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Fuzzy Multi Assignment Problem by Matrix's One Method

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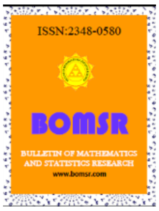

ABSTRACT

Assignment problem is an important concept in Operations Research, as we discuss this in real physical world. In this paper, we have studied some new methods of solving the Assignment Problem to analyse and compare them to identify the best method. Then, we have used the Fuzzy Multi Number in Assignment Problem with the aim to find the optimal solution for the decision-making criteria with multi constraints. Few numerical examples are also considered to check the efficiency of the Fuzzy Multi Number concept in Assignment Problems.

Keywords: Assignment Problem, HA-Method, Proposed method, Assignment method of matrix one, Alternative Method, Optimization problem, Fuzzy Set, Multi Set, Triangular Fuzzy Multi Number, Trapezoidal Fuzzy Multi Number.

1. INTRODUCTION

We know that the aim of an assignment problem is to assign a number of possessions to an equal number of happenings so as to minimize total cost or maximize total profit of allocation. And in (1995), Kuhn [2] developed the Hungarian method of the Assignment Problem, the reason for naming it with this name is because its basis lies by the effort of the Hungarian mathematician Egeraryin. Many researchers have advanced various techniques to solve generalized assignment problems. The assignment problem is how to make assignments to improve the desired goal [11] Zaman. K., and Saha. S. K, it is known as the degenerate problem of the transportation problem (TP), because in a cost matrix (m×m) of the assignment problem the gross number of allocated cells is, this problem refers to the cost allocated to the person to carry out the job assigned to him. In this project work, we have first studied some new methods of solving the Assignment Problem. There are many methods to develop an easy computational technique for such problem. Hungarian method is one of them P.K. Gupta [5] for the history of this method. We have analyzed and compared the new methods to identify the best method.

The Fuzzy Set (FS) was introduced by Zadeh [10], it is a class of objects with a of membership between 0 and 1. Zimmermann [12] proposed the basic definitions of fuzzy sets and algebraic operations. Yun et.al1. [9] generalized the triangular fuzzy number and calculated four operations based on the Zadeh's extension principle. Nagoor Gani & Mohamed Assarudeen [3] introduced a new method to find the fuzzy optimal solution of fully fuzzy linear programming problems with triangular fuzzy number. Yager [8] first discussed fuzzy multisets, he uses the term of fuzzy bag; an element of X may occur more than once with possibly of the same or different membership values. Ranking the fuzzy numbers is an important aspect of decision making in a fuzzy environment for practical applications. The Fuzzy Multi Numbers are transformed to crisp data by the ranking technique. The efficiency of the Fuzzy Multi Assignment Problem is analyzed with the numerical examples, and it is clear that, this ranking measure are well suited for multi criteria decisions.

2. PRELIMINARIES

In this section reviewed the basis notions, concepts and definitions.

Definition: 2.1 [4]

The application of scientific and especially mathematical methods to the study and analysis of problems involving complex systems called also **Operational Research**.

Definition: 2.2 [4]

Suppose there are *n* jobs to be performed and n persons are available for doing these jobs. Assume that each person can do each job at a term, though with varying degree of efficiency, let C_{ij} be the cost if the i^{th} person is assigned to the j^{th} job. The problem is to find an assignment (which job should be assigned to which person one on-one basis) So that the total cost of performing all jobs is minimum, problem of this kind is known as **Assignment Problem** (AP).

Definition: 2.3 [4]

Although an assignment problem can be formulated as a linear programming problem, it is solved by a special method known as **Hungarian Method** because of its special structure. If the time of completion or the costs corresponding to every assignment is written down in a matrix form, it is referred to as a **Cost matrix.** The Hungarian Method is based on the principle that if a constant is added to every element of a row and/or a column of cost matrix, the optimum solution of the resulting assignment problem is the same as the original problem.

Definition: 2.4 [10]

Let X be a nonempty set. A **fuzzy set** A of X is defined as A = { < x, $\mu_A(x)$ > / x∈ X }, where $\mu_A(x)$ is called membership function and it maps each element of X to a value between 0 and 1.

Definition: 2.5 [11]

A **fuzzy number** is a generalization of a regular real number. It does not refer to a single value but rather to a connected set of possible values, where each possible value has its weight between 0 and 1. The weight is called the membership function. A fuzzy number A is a convex normalized fuzzy set on the real line R such that there exists at least one x ∈ R with $\mu_A(x)$ = 1 and $\mu_A(x)$ is piecewise continuous.

Definition: 2.6 [8]

A **Multi Set** is an unordered collection of objects in which, unlike an ordinary set, objects are allowed to repeat. Each individual occurrence of an object is a **multi-set** which is called its element.

Definition: 2.7 [6]

Let X be a nonempty set. A **Fuzzy Multi Set** A in X is characterized by the count membership function Mc such that Mc: $X \to$ Q where Q is the set of all crisp multi sets in [0,1]. Hence, for any $x \in X$, Mc(x) is the crisp multi set from [0, 1].

The membership sequence is defined as $(\mu_A^1(x), \mu_A^2(x), \ldots \ldots \ldots \mu_A^p(x))$ where $\mu_A^1(x) \geq \mu_A^2(x) \geq \cdots \geq \mu_A^p(x)$.

Therefore, **FMS A** is given by $A = \left\{ \langle x, (\mu_A^1(x), \mu_A^2(x), \ldots \ldots \ldots \mu_A^p(x)) \rangle / x \in X \right\}$

Definition: 2.8 [3]

A **Triangular fuzzy number** A is denoted by 3 – tuples (a, b, c), where a, b and c are real numbers and $a \leq b \leq c$ with membership function defined as

$$\mu_A(x) = \begin{cases} 0 & for \quad x \leq a \\ \dfrac{x-a}{b-a} & for\ a \leq x \leq b \\ 1 & for\ x = b \\ \dfrac{c-x}{c-b} & for\ b \leq x \leq c \\ 0 & for \quad x \geq c \end{cases}$$

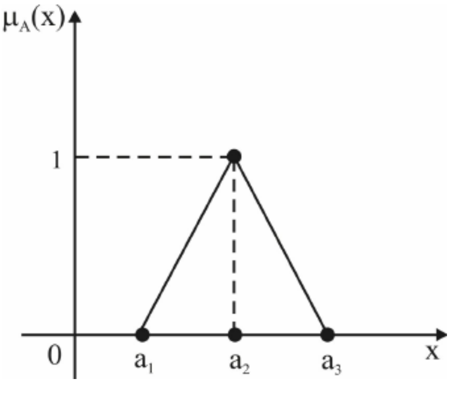

Figure 1 : Triangular Fuzzy Number

Definition: 2.9 [3]

The **ranking** function is approach of ordering fuzzy numbers which is an efficient. The ranking function is denoted by F (ℝ), where ℝ: F(ℝ) → ℝ , and F (ℝ) is the set of fuzzy numbers defined on a real line, where a natural order exist.

Let $a, b \in \mathbb{R}$, then **ranking function** for real numbers a, b is defined as

(i) $D(a,b) > 0 \Leftrightarrow D(a,0) > D(b,0) \Leftrightarrow a > b$
(ii) $D(a,b) < 0 \Leftrightarrow D(a,0) < D(b,0) \Leftrightarrow b < a$
(iii) $D(a,b) = 0 \Leftrightarrow D(a,0) = D(b,0) \Leftrightarrow b = a$

Definition: 2.10 [1]

The Ranking Measure of **Graded Mean Integration Representation** proposed by **Chen and Hsieh [1]** is as follows for the

Triangular fuzzy number (a, b, c) is $\frac{(a + 4b + c)}{6}$

Definition: 2.11 [6]

The **Cardinality** of the membership function Mc(x) is the length of an element x in the Fuzzy Multi Set *A* denoted as η, defined as η = Mc(x)

If A, B, C are the *FMS* defined on *X*, then their cardinality η = Max { η(A), η(B), η(C) }.

Definition: 2.12

A **Triangular fuzzy multi number** A_i is denoted by 3 – tuples (a_i, b_i, c_i), where $a_i,\ b_i\ and\ c_i$ are real numbers and $a_i \leq b_i \leq c_i$ with membership function defined as

$$\mu_{A_i}(x) = \begin{cases} 0 & for \quad x \leq a_i \\ \dfrac{x - a_i}{b_i - a_i} & for\ a_i \leq x \leq b_i \\ \dfrac{c_i - x}{c_i - b_i} & for\ b_i \leq x \leq c_i \\ 0 & for \quad x \geq c_i \end{cases}$$

Definition: 2.13

The membership sequence is in the form $(\mu_A^1(x), \mu_A^2(x), \ldots\ldots\ldots \mu_A^p(x))$ where $\mu_A^1(x) \geq \mu_A^2(x) \geq \cdots \geq \mu_A^p(x)$ with the Cardinality, the length of an element x in the Fuzzy Multi Set *A* denoted as η.

Then, **the Ranking Measure** by the **Graded Mean Integration Representation of** fuzzy multi number A_i is defined as

$\frac{1}{\eta}\sum_{i=1}^{\eta}\frac{(a_i + 4b_i + c_i)}{6}$ for the Triangular fuzzy multi numbers (a_i, b_i, c_i).

3. VARIOUS METHODS OF SOLVING THE ASSIGNMENT PROBLEM

HUNGARIAN METHOD

Subtract the smallest element of each row and column from every element of the corresponding row and column. Then, the optimal solution is determined.

ALTERNATIVE HUNGARIAN METHOD

Subtract the smallest element of each row and column from every element of the corresponding row and column. Consider the location of zero at each row. If row contain only one zero then assign it for the corresponding row and delete the corresponding row and column after allocation. Otherwise read the location of zero below for further process and find the optimal solution.

AI-SAEEDI'S METHOD 1

For each row and column of the cost matrix table, we define the two lowest costs available and find the difference between these two costs and place it to the right of this row in a new column and we define the two lowest costs available and find the difference between these two costs and place it to the right of this column in a new row. Then we choose the largest difference value out of all the difference values mentioned in the two penalties. Calculate the total cost to AP by applying the following formula.

AI-SAEEDI'S METHOD 2

For each row and column of the cost matrix table, we define the two lowest costs available and find the difference between these two costs and place it to the right of this row in a new column and for each column we select the greatest and lowest cost define the two lowest costs and find the difference between these two costs and place it to the right of this column in a new row. Then we choose the largest difference value out of all the difference values mentioned in the two penalties. Calculate the total cost to AP by applying the following formula.

MATRIX ONE'S ASSIGNMENT METHOD

Find the minimum cost of each row and column and then divide each element of each row and column of the matrix by its minimum cost. Then we assigned the following matrix of the problem and find the optimal solution.

3.1 : Numerical Example

Four different jobs (A, B, C and D) can be done on four different machines. The setup and take down time costs are assumed to be prohibitively high for change overs. The matrix below gives the cost in rupees of processing job i on machine j. How should the jobs be assigned to the various machines so that the total cost is minimized.

		Job			
		A	**B**	**C**	D
Machines	**1**	8	20	15	17
	2	15	16	12	10
	3	22	19	16	30
	4	25	15	12	9

The optimal solution of the problem is represented above in a tabular form for all the new methods studied to identify the best method.

3.2 : Numerical Example

A company wishes to assign 3 jobs to 3 man in such a way that each job is assigned to some machine and no machine works on more than one job. The cost of assigning job i to machine j is given by the matrix below (ij^{th} entry).

	Job			
		1	**2**	**3**
Man	**A**	7	3	2
	B	2	1	3
	C	3	4	6

The following table list outs the optimal solution of the defined problem for all the new methods to analyze & compare for the determination of the best method for any Assignment Problems in future.

Methods	**Optimal Solutions**	**Total cost**
Hungarian Method	1-C, 2-B, 3-A	19
Alternative Hungarian Method	1-B, 2-A, 3-C	19
Al-Saeedi's Method 1	1-A, 2-B, 3-C	19
Al-Saeedi's Method 2	1-C, 2-B, 3-A	19
Matrix One Assignment Method	1-A, 2-B, 3-C	19

From the above **Case Studies A & B** (**3.1 & 3.2)**, It is found that the optimal solution is the same as the Hungarian method and other four different new methods. We have noted the effectiveness of the new methods and that the results are equal and with fewer solution steps. We can conclude that the solution with the new method **Matrix One Assignment** is best from other methods, as we get the optimal solution with **less number steps & less consumption time** which will be very useful for decision makers.

Methods	**Optimal Solutions**	**Total cost**
Hungarian Method	1-A, 2-B, 3-C, 4-D	48
Alternative Hungarian Method	1-A, 2-C, 3-B, 4-D	48
Al-Saeedi's Method 1	1-B, 2-C, 3-A, 4-D	48
Al-Saeedi's Method 2	1-B, 2-C, 3-A, 4-D	48
Matrix One Assignment Method	1-A, 2-D, 3-C, 4-B	48

4. FUZZY MULTI NUMBERS IN ASSIGNMENT PROBLEMS

Triangular fuzzy numbers are used to represent uncertain and incomplete information in decision-making, risk evaluation, and expert systems. Measurement of similarity should keep some parameters of triangular fuzzy numbers. Shape and midpoint are important metrics of a triangular fuzzy number. Therefore, they should be taken into consideration when measuring the similarity of triangular fuzzy numbers.

Suppose there are n works to be performed and n persons are available for doing the works. Assume that each person can do each work at a time, though with unreliable grade of efficiency. Let ij be the fuzzy cost if the i^{th} person is assigned the j^{th} work, the problem is to find a minimum fuzzy cost with fuzzy assignment.

4.1 : Numerical Example

Four persons are available to do four different jobs. From past records, the cost (in Rupees) that each person takes to do each job is known and is represented by triangular fuzzy multi numbers and is shown in following Table.

		Job			
		A	**B**	**C**	**D**
Machines	**1**	{(1,2,3) (5,6,7) (4,5,6) (7,9,11)}	{(3,6,9) (7,9,11) (5,7,9) (5,6,7)}	{(8,9,10) (9,10,11) (11,13,15) (11,12,13)}	{(3,4,5) (3,5,7) (4,5,6) (9,10,11)}
	2	{(1,5,9) (7,8,9) (7,9,11) (9,10,11)}	{(1,2,3) (3,6,9) (4,5,6) (5,7,9)}	{(5,6,7) (6,9,12) (8,9,10) (10,11,12)}	{(3,4,5) (3,6,9) (4,5,6) (8,9,10)}
	3	{(1,2,3) (2,3,4) (3,6,9) (4,5,6)}	{(3,6,9) (4,5,6) (4,5,6) (9,10,11)}	{(6,7,8) (9,10,11) (4,5,6) (11,12,13)}	{(3,6,9) (5,7,9) (6,7,8) (7,8,9)}
	4	{(7,8,9) (8,9,10) (10,11,12) (11,12,13)}	{(1,2,3) (2,3,4) (3,5,7) (5,6,7)}	{(5,6,7) (6,7,8) (8,9,10) (7,10,13)}	{(0,1,2) (1,2,3) (2,3,4) (3,6,9)}

The assignment problem 4×4 matrix in of the triangular fuzzy multi number is considered which has four repeated fuzzy triangular numbers in each cell. These measures are considered for **four different times in day (morning, afternoon, evening and night).** We have to convert the fuzzy multi number to a corresponding crisp number and for this we use the concept of ranking measure.

An approach is proposed for fuzzy multi assignment problem into a crisp assignment problem using the ranking function for the fuzzy multi assignment cost matrix. By using the formula of the **Ranking Measure of the Triangular fuzzy multi numbers (a_i, b_i, c_i),** we can get the crisp data, which is $\frac{1}{\eta}\sum_{i=1}^{\eta}\frac{(a_i+4b_i+c_i)}{6}$. Here, the cardinality $\eta = 4$.

	Job				
		A	B	C	D
Machines	1	5	7	11	6
	2	8	5	9	6
	3	4	7	10	7
	4	10	4	8	3

Using the **Matrix One Assignment Method,** we have determined the optimal solution as this method was found to be the best one from the chapter 3.

1-C, 2-B, 3-A, 4-D is the assignment solution for the multi criterion options with the total cost of **22 Rs.**

4.2 : Numerical Example

Three persons (A, B, C) are available to do Three different jobs (1, 2, 3). From past records, the cost that each person takes to do each job is known and is represented by triangular fuzzy numbers with four different times in day (morning, afternoon, evening and night) and is represented in the following Table.

		Job		
		1	2	3
Man	A	{(4,5,6,)	{(3,6,9)	{(3,4,5)
		(7,8,9,)	(4,5,6)	(3,6,9)
		(8,9,10)	(5,7,9)	(4,5,6)
		(9,10,11)}	(9,10,11)}	(8,9,10)}
	B	{(1,2,3)	{(3,5,7)	{(8,9,10)
		(3,6,9)	(8,9,10)	(5,6,7)
		(4,5,6)	(4,5,6)	(6,7,8)
		(5,7,9)}	(7,9,11)}	(7,10,13)}
	C	{(3,6,9)	{(1,5,9)	{(3,6,9)
		(5,7,8)	(7,8,9)	(5,7,9)
		(3,4,5)	(9,10,11)	(6,7,8)
		(5,7,9)}	(7,9,11)}	(7,8,9)}

Ranking a fuzzy multi number involves in the assignment problem 3×3 matrix into the triangular fuzzy multi number, and whereby the fuzzy multi number is renewed to an approximated crisp number. Just as the decision maker takes two concepts that are the same, similarly, for this problem we have to convert the fuzzy multi number to a corresponding crisp number and compare the number on the basis of crisp values. An approach is proposed for fuzzy multi assignment problem into a crisp assignment problem using the ranking function for the fuzzy multi assignment cost matrix. By using the following formula, we get the crisp data.

Ranking Measure of the Triangular fuzzy multi numbers (a_i, b_i, c_i) is $\frac{1}{\eta}\sum_{i=1}^{\eta}\frac{(a_i+4b_i+c_i)}{6}$ where the cardinality is 4 as we have the repeated triangular fuzzy numbers

	Job			
		1	2	3
Man	A	8	7	6
	B	5	7	8
	C	6	8	7

The optimal solution of the multi criterion assignment problem is 1-C, 2-B, 3-A of 22 Rs is obtained by applying the Matrix One Assignment Method.

5. CONCLUSION

Assigning the right job to the right worker is necessary and very important to complete any job with high efficiency and this paper seeks to solve a Fuzzy Multi Assignment Problem. It was observed that the solution is achievable, a systematic and transparent. The Fuzzy Multi Assignment Problem of triangular fuzzy multi numbers has been transformed into crisp assignment problem using Ranking Measure and to solve, we have used the Matrix One's Assignment Method. Our study of new alternative methods of Hungarian method are Alternative Hungarian method, Al-Saeedi's Method 1 and 2 and Matrix One Assignment methods and we have found that the Matrix One's is the best one.

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