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RESEARCH ARTICLE

Face colouring of Fuzzy Magic Planar Graphs

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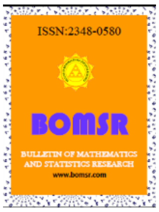


ABSTRACT

The ambiguity in most of the world wide problems formulates a void with which it is hard to examine its consequences for analysis either in its description or in its relationship among the factors. This structures the need for the introduction of fuzzy graph theory. In reality, fuzzy graph colouring outlines the central core problem in the fuzzy graph theory and the striking and most prominent fact about it occur with its practical application in the field of combinatorial optimization. Almost every graph has two kinds of coloring, for example, vertex colouring and edge colouring. In this paper, we discussed about the face coloring of the certain types of fuzzy magic graph.

Keywords: Fuzzy graph, Graph labeling, Fuzzy magic graph, Face coloring.

1. Introduction

Graphs constitute one of the major sources of information that can effectively elucidate the relation between two objects. The significance of graphs in the current scenario can be seen in practically varying backgrounds such as craft scheduling, computer network and also in automatic channel allocation. Such an interesting concept of graph was discussed by Haray E., in 1969 [1]. The ideology of fuzzy ordering was discussed by Zadeh, L.A. in 1971 [2]. Furthermore, the concept of fuzzy graph was discussed in detail by Bhattacharya, P.(1987)[3].

Graph labeling has been the currently emerging area within the research of graph theory. A graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions. The idea of a graph is claimed to be a fuzzy labeling graph if it has fuzzy labeling. A labeling of a graph is an assignment of a value to the vertices and the edges of a graph. The concept of graph labeling was discussed in [4]. Later, the magic graphs were described and studied in detail by Jamil et.al [5] and Sobha et.al [6]. Fuzzy magic graph and face magic labeling have been considerably studied in later years [7].

The idea of a coloring of the faces of a map is to assign each face of the graph a color in such a way that two faces that share an edge must get different colors. Faces that meet only at a vertex are allowed to be colored the same color. In addition to general coloring of a map, mathematicians are interested in coloring that use the minimum number of colors. The concept of fuzzy total coloring and fuzzy total chromatic number of a fuzzy graph were introduced [8], [9]. The (face) *chromatic number* of a map is the smallest numbers of colors that can be used to color the map subject to our rule, that faces with an edge in common get different colors. The concept of chromatic number of fuzzy graph was introduced by Munoz et.al. [10]..

2. PRELIMINARIES

2.1 Fuzzy Labelling Graph:

A graph $G = (V, \mu, \sigma)$ is said to be a fuzzy labelling graph if $\mu: V \rightarrow [0,1]$ and $\sigma : V \times V \rightarrow [0,1]$, is bijective such that the membership value of edges and vertices are distinct and

$\sigma(x, y) \leq \mu(x) \wedge \mu(y)$ for all $x, y \in V$.

2.2 Fuzzy Magic Graph:

A fuzzy graph G=(V, μ, σ) is called a fuzzy magic graph if there are two bijective functions μ:V → [0,1] and $\sigma : V \times V \rightarrow [0,1]$, with restricted the conditions σ (u,v) < μ (u) + μ (v) and μ (u)+ σ (uv)+ μ (v) = m(G) ≤ 1 where , m(G) is a real constant for all u,v∈ G

Example

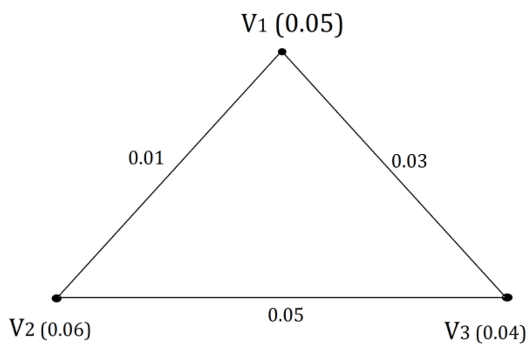


Figure 1. (Fuzzy Magic Graph)

$\sigma(v_1,v_2) < \mu(v_1) + \mu(v_2)$

$\sigma(v_1,v_3) < \mu(v_1) + \mu(v_3)$

and $\mu(v_1)+ \sigma(v_1 v_2)+ \mu(v_2) = 0.12 \leq 1$

$\mu(v_1)+ \sigma(v_1 v_3)+ \mu(v_3) = 0.12 \leq 1$

2. 3 Fuzzy Vertex Magic Graph:

A fuzzy labelling graph is said to be a fuzzy vertex magic graph if $\mu\ (u) + \sigma\ (uv) + \mu\ (v)$ has a same magic value for all $u, v \in V$ which is denoted as $m_0(G)$.

2.4 Fuzzy Edge Magic Graph:

A fuzzy labelling graph is said to be a fuzzy edge magic graph if $\mu\ (u) + \sigma\ (uv) + \mu\ (v)$ has a same magic value for all $u, v \in V$ which is denoted as $M_0(G)$.

2.5 Fuzzy Totally Magic Graph:

A fuzzy labelling graph is said to be a fuzzy totally magic graph if and only if there exist a fuzzy labelling which is both fuzzy edge magic with magic value M_0 and fuzzy vertex magic with magic value m_0. A fuzzy labelling graph is said to be a fuzzy magic graph if $\mu\ (u) + \sigma\ (uv) + \mu\ (v)$ has a same value for all $u, v \in V$ which is denoted as $m_0(G)$. It is not required that $M_0 = m_0$.

Every fuzzy magic graph is a fuzzy labelling graph, but the converse is not true.

2.6 Face Coloring

Graph coloring is to assign a color to elements (vertex, edge, and face) of graph such that two adjacent elements has different color. The chromatic number of a graph is the smallest numbers of color needed in graph coloring.

Face coloring of a planar graph assigns a color to each face so that no two faces that share an edge have the same color.

Basic Notation: The fuzzy face chromatic number of a fuzzy magic graph G is the minimum number of colors needed for a proper fuzzy face coloring of G. It is denoted by $\chi''\ (G)$.

2.7 Butterfly Graph :

The butterfly graph (Figure 2) is a planar undirected graph with 5 vertices and 6 edges.

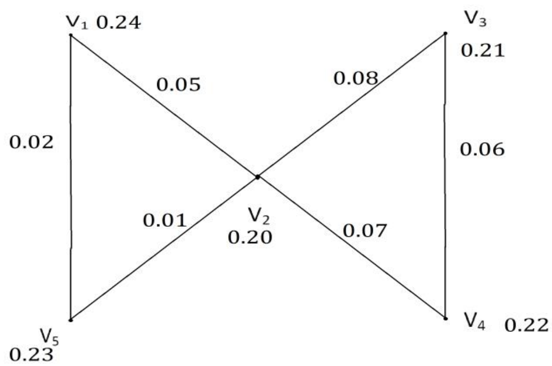


Figure 2. Butterfly Graph

2.8 Pan graph

The Pan graph is the graph obtained by joining a cycle graph to a singleton graph with a bridge. The special case of the 3-pan graph is sometimes known as the paw graph (Figure 3) and the 4-pan graph as the banner graph (Figure 4).

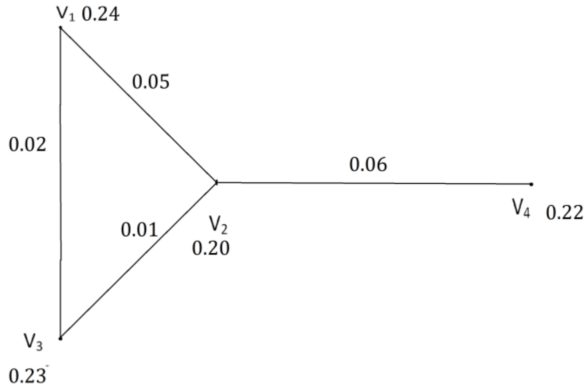


Figure 3. Paw graph (3-pan graph)

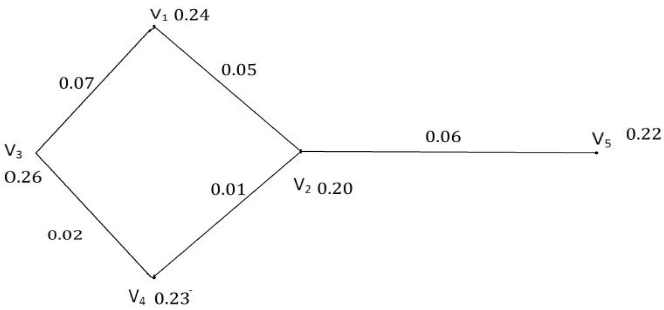


Figure 4. Banner graph (4-pan graph)

2.9 Wheel Graph

A Wheel graph (Figure 5) is a graph formed by connecting a single vertex to all the vertices of a cycle. For $n \geq 4$ the wheel W_n is defined to be the graph $K_1 + C_{n-1}$.

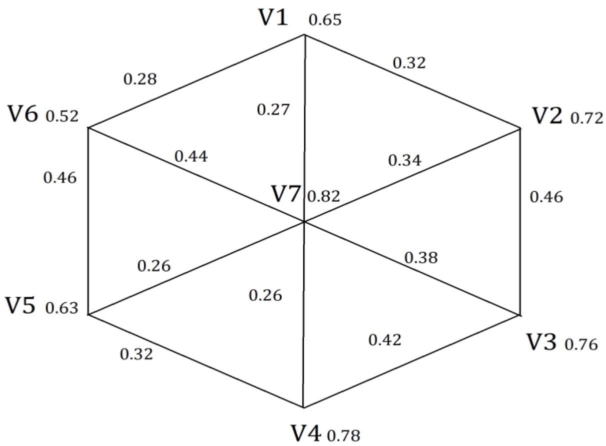


Figure 5. Wheel graph (W_6)

2.10 Fan Graph

A Fan graph $F_{m,n}$(Figure 6) is defined as the graph join $K_m + P_n$, where K_m is the empty graph on m vertices and P_n is the path graph on n vertices.

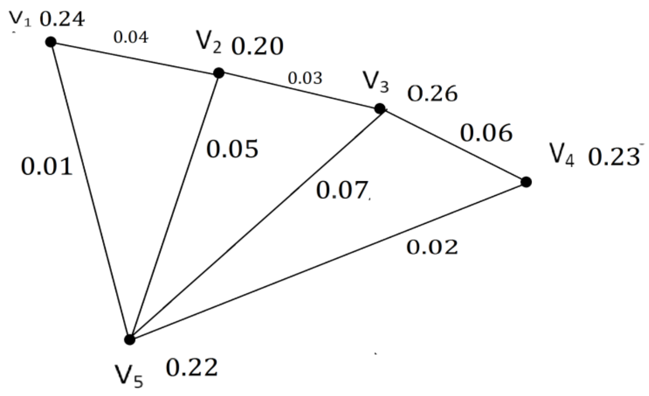


Figure 6. Fan graph $F_{1,4}$

2.11 Helm Graph

The helm graph (Figure 7) is the graph obtained from a wheel graph by adjoining a pendent edge at each vertex of the cycle.

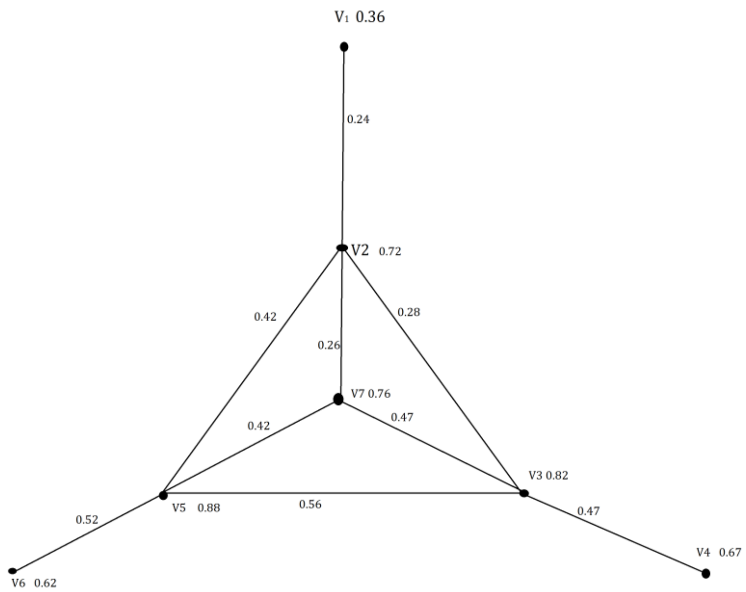


Figure 7. Helm graph

2.12 Bull Graph

The bull graph (Figure 8) is a planar undirected graph with 5 vertices and 5 edges, in the form of a triangle with two disjoint pendant edges.

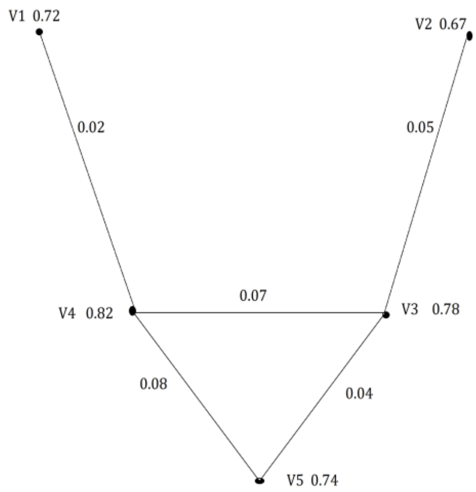


Figure 8. Bull Graph

2.13 Ladder Graph

The ladder graph (Figure 9) Ln is a planar undirected graph with $2n$ vertices and $3n - 2$ edges.

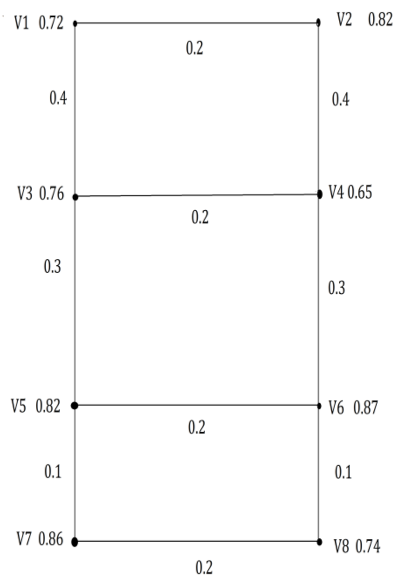


Figure 9. Ladder Graph L_4

2.14 Circular Ladder Graph

The circular ladder graph CLn is constructible by connecting the four 2-degree vertices in a *straight* way, It has $2n$ vertices and $3n$ edges. Like the ladder graph, it is connected, planar and Hamiltonian, but it is bipartite if and only if *n* is even.

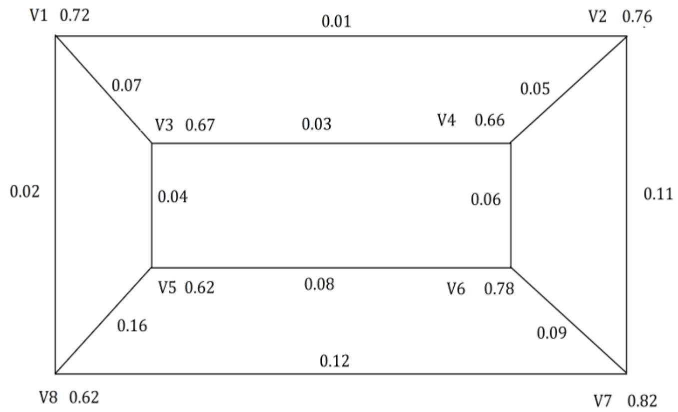


Figure 10. Circular Ladder graph CL_4

Ideology:

Here, chromatic number of a fuzzy magic graph in wheel graph (W_n), ladder graph (L_n), and circular ladder graph is discussed.

3 Face colouring for wheel graphs (Fan graphs, Helm graphs too):

Discussion 1:

Let $n > 3$ and W_{n-1} be a wheel graph. Then, $\chi''\ (W_{n-1})=\begin{cases} 4; & for\ f_n\ is\ even \\ 3; & for\ f_n\ is\ odd \end{cases}$,

Here, $|V(W_{n-1})| = n$; $|E(W_{n-1})| = 2n - 2$; $|F(W_{n-1})| = n$

Observation:

Case 1: For n is odd.

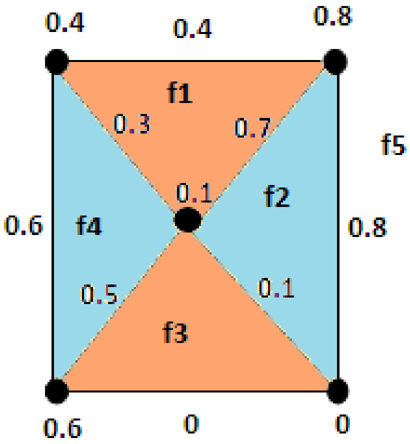
The chromatic number of face coloring in wheel graph is at least 2, because wheel graph has at least two adjacent faces and outer face has 1 different colour (say f_3) .Hence,

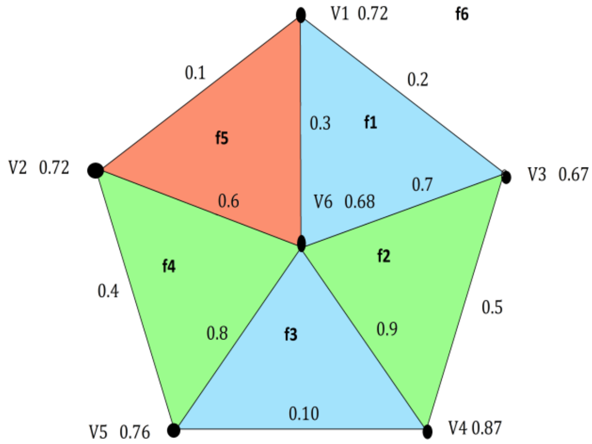
$$\chi''\ (W_{n-1}) = 2 + 1 = 3.$$

Case 2: For n is even.

The chromatic number of face coloring in wheel graph is at least 3, because wheel graph has at least two adjacent faces. For $n = 3, c(f_1) \neq c(f_2), c(f_2) \neq c(f_3), and\ c(f_3) \neq c(f_1)$. We can see that every faces of W_3 must have 3 different colors. and outer face has 1 different colour (say f_4) Hence, $\chi''\ (W_{n-1}) = 3 + 1 = 4$.

Examples:





(a) W_4 (b) W_5

3.1 Face coloring for Ladder graphs:

Discussion 2:

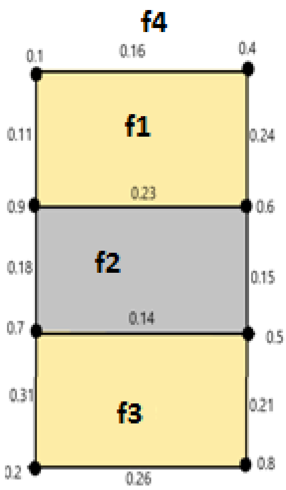
Let $n \geq 2$ and L_n be a ladder graph. Then, $\chi''\ (L_n)$=3,

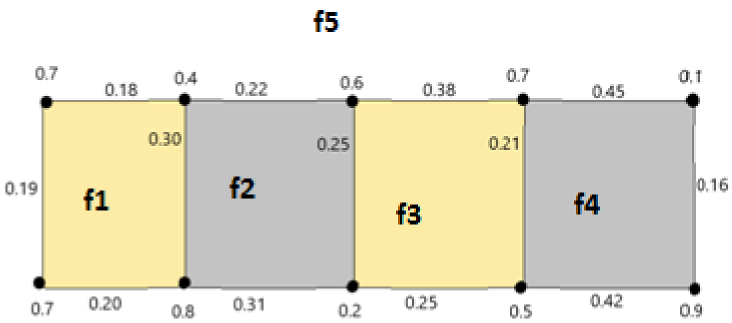
Here, $|V(L_n)| = 2n$; $|E(L_n)| = 3n - 2$; $|F(L_n)| = n$

Observation:

The chromatic number of face coloring in ladder graph is at least 2, because ladder graph has at least two adjacent faces and outer face has 1 different colour (say f_3) . Hence, $\chi''\ (L_n)$=2+1=3.

Examples:





(a)L_4 (b)L_5

3.2 Face coloring for circular ladder:

Discussion 3:

Let $n \geq 3$ and CL_n be a Circular ladder graph. Then, $\chi''\ (CL_n)=\begin{cases} 3; & for\ n\ is\ even \\ 4; & for\ n\ is\ odd \end{cases}$,

Here, $|V(CL_n)| = 2n;\ |E(CL_n)| = 3n;\ \ |F(CL_n)| = n + 1$

Observation:

Case 1: For n is even.

The chromatic number of face coloring in circular ladder graph is at least 3, because circular ladder graph has at least two adjacent faces. And there exists an extra face (in centre) in the circular ladder graph, which has its all adjacent faces the circular ladder. Thus, there arises a new color, $c(f_3)$. and outer face has the same colour as the centre face.

(i.e) $c(f_1) \neq\ c(f_2) \neq c(f_3)$.

Hence, $\chi''\ (CL_n) =\ 2 + 1 = 3$.

Case 2: For n is odd.

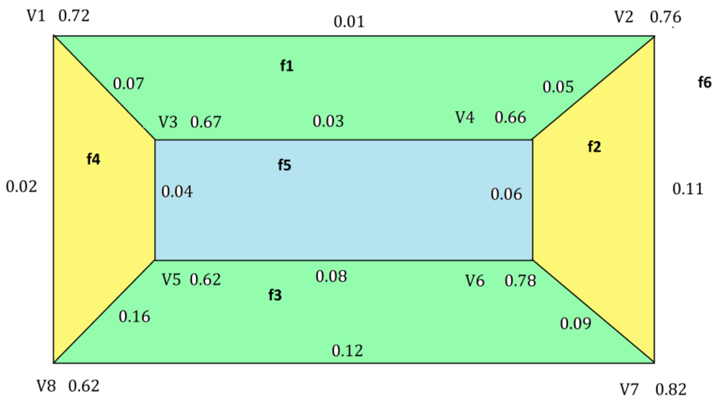
The chromatic number of face coloring in circular ladder graph is at least 4, because circular ladder graph has at least two adjacent faces. For $f_i\ =\ 5$. We can see that every faces of CL_3must have 3 different colors. Also similar to the case 1, there exists an extra face (in centre) in the circular ladder graph, which has its all adjacent faces the circular ladder. Thus, there arises a new color, $c(f_4)$ and outer face has the same colour as the centre face

(i.e) $(f_1) \neq c(f_2) \neq\ c(f_3) \neq c(f_c), c_4(f_o)$.

f_c, f_o where centre and outer faces

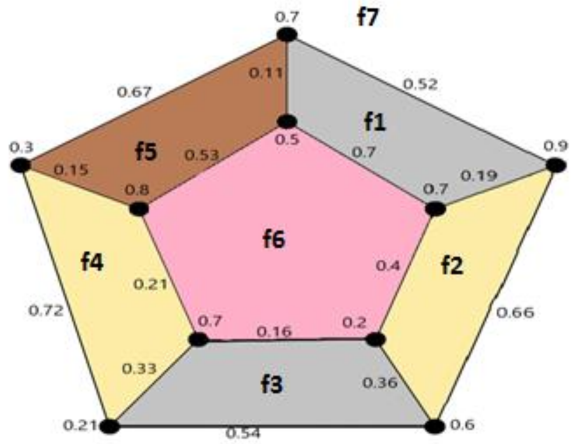
Hence, $\chi''\ (CL_n) =\ 4$.

Examples:



$c(f_1)c(f_3) \neq c(f_2)c(f_4) \neq\ c(f_c = f_5), c(f_o = f_6)$.

(a) CL_4



$$c(f_1)c(f_3) \neq c(f_2)c(f_4) \neq c(f_5) \neq \ c(f_c = f_6), c(f_o = f_7)$$

(b) CL_5

4.Conclusion:

In this paper, we discussed the basic definitions of the fuzzy magic graphs, various common types of the fuzzy magic graphs and finally about the about the face coloring of the certain types of fuzzy magic graph.

We determined face chromatic number for certain types of fuzzy magic graphs such as wheel graph (Fan graphs, Helm graphs too), ladder graph, and circular ladder graph and also observed with various examples as required.

In future we try to study the magic and anti-magic fuzzy planar graphs.

REFERENCES

[1]. Haray E., Graph theory, Addition Wesley, Reading, MA (1969).

[2]. Zadeh, L. A. (1971). Similarity relations and fuzzy orderings. Information Sciences, 3(2), 177–200.

[3]. Bhattacharya, P. (1987). Some remarks on fuzzy graphs. Pattern Recognition Letters, 6(5), 297–302.

[4]. Petr Kovář (2004), Magic Labelings of Graphs, Ph.D. Thesis, VSB – Technical University of Ostrava Faculty of Electrical Engineering and Computer Science

[5]. Jamil R.N., Javaid M., ur Rehman M.A., Kirmani K.N. (2016), On the construction of fuzzy magic graphs, Science International 28(3).

[6]. K.R. Sobha, S. Chandra Kumar and R.S. Sheeba (2018), Fuzzy Magic Graphs-A Brief Study, International Journal of Pure and Applied Mathematics, Volume 119 No. 15 2018, 1161-1170, Volume 119 No. 15 2018, 1161-1170.

[7]. Stephen John. B & Anlin Jeni . J,(2019), face magic labeling, IJRAR- International Journal of Research and Analytical Reviews, E ISSN 2348 –1269, PRINT ISSN 2349-5138.

[8]. Agustin I H, Alfarisi R, Dafik, Kristiana A I, Prihandini R M and Kurniawati E Y, (2018), On Super Local Antimagic Total Edge Coloring of Some Wheel Related Graphs, AIP Conference Proceedings 2014.

[9]. R Nisviasari, Dafik , T K Maryati , I H Agustin , E Y Kurniawati (2019), Local super antimagic total face coloring of planar graphs, ICEGE 2018, IOP Conf. Series: Earth and Environmental Science 243 012117, IOP Publishing.

[10]. .Susana Munoz, M.Teresa Ortuno, Javier Ramirez, Javier Yanez, Coloring fuzzy graphs ELSEVIER, pp: 211-221 (2004).

[11]. R. Shanmugapriya and P. K. Hemalatha(2021), Application of Fuzzy Vertex Magic graph, Proceedings of First International Conference on Mathematical Modeling and Computational Science, Advances in Intelligent Systems and Computing 1292, https://doi.org/10.1007/978- 981-33-4389-4_53

[12]. [Shanmugapriya, R., Hemalatha, P. K., & Suba, M. (2019, July). Fuzzy graceful labeling on the double fan graphs and the double wheel graphs. IJITEE, 8(9).