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RESEARCH ARTICLE

On Ranking Measures of Intuitionistic Fuzzy Multi Numbers

E. Vivek[1], N. Uma[2], M. Keerthika[3]

[1,2]Department of Mathematics, Sri Ramakrishna College of Arts and Science (Formerly SNR Sons College), Coimbatore, Tamil Nadu. (INDIA)

Email:vivek@srcas.ac.in[1], uma.n@srcas.ac.in[2]

[3]M.Sc. Student, Department of Mathematics, Sri Ramakrishna College of Arts and Science (Formerly SNR Sons College), Coimbatore, Tamil Nadu. (INDIA).

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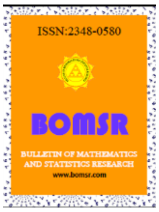

ABSTRACT

Two Ranking techniques of Intuitionistic Fuzzy Multi Numbers are introduced and compared in this paper. To transform the Intuitionistic Fuzzy Multi Numbers to crisp data these Ranking technique can be used for any multi decision analysis. The efficiency of the defined technique is illustrated with the numerical examples. Here, we have considered the Triangular, Trapezoidal, Pentagonal, Hexagonal and Octagonal Intuitionistic Fuzzy Multi Numbers with its membership and non-membership functions.

Keywords: Intuitionistic Fuzzy Set, Intuitionistic Fuzzy Multi Set, Triangular Intuitionistic Fuzzy Multi Number, Trapezoidal Intuitionistic Fuzzy Multi Number, Pentagonal Intuitionistic Fuzzy Multi Number, Hexagonal Intuitionistic Fuzzy Multi Number, Octagonal Intuitionistic Fuzzy Multi Number, Ranking Function.

2020 MSC: 03B52; 03E72.

I. INTRODUCTION

Zadeh [23] introduced the Fuzzy Set (FS) & Later, **Zimmermann [24]** proposed the basic definitions of fuzzy sets, types of fuzzy sets, algebraic operations defined on fuzzy sets and its extension. **Atanassov [2, 3]** proposed the Intuitionistic Fuzzy sets (IFS) as the generalization of the

Fuzzy set. The IFS represent the uncertainty with respect to both membership ($\mu \in [0,1]$) and non membership ($\vartheta \in [0, 1]$) such that $\mu + \vartheta \leq 1$. Intuitionistic fuzzy sets are suitable to handle problems with imprecise information as they are characterized by its membership and non-membership values. Both the theories of Fuzzy and Intuitionistic fuzzy sets were applied in many real-life decision-making problems.

The Fuzzy Number was defined by **Dijkman et.all [9]**, which is a quantity whose value is imprecise, rather than exact as is the case with "ordinary" (single-valued) numbers and its membership function is monotonic on both sides of the highest membership values. Ranking the fuzzy numbers is an important aspect of decision making in a fuzzy environment for practical applications. **Chen [5]** introduces the concept of ordering (ranking) value of each fuzzy number and uses these values to determine the order of the *n* fuzzy numbers. **Bharati [4] & Kumar [11]** define the intuitionistic fuzzy number and considers a fuzzy origin and measures distance of each intuitionistic fuzzy number from fuzzy origin and then compare distance between them. They have also defined new ranking function for Intuitionistic fuzzy numbers and verified its axioms.

For ranking of Intuitionistic fuzzy number with efficiency, the five types of Triangular, Trapezoidal, Pentagonal, Hexagonal and Octagonal Intuitionistic Fuzzy numbers plays an important role. **Deng-Feng Li [8]** defined the concept of a triangular intuitionistic fuzzy numbers and develop a new methodology for ranking on the concept of a ratio of the value index to the ambiguity index. **Das [7]** proposed a new ranking procedure for trapezoidal intuitionistic fuzzy number by taking sum of value and ambiguity index. **Annie Christi [1]** defined pentagonal intuitionistic fuzzy numbers for transportation problem in an intuitionistic fuzzy environment by using the accuracy function. **Sahaya Sudha [18] and Thamaraiselvi [20]** introduced the intuitionistic hexagonal fuzzy numbers focusing on alpha cuts without affecting its originality. **Menaka [12]** introduced the octagonal intuitionistic fuzzy numbers for finding an optimal solution for intuitionistic fuzzy transportation problem.

Yager [22] introduced fuzzy multisets, he uses the term fuzzy bag; an element of X may occur more than once with possibly of the same or different membership values. **Shinoj [19]** proposed a new concept of Intuitionistic Fuzzy Multi Sets (IFMS, which allows the repeated occurrences of different membership and non-membership functions. Later, various Distance and Similarity Measures of IFMS are extended from the IFS, which shows that these measures may be applied in many decision-making situations **[13],[14],[15],[16] [17], and [21].** In this paper, we would like to introduce the Ranking Measures of IFMN. The numerical evaluation shows that the proposed are well suited to use for any linguistic variables.

II PRELIMINARIES

(a) The basic concepts and definitions from various articles applicable to this study are reviewed in this section.

Definition: 2.1 [23]

Let X be a nonempty set. A **fuzzy set** A of X is defined as A = { < x, $\mu_A(x)$ > / x∈ X }, where $\mu_A(x)$ is called membership function and it maps each element of X to a value between 0 and 1. The generalizations of fuzzy sets are the Intuitionistic fuzzy *(IFS)* set with independent membership and non-membership function.

Definition: 2.2 [2], [3]

Let X is a non-empty set. An **intuitionistic fuzzy set** A of X is given by A = {< x, μ_A (x), ϑ_A(x) > / x∈ X} where μ_A : X → [0,1] and ϑ_A : X → [0,1] with the condition $0 \le \mu_A(x) + \vartheta_A(x) \le 1$, $\forall x \in X$ Here, $\mu_A(x)$ *and* $\vartheta_A(x)$ ∈ [0,1] denote the membership and the non-membership functions.

Definition: 2.4 [5], [9] [10]

A **fuzzy number** is a generalization of a regular real number. A fuzzy number A is a convex normalized fuzzy set on the real line R such that there exist at least one x ∈ R with $\mu_A(x)$ = 1 and $\mu_A(x)$ is piecewise continuous.

Definition: 2.5 [1],[7],[11],[12], [18], [20]

An Intuitionistic Fuzzy Subset A = { < x, μ_A (x), ϑ_A(x) > / x∈ X } of the real line R is called an **intuitionistic fuzzy number**, if there exists m in R such that $\mu_A(m) = 1$ *and* $\vartheta_A(m)$= 0, μ_A (x) is a continuous function from R→ [0,1] and ϑ_A(x) is a continuous function from

R→ [0,1] such that $0 \le \mu_A(x) + \vartheta_A(x) \le 1$, $\forall x \in X$

Definition: 2.6 [13],[14],[15],[16],[17],[19]

Let X be a nonempty set. A **intuitionistic fuzzy multi set (IFMS)** *A* in X is characterized by two functions namely count membership function Mc and count non membership function NMc such that Mc : X → Q and NMc : X → Q where Q is the set of all crisp multi sets in [0,1]. Hence, for any $x \in X$, Mc(x) is the crisp multi set from [0, 1] whose membership sequence is defined as ($\mu_A^1(x)$, $\mu_A^2(x)$, ……… $\mu_A^p(x)$) where $\mu_A^1(x) \ge \mu_A^2(x) \ge \cdots \ge \mu_A^p(x)$ and the non-membership sequence NMc(x) is defined as ($\vartheta_A^1(x), \vartheta_A^2(x), \ldots\ldots\ldots \vartheta_A^p(x)$) where the non-membership can be either decreasing or increasing function, such that

$0 \le \mu_A^i(x) + \vartheta_A^i(x) \le 1$, $\forall x \in X$ and $i = 1,2, \ldots p$. Therefore, ***IFMS A*** is given by

$A = \{\langle x, (\mu_A^1(x), \mu_A^2(x), \ldots\ldots\ldots \mu_A^p(x)), \vartheta_A^1(x), \vartheta_A^2(x), \ldots\ldots\ldots \vartheta_A^p(x)) \rangle / x \in X\}$ where $\mu_A^1(x) \ge \mu_A^2(x) \ge \cdots \ge \mu_A^p(x)$

Definition: 2.7 [13],[14],[15],[16],[17],[19]

The **cardinality** of the membership function Mc(x) is the length of an element x in the Fuzzy Multi Set *A* denoted as η, defined as η = Mc(x)

If A, B, C are the *FMS* defined on *X*, then their cardinality η = Max { η(A), η(B), η(C) }.

Definition: 2.8 [11]

The **ranking** function is approach of ordering fuzzy numbers which is an efficient. The ranking function is denoted by F (ℝ), where ℝ: F(ℝ) → ℝ , and F (ℝ) is the set of fuzzy numbers defined on a real line, where a natural order exist.

Let $a, b \in \mathbb{R}$, then **ranking function** for real numbers a, b is defined as

(i) $D(a,b) > 0 \Leftrightarrow D(a,0) > D(b,0) \Leftrightarrow a > b$
(ii) $D(a,b) < 0 \Leftrightarrow D(a,0) < D(b,0) \Leftrightarrow b < a$
(iii) $D(a,b) = 0 \Leftrightarrow D(a,0) = D(b,0) \Leftrightarrow b = a$

(b) The basic concepts and definitions which are introduced to this study are as follows

Definition: 2.9

A **fuzzy multi number** is a generalization of a regular real number. It does not refer to a single value but rather to a connected set of possible values, where each possible value has its weight between 0 and 1. The weight is called the membership function. The membership sequence is in the form $(\mu_A^1(x), \mu_A^2(x), \ldots\ldots\ldots \mu_A^p(x))$ where $\mu_A^1(x) \geq \mu_A^2(x) \geq \cdots \geq \mu_A^p(x)$

Definition: 2.10

A **intuitionistic fuzzy multi number** is an subset of intuitionistic fuzzy set

A = $\{ < x, \mu_A(x), \vartheta_A(x) > / x \in X \}$ of the real line R , if there exists m in R such that

$\mu_A(m) = 1$ *and* $\vartheta_A(m) = 0$, $\mu_A(x)$ is a continuous function from R→ [0,1] and $\vartheta_A(x)$ is a continuous function from R→ [0,1] such that $0 \leq \mu_A(x) + \vartheta_A(x) \leq 1$, $\forall x \in X$

The membership sequence is in the form
$\{(\mu_A^1(x), \mu_A^2(x), \ldots\ldots\ldots \mu_A^p(x)), (\vartheta_A^1(x), \vartheta_A^2(x), \ldots\ldots\ldots \vartheta_A^p(x))\}$
where $\mu_A^1(x) \geq \mu_A^2(x) \geq \cdots \geq \mu_A^p(x)$ and $\vartheta_A^1(x) \geq \vartheta_A^2(x) \geq \cdots \geq \vartheta_A^p(x)$.

Definition: 2.10

A **triangular intuitionistic fuzzy multi number** A_i is denoted by

$A_i = \{(a_1^i, a_2^i, a_3^i)\,(b_1^i, b_2^i, b_3^i)\}$ where $a_1^i,\ a_2^i,\ a_3^i,\ b_1^i,\ b_2^i$ *and* b_3^i are real numbers and $b_1^i \leq a_1^i \leq a_2^i \leq a_3^i \leq b_3^i$ with the following membership $\mu_{A_i}(x)$ and non-membership function $\vartheta_{A_i}(x)$ defined as

$$\mu_{A_i}(x) = \begin{cases} 0 & for \quad x \leq a_1^i \\ \dfrac{x - a_1^i}{a_2^i - a_1^i} & for\ a_1^i \leq x \leq a_2^i \\ 1 & for\ x = a_2^i \\ \dfrac{a_3^i - x}{a_3^i - a_2^i} & for\ a_2^i \leq x \leq a_3^i \\ 0 & for \quad x \geq a_3^i \end{cases}$$

$$\vartheta_{A_i}(x) = \begin{cases} 1 & for \quad x \leq b_1^i \\ \dfrac{b_2^i - x}{b_2^i - b_1^i} & for\ b_1^i \leq x \leq b_2^i \\ 0 & for\ x = b_2^i \\ \dfrac{x - b_2^i}{b_3^i - b_2^i} & for\ b_2^i \leq x \leq b_3^i \\ 1 & for \quad x \geq b_3^i \end{cases}$$

Definition: 2.11

A **trapezoidal intuitionistic fuzzy multi number** A_i is denoted by

$A_i = \{(a_1^i, a_2^i, a_3^i, a_4^i)\,(b_1^i, b_2^i, b_3^i, b_4^i)\}$ where $a_1^i,\ a_2^i,\ a_3^i,\ a_4^i,\ b_1^i,\ b_2^i, b_3^i$ *and* b_4^i are real numbers and $b_1^i \leq a_1^i \leq a_2^i \leq a_3^i \leq a_4^i \leq b_4^i$ with the following membership $\mu_{A_i}(x)$ and non-membership function $\vartheta_{A_i}(x)$ defined as

$$\mu_{A_i}(x) = \begin{cases} 0 & for \quad x \leq a_1^i \\ \dfrac{x - a_1^i}{a_2^i - a_1^i} & for\, a_1^i \leq x \leq a_2^i \\ 1 & for \ a_2^i \leq x \leq a_3^i \\ \dfrac{a_4^i - x}{a_4^i - a_3^i} & for \ a_3^i \leq x \leq a_4^i \\ 0 & for \quad x \geq a_4^i \end{cases}$$

$$\vartheta_{A_i}(x) = \begin{cases} 1 & for \quad x \leq b_1^i \\ \dfrac{b_2^i - x}{b_2^i - b_1^i} & for \ b_1^i \leq x \leq b_2^i \\ 0 & for \ b_2^i \leq x \leq b_3^i \\ \dfrac{x - b_4^i}{b_4^i - b_3^i} & for \ a_2^i \leq x \leq b_3^i \\ 1 & for \quad x \geq b_3^i \end{cases}$$

Definition: 2.12

A **pentagonal intuitionistic fuzzy multi number** A_i is denoted by

$A_i = \{(a_1^i, a_2^i, a_3^i, a_4^i, a_5^i)\,(b_1^i, b_2^i, b_3^i, b_4^i, b_5^i)\}$ where $a_1^i, a_2^i, a_3^i, a_4^i, a_5^i, b_1^i, b_2^i, b_3^i, b_4^i$ and b_5^i are real numbers and $b_1^i \leq a_1^i \leq a_2^i \leq b_2^i \leq a_3^i \leq a_4^i \leq b_4^i \leq a_5^i \leq b_5^i$ with the following membership $\mu_{A_i}(x)$ and non-membership function $\vartheta_{A_i}(x)$ defined as

$$\mu_{A_i}(x) = \begin{cases} 0 & for \quad x \leq a_1^i \\ \dfrac{x - a_1^i}{a_2^i - a_1^i} & for\, a_1^i \leq x \leq a_2^i \\ \dfrac{x - a_2^i}{a_3^i - a_2^i} & for\, a_2^i \leq x \leq a_3^i \\ 1 & for \ x = a_3^i \\ \dfrac{a_4^i - x}{a_4^i - a_3^i} & for \ a_3^i \leq x \leq a_4^i \\ \dfrac{a_5^i - x}{a_5^i - a_4^i} & for \ a_4^i \leq x \leq a_5^i \\ 0 & for \quad x \geq a_5^i \end{cases}$$

$$\vartheta_{A_i}(x) = \begin{cases} 1 & for \quad x \leq b_1^i \\ \dfrac{b_2^i - x}{b_2^i - b_1^i} & for\, b_1^i \leq x \leq b_2^i \\ \dfrac{b_3^i - x}{b_3^i - b_2^i} & for\, b_2^i \leq x \leq b_3^i \\ 0 & for \ x = b_3^i \\ \dfrac{x - b_3^i}{b_4^i - b_3^i} & for \ b_3^i \leq x \leq b_4^i \\ \dfrac{x - b_4^i}{b_5^i - b_4^i} & for \ b_4^i \leq x \leq b_5^i \\ 1 & for \quad x \geq b_5^i \end{cases}$$

Definition : 2.13

A **hexagonal intuitionistic fuzzy multi number** A_i is denoted by

$A_i = \{(a_1^i, a_2^i, a_3^i, a_4^i, a_5^i, a_6^i)(b_1^i, b_2^i, b_3^i, b_4^i, b_5^i, b_6^i)\}$ where

$a_1^i,\ a_2^i,\ a_3^i,\ a_4^i,\ a_5^i,\ b_1^i,\ b_2^i, b_3^i, b_4^i\ and\ b_5^i$ are real numbers and

$b_1^i \le a_1^i \le a_2^i \le b_2^i \le a_3^i \le a_4^i \le a_5^i \le b_5^i \le a_6^i \le b_6^i$ with the following membership $\mu_{A_i}(x)$ and non-membership function $\vartheta_{A_i}(x)$ defined as

$$\mu_{A_i}(x) = \begin{Bmatrix} 0 & for\quad x \le a_1^i \\ \frac{1}{2}\left(\frac{x - a_1^i}{a_2^i - a_1^i}\right) & for\ a_1^i \le x \le a_2^i \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x - a_2^i}{a_3^i - a_2^i}\right) & for\ a_2^i \le x \le a_3^i \\ 1 & for\ a_3^i \le x \le a_4^i \\ 1 - \frac{1}{2}\left(\frac{x - a_4^i}{a_5^i - a_4^i}\right) & for\ a_4^i \le x \le a_5^i \\ \frac{1}{2}\left(\frac{a_6^i - x}{a_6^i - a_5^i}\right) & for\ a_5^i \le x \le a_6^i \\ 0 & for\quad x \ge a_6^i \end{Bmatrix}$$

$$\vartheta_{A_i}(x) = \begin{Bmatrix} 1 & for\quad x \le b_1^i \\ 1 - \frac{1}{2}\left(\frac{x - b_1^i}{b_2^i - b_1^i}\right) & for\ b_1^i \le x \le b_2^i \\ \frac{1}{2}\left(\frac{b_3^i - x}{b_3^i - b_2^i}\right) & for\ b_2^i \le x \le b_3^i \\ 0 & for\ b_3^i \le x \le b_4^i \\ \frac{1}{2}\left(\frac{x - b_4^i}{b_5^i - b_4^i}\right) & for\ b_4^i \le x \le b_5^i \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x - b_5^i}{b_6^i - b_5^i}\right) & for\ b_5^i \le x \le b_6^i \\ 1 & for\quad x \ge b_6^i \end{Bmatrix}$$

Definition: 2.14

An **octagonal intuitionistic fuzzy multi number** A_i is denoted by

$A_i = \{(a_1^i, a_2^i, a_3^i, a_4^i, a_5^i, a_6^i, a_7^i, a_8^i)(b_1^i, b_2^i, b_3^i, b_4^i, b_5^i, b_6^i, b_7^i, b_8^i)\}$ where

$a_1^i,\ a_2^i,\ a_3^i,\ a_4^i,\ a_5^i, a_6^i, a_7^i, a_8^i,\ b_1^i,\ b_2^i, b_3^i, b_4^i, b_5^i, b_6^i,\ b_7^i\ and\ b_8^i$ are real numbers and $b_1^i \le a_1^i \le a_2^i \le b_2^i \le a_3^i \le b_3^i \le a_4^i \le a_5^i \le a_6^i \le b_6^i \le a_7^i \le b_7^i \le a_8^i \le b_8^i$ with the following membership $\mu_{A_i}(x)$ and non-membership function $\vartheta_{A_i}(x)$ defined as

$$\mu_{A_i}(x) = \begin{cases} 0 & for \quad x \le a_1^i \\ \frac{1}{2}\,(\frac{x-a_1^i}{a_2^i-a_1^i}) & for\ a_1^i \le x \le a_2^i \\ \frac{1}{2} & for\ a_2^i \le x \le a_3^i \\ \frac{1}{2} + \ (1-\frac{1}{2})(\frac{x-a_3^i}{a_4^i-a_3^i}) & for\ a_3^i \le x \le a_4^i \\ 1 & for\ a_4^i \le x \le a_5^i \\ \frac{1}{2} + \left(1-\frac{1}{2}\right)(\frac{a_6^i-x}{a_6^i-a_5^i}) & for\ a_5^i \le x \le a_6^i \\ \frac{1}{2} & for\ a_6^i \le x \le a_7^i \\ \frac{1}{2}\,(\frac{a_8^i-x}{a_8^i-a_7^i}) & for\ a_7^i \le x \le a_8^i \\ 0 & for \quad x \ge \quad a_8^i \end{cases}$$

$$\vartheta_{A_i}(x) = \begin{cases} 1 & for \quad b_1^i \le x \\ \frac{1}{2} + \ (1-\frac{1}{2})(\frac{b_2^i-x}{b_2^i-b_1^i}) & for\ b_1^i \le x \le b_2^i \\ \frac{1}{2} & for\ b_2^i \le x \le b_3^i \\ \frac{1}{2}\,(\frac{b_4^i-x}{b_4^i-b_3^i}) & for\ b_3^i \le x \le b_4^i \\ 0 & for\ b_4^i \le x \le b_5^i \\ \frac{1}{2}\left(\frac{x-\ b_5^i}{b_6^i-b_5^i}\right) & fo\ r\ b_5^i \le x \le b_6^i \\ \frac{1}{2} & for\ b_6^i \le x \le b_7^i \\ \frac{1}{2} + (\,1-\frac{1}{2}\,)\,(\frac{x-\ b_7^i}{b_8^i-b_7^i}) & for\ b_7^i \le x \le b_8^i \\ 1 & for \quad x \ge \quad b_8^i \end{cases}$$

III. RANKING MEASURES OF INTUITIONISTIC FUZZY MULTI NUMBERS

Many Ranking Measures are available for Intuitionistic Fuzzy Numbers (IFN), where the membership and non-membership functions only once. But in Intuitionistic Fuzzy Multi Numbers (IFMN), it should be considered more than once; because of their multi membership and non-membership functions. And, their considerations are combined together by means of Summation concept based on their cardinality. In this paper we have extended two ranking methods of IFN to IFMN.

Method I : The Ranking Measure by the **Graded Mean Integration Representation [6]** of Intuitionistic Fuzzy Multi Number (IFMN) A_i is defined for Triangular, Trapezoidal, Pentagonal, Hexagonal and Octagonal with its membership and non-membership functions.

- Triangular intuitionistic fuzzy multi numbers $\{(a_1^i, a_2^i, a_3^i)\ (b_1^i, b_2^i, b_3^i)\}$ is $\frac{1}{\eta}\sum_{i=1}^{\eta} Max\ [\frac{(a_1^i + 4\ a_2^i + a_3^i)}{6}, \frac{(b_1^i + 4\ b_2^i + b_3^i)}{6}]$
- Trapezoidal intuitionistic fuzzy multi numbers $\{(a_1^i, a_2^i, a_3^i, a_4^i)\ \ (b_1^i, b_2^i, b_3^i, b_4^i)\}$ is $\frac{1}{\eta}\sum_{i=1}^{\eta} Max\ [\frac{(a_1^i+2\ a_2^i+ 2a_3^i + a_4^i)}{6}, \frac{(b_1^i+ 2b_2^i+ 2b_3^i + b_4^i)}{6}]$
- Pentagonal intuitionistic fuzzy multi numbers $\{(a_1^i, a_2^i, a_3^i, a_4^i, a_5^i)\ \ (b_1^i, b_2^i, b_3^i, b_4^i, b_5^i)\}$ is $\frac{1}{\eta}\sum_{i=1}^{\eta} Max\ [\frac{(a_1^i+ a_2^i+ 4a_3^i + a_4^i + a_5^i)}{8}, \frac{(b_1^i+ b_2^i+ 4b_3^i + b_4^i+ b_5^i)}{8}]$
- Hexagonal intuitionistic fuzzy multi numbers $\{(a_1^i, a_2^i, a_3^i, a_4^i, a_5^i, a_6^i)\ \ (b_1^i, b_2^i, b_3^i, b_4^i, b_5^i, b_6^i)\}$ is $\frac{1}{\eta}\sum_{i=1}^{\eta} Max\ [\frac{(a_1^i+ a_2^i+ 2a_3^i +2\ a_4^i + a_5^i+ a_6^i)}{8}, \frac{(b_1^i+ b_2^i+ 2b_3^i + 2b_4^i+ b_5^i+ b_6^i)}{8}]$
- Octagonal intuitionistic fuzzy multi numbers $\{(a_1^i, a_2^i, a_3^i, a_4^i, a_5^i, a_6^i, a_7^i,\ a_8^i)\ (b_1^i, b_2^i, b_3^i, b_4^i, b_5^i, b_6^i, b_7^i,\ b_8^i)\}$ is $\frac{1}{\eta}\sum_{i=1}^{\eta} Max\ [\frac{(a_1^i+ a_2^i+ a_3^i +2\ a_4^i + 2a_5^i+ a_6^i + a_7^i + a_8^i)}{10}, \frac{(b_1^i+ b_2^i+ b_3^i + 2b_4^i+ 2b_5^i+ b_6^i + b_7^i+ b_8^i)}{10}]$

Method II

Here, we have introduced another method using the **Helipern's Expected Value[10]** using the notions of the expected value which are based on the lower and upper expected values of A_i, Intuitionistic Fuzzy Multi Number for Triangular, Trapezoidal, Pentagonal, Hexagonal and Octagonal with its membership and non-membership functions.

- Triangular intuitionistic fuzzy multi numbers $\{(a_1^i, a_2^i, a_3^i)\ (b_1^i, b_2^i, b_3^i)\}$ is $\frac{1}{\eta}\sum_{i=1}^{\eta} Max\ [\frac{(a_1^i + 2\ a_2^i + a_3^i)}{4}, \frac{(b_1^i + 2\ b_2^i + b_3^i)}{4}]$
- Trapezoidal intuitionistic fuzzy multi numbers $\{(a_1^i, a_2^i, a_3^i, a_4^i)\ \ (b_1^i, b_2^i, b_3^i, b_4^i)$ is $\frac{1}{\eta}\sum_{i=1}^{\eta} Max\ [\frac{(a_1^i+ a_2^i+ a_3^i + a_4^i)}{4}, \frac{(b_1^i+ b_2^i+ b_3^i + b_4^i)}{4}]$
- Pentagonal intuitionistic fuzzy multi numbers $\{(a_1^i, a_2^i, a_3^i, a_4^i, a_5^i)\ \ (b_1^i, b_2^i, b_3^i, b_4^i, b_5^i)\}$ is $\frac{1}{\eta}\sum_{i=1}^{\eta} Max\ [\frac{(a_1^i+ a_2^i+ 2a_3^i + a_4^i + a_5^i)}{6}, \frac{(b_1^i+ b_2^i+ 2b_3^i + b_4^i+ b_5^i)}{6}]$
- Hexagonal intuitionistic fuzzy multi numbers $\{(a_1^i, a_2^i, a_3^i, a_4^i, a_5^i, a_6^i)\ \ (b_1^i, b_2^i, b_3^i, b_4^i, b_5^i, b_6^i)\}$ is $\frac{1}{\eta}\sum_{i=1}^{\eta} Max\ [\frac{(a_1^i+ a_2^i+ a_3^i + a_4^i + a_5^i+ a_6^i)}{6}, \frac{(b_1^i+ b_2^i+ b_3^i + b_4^i+ b_5^i+ b_6^i)}{6}]$
- Octagonal intuitionistic fuzzy multi numbers $\{(a_1^i, a_2^i, a_3^i, a_4^i, a_5^i, a_6^i, a_7^i,\ a_8^i)\ (b_1^i, b_2^i, b_3^i, b_4^i, b_5^i, b_6^i, b_7^i,\ b_8^i)\}$ is $\frac{1}{\eta}\sum_{i=1}^{\eta} Max\ \left[\frac{(a_1^i+ a_2^i+ a_3^i + a_4^i + a_5^i+ a_6^i + a_7^i + a_8^i)}{8}, \frac{(b_1^i+ b_2^i+ b_3^i + b_4^i+ b_5^i+ b_6^i + b_7^i+ b_8^i)}{8}\right]$

IV. NUMERICAL ILLUSTRATION OF THE INTUITIONISTIC FUZZY MULTI RANKING MEASURES

Example 1 : The Ranking Measure of the given **triangular intuitionistic fuzzy multi numbers** of cardinality **η = 4** is represented in the following table for the two extended methods of by **Chen & Hsieh** and by **Helipern.** This shows that these measures are well suited to use for any multi linguistic variables.

A = {(6,8,10) (5,8,11) ; (7,9,11) (6,9,13) ; (5,7,9) (4,7,9) ; (7,8,9) (6,8,9)}

B = { (2,3,4) (1,3,5) ; (3,4,5) (2,4,7) ; (1,2,3) (1,2,6) ; (2,4,6) (1,4,6)}

C = {(7,9,10) (6,9,11) ; (6,8,9) (5,8,10) ; (4,5,9) (4,5,9) ; (9,10,12) (8,10,14)}

Triangular Intuitionistic Fuzzy Multi Ranking Measure	A	B	C
Method 1 : Graded Mean Integration Representation	8.04	3.455	8.1225
Method 2 : Expected Value	8.0625	3.5625	8.1875
Ranking Result : C > A > B			

Example 2 : The determination of the Ranking Measure of the **trapezoidal intuitionistic fuzzy multi numbers** of cardinality **η = 3** is in the following table using the two extended methods of Graded Mean Integration Representation Method and Expected Value Methods

A = { (2,3,5,6) (0,3,5,7) ; (1,2,4,5) (1,2,4,9) ; (3,5,6,8) (3,5,6,9)}

B = {(4,5,9,11) (2,5,9,14) ; (6,11,13,15) (4,11,13,15) ; (2,3,6,8) (1,3,6,14)}

C = { (2,6,9,12) (2,6,9,16) ; (1,5,7,9) (1,5,7,15) ; (4,7,9,13) (3,7,9,18)}

Trapezoidal Intuitionistic Fuzzy Multi Ranking Measure	**A**	**B**	**C**
Method 1 : Graded Mean Integration Representation	4.433	8.11	8.1633
Method 2 : Expected Value	4.583	8.25	8.66
Ranking Result : C > A > B			

Example 3 : The Ranking Measure of **pentagonal intuitionistic fuzzy multi numbers with** cardinality **η = 3** is is illustrated in the following tabular column for the two defined methods of **Chen & Hsieh** and **Helipern**.

A = {(1,3,5,7,10) (0,3,5,6,11) ; (2,2,4,6,9) (2,4,4,7,13) ; (3,5,6,8,12) (1,2,6,8,12)}

B = {(2,4,8,13,15) (1,3,8,12,15) ; (2,3,7,11,15) (2,4,7,11,17) ; (4,5,8,10,14) (0,5,8,13,16)}

C = {(4,7,9,12,15) (3,6,9,11,16) ; (3,6,8,10,11) (2,5,8,9,16) ; (2,8,9,15,16) (2,7,9,13,17)}

Pentagonal Intuitionistic Fuzzy Multi Ranking Measure	**A**	**B**	**C**
Method 1 : Graded Mean Integration Representation	5.625	8.0	9.25
Method 2 : Expected Value	5.826	8.106	9.443
Ranking Result : C > A > B			

Example 4 : The Ranking Measure of the given **hexagonal intuitionistic fuzzy multi numbers** of cardinality **η = 3** is defined in the following tabular column for the methods of Graded Mean Integration Representation Method and Expected Value Method which shows that these measures are efficient.

A = {(1,3,5,6,8,11)(1,5,5,6,11,12); (0,3,5,7,9,13)(0,4,5,7,13,14); (2,3,7,9,12,13)(2,5,7,9,11,14)}

B = {(6,7,8,9,11,13)(2,3,8,9,9,13); (5,6,8,10,12,14)(4,5,8,10,12,14); (2,3,5,9,11,15)(0,3,5,9,10,16)}

C = {(1,2,3,4,5,6) (1,3,3,4, 9,14) ; (2,5,7,9,11,13) (2,6,7,9,15,16) ; (2,4,8,9,13,14) (2,5,8,9,12,15)}

Hexagonal Intuitionistic Fuzzy Multi Ranking Measure	A	B	C
Method 1 : Graded Mean Integration Representation	7.2916	8.4583	7.625
Method 2 : Expected Value	7.55	8.5533	7.9433
Ranking Result : B > C > A			

Example 5 : The two defined ranking methods of Graded Mean Integration Representation Method by **Chen & Hsieh** and Expected Value Method by **Helipern** of the given **octagonal intuitionistic fuzzy multi numbers** of cardinality **η = 3** is explained in the following table shows that, these measures are well suited for any multi criteria decision making applications.

A = {(7, 8, 9, 10, 11, 12, 13, 14) (3, 6, 7, 10, 11, 11, 12, 15) ;

(6, 7, 8, 9, 11, 13, 14, 15) (0, 2, 3, 9, 11, 12, 13, 16) ;

(5, 7, 9, 10, 12, 14, 15, 16) (2, 4, 5, 10, 12, 13, 15, 17)}

B = {(2, 4, 6, 7, 8, 9, 10, 11) (1, 2, 3, 7, 8, 8, 9, 12) ;

(1, 2, 3, 4, 6, 8, 11, 14) (1, 3, 4, 4, 6, 9, 11, 14) ;

(0, 3, 4, 7, 8, 12, 13, 15) (0, 5, 7, 7, 8, 14, 15, 16)}

C = {(1, 2,3, 6, 8, 9, 9, 10) (1, 5, 6, 6, 8, 9, 10, 13) ;

(0, 1, 2, 5, 6, 7, 9, 10) (0, 4, 5, 5, 6,12, 13 , 14) ;

(3, 4, 5, 7, 8, 9, 11, 13) (2, 4, 5, 7, 8, 10, 11, 13)}

Octagonal Intuitionistic Fuzzy Multi Ranking Measure	A	B	C
Method 1 : Graded Mean Integration Representation	10.6	7.43	7.36
Method 2 : Expected Value	10.625	7.625	7.5416
Ranking Result : A > B > C			

Analysis of the Introduced Ranking Measures

The extended ranking methods of Chen & Hsieh - Graded Mean Integration Representation Method and Helipern - Expected Value Method from the fuzzy ranking measures is analyzed by considering various cardinality for the IFMNs using the numerical example. Also, the defined ranking measures of IFMNs satisfies all the properties of general ranking measure conditions. The numerical examples clearly show the that the two **measures values are closer** and gives the **same ranking result** for triangular, trapezoidal, pentagonal, hexagonal and octagonal IFMNs.

V. CONCLUSION

Two **Ranking measures** of Graded Mean Integration Representation Method and Expected Value Method were introduced by for the **Intuitionistic Fuzzy Multi Numbers**. By the introduced Ranking Measures, the IFMNs are transformed to crisp data, which can used for any multi criteria decision making problems. We have introduced the Ranking Measures of IFMNs with its membership and non-membership function for the Triangular, Trapezoidal, Pentagonal, Hexagonal and Octagonal Numbers from the extension for fuzzy ranking measures. The efficiency of the Ranking Techniques is analyzed with the numerical examples, and it is clear that all two methods are well suited as their ranking measures are closer to one another.

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