∞ /FCFS) Queuing model [7]. A.R Sudamani Ramasamy and B. Vennila (2012) studied random queue length control chart((M / M / c): (∞ / FCFS) using skewness [1] and N. Pukazhendhi and S. Poornima (2018) constructed and developed waiting time control chart for the (M/M/S): (∞/FCFS) queueing model using process capability [8].

SIX SIGMA

The idea of six sigma, a system made up of techniques to get rid of errors, boost quality, and attain operational excellence, is introduced by Motorola (1980)

Six Sigma quality levels were constructed and developed by Radhakrishnan and Sivakumaran (2008) [10]. The research work on Six Sigma-based control charts by Radhakrishnan and Balamurugan (2010) paved the way for this work on Six Sigma-based control charts for the number of defects [9]. The process capability index measures the accuracy and consistency of performance of the system under consideration.

Expected No. of Customers in the Queue $\left(L_q\right)$ or [E(Q)]

$$E(Q) = \sum_{n=1}^{\infty}(n-1)P_n = E(Q) = \frac{\rho_q-\rho_q(1-\rho_q)}{1-\rho_q} \qquad (1)$$

$$E(Q^2) = \sum_{n=1}^{\infty}(n^2-1)\ P_n = \rho_q\ \frac{(3\rho_q-\rho_q^2)}{(1-\rho_q)^2} \qquad (2)$$

$$V(Q) = E(Q^2) - [\,E(Q)]^2 = \frac{\rho_q^2[3-\rho_q-\rho_q^2]}{(1-\rho_q)^2} \qquad (3)$$

$$\sigma_Q = \sqrt{\frac{{\rho_q}^2[3-\rho_q-\rho_q^2]}{(1-\rho_q)^2}} = \frac{\rho_q}{1-\rho_q}\sqrt{3-\rho_q-\rho_q^2} \qquad (4)$$

Shewhart Control chart performance of the system is

$$UCL_s = E(Q) + 3\times\sqrt{Var(Q)} = \frac{\rho_q-\rho_q(1-\rho_q)}{1-\rho_q} + 3\times\sqrt{\frac{\rho_q^2[3-\rho_q-\rho_q^2]}{(1-\rho_q)^2}} \qquad (5)$$

$$CL_s = E(Q) = \frac{\rho_q-\rho_q(1-\rho_q)}{1-\rho_q} \qquad (6)$$

$$LCL_s = E(Q) - 3 \times \sqrt{Var(Q)} == \frac{\rho_q - \rho_q(1-\rho_q)}{1-\rho_q} - 3 \times \sqrt{\frac{\rho_q^2[3-\rho_q-\rho_q^2]}{(1-\rho_q)^2}} \quad \text{\_\_\_\_\_\_\_ (7)}$$

Six Sigma based Control chart performance of the system by using process capability is given by:

Cp(process capability) = $\frac{TL}{6}$ where TL is tolerance level in the difference between highest and lowest value between the range of expected values of the standard deviations for the assumed λ & μ .

$$UCL_{6\sigma} = E(Q) + \sqrt{V(Q)} \times \sigma_{pc}$$

$$CL_{6\sigma} = E(Q)$$

$$LCL_{6\sigma} = E(Q) - \sqrt{V(Q)} \times \sigma_{pc}$$

Table 1: Shewhart Control chart and Six Sigma based Control chart using Process capability for fixed μ

λ_q	μ_q	ρ_q	σ_Q	SHEWHART			σ_{pc}	
				LCL	CL	UCL	LCL	UCL
2	11	0.1818	0.3709	-1.0722	0.0404	1.1530	-0.7246	0.8054
2.5	11	0.2273	0.4852	-1.3887	0.0668	1.5223	-0.6982	0.8318
3	11	0.2727	0.6108	-1.7301	0.1023	1.9346	-0.6627	0.8673
3.5	11	0.3182	0.7497	-2.1005	0.1485	2.3975	-0.6165	0.9135
4	11	0.3636	0.9043	-2.5050	0.2078	2.9206	-0.5572	0.9728
4.5	11	0.4091	1.0778	-2.9501	0.2832	3.5165	-0.4818	1.0482
5	11	0.4545	1.2744	-3.4445	0.3788	4.2021	-0.3862	1.1438
5.5	11	0.5000	1.5000	-4.0000	0.5000	5.0000	-0.2650	1.2650
6	11	0.5455	1.7624	-4.6327	0.6545	5.9418	-0.1105	1.4195
6.5	11	0.5909	2.0731	-5.3658	0.8535	7.0729	0.0885	1.6185
7	11	0.6364	2.4492	-6.2339	1.1136	8.4612	0.3486	1.8786

Figure:1

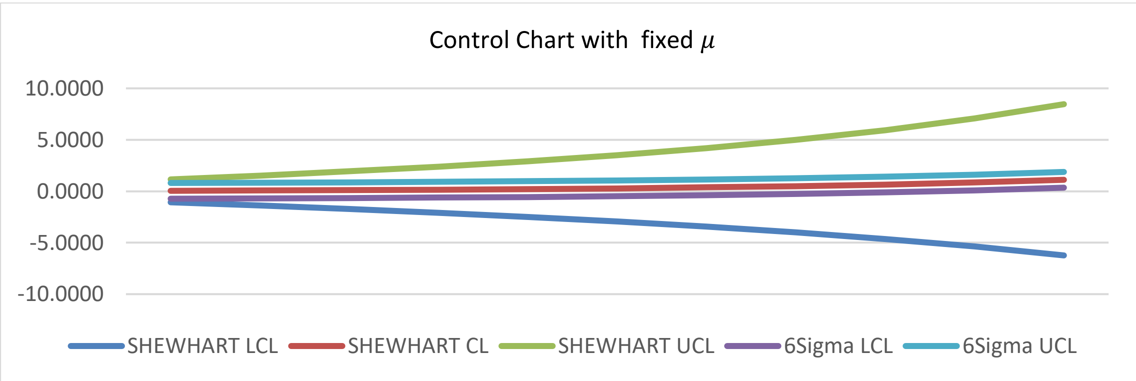
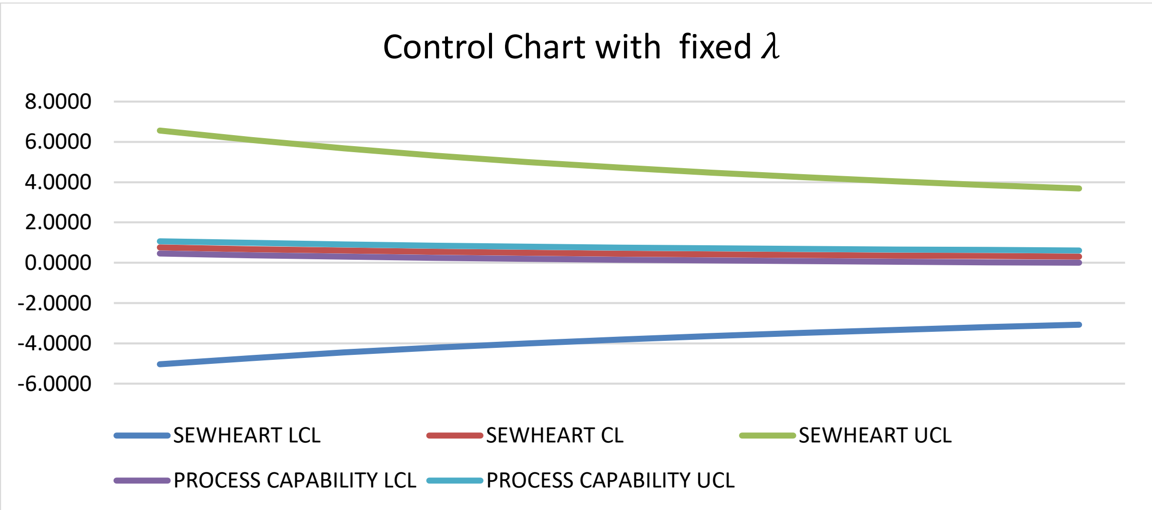


Table 2: Shewhart Control chart and Six Sigma based Control chart using Process capability for fixed λ

λ_q	μ_q	ρ_q	σ_Q	SHEWHEART			σ_{pc}	
				LCL	CL	UCL	LCL	UCL
4	7	0.5714	1.9331	-5.0375	0.7619	6.5613	0.4595	1.0643
4	7.25	0.5517	1.8021	-4.7272	0.6790	6.0853	0.3766	0.9814
4	7.5	0.5333	1.6883	-4.4553	0.6095	5.6743	0.3071	0.9119
4	7.75	0.5161	1.5884	-4.2147	0.5505	5.3157	0.2481	0.8529
4	8	0.5000	1.5000	-4.0000	0.5000	5.0000	0.1976	0.8024
4	8.25	0.4848	1.4212	-3.8072	0.4563	4.7198	0.1539	0.7587
4	8.5	0.4706	1.3504	-3.6329	0.4183	4.4695	0.1159	0.7207
4	8.75	0.4571	1.2865	-3.4745	0.3850	4.2444	0.0826	0.6874
4	9	0.4444	1.2285	-3.3299	0.3556	4.0410	0.0532	0.6580
4	9.25	0.4324	1.1756	-3.1972	0.3295	3.8561	0.0271	0.6319
4	9.5	0.4211	1.1271	-3.0750	0.3062	3.6874	0.0038	0.6086

Figure 2:



CONCLUSION

Application of control charts in queuing theory is an emerging area in statistical quality control. Customer satisfaction is mainly based on the no. of customers in the queue and time taken to serve a customer. Hence this paper studies the single server infinity capacity queue focussing on the queue length using six sigma based control charts. When we use six sigma initiatives, there is reduction in the variation and no. of customers waiting in the queue false with in the optimal range. This increases the system effectiveness.

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