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RESEARCH ARTICLE

ON CERTAIN GENERATES FOR MODIFIED VERMA INFORMATION MEASURES AND THEIR CORRESPONDING CONGENIAL FUZZINESS

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ABSTRACT

Some generating functions are introduced which generates the modified version of verma information measures and their corresponding information measures in fuzzy set under discrete probability distribution.

Key words: Measure of Entropy , Directed Divergence, Generating Function.

1. INTRODUCTION

In 1966, Golomb [1] defined

$$f(t) = -\sum_{i=1}^{n} p_i^t \quad (1.1)$$

as generating function with the property that

$$f'(1) = -\sum_{i=1}^{n} p_i \ln p_i \quad (1.2)$$

That is Shannon's [6] measure of entropy.

Later in 1985, Guiasu and Reisher [2] defined the generating function $g(t)$ for relative information or cross-entropy or directed divergence of one probability distribution from $P = (p_1, p_1, \dots \dots \dots \dots \dots, p_n)$ another probability distribution $Q = (q_1, q_2, \dots \dots \dots, q_n)$ by

$$g(t) = \sum_{i=1}^{n} q_i (p_{i/} q_i)^t \quad (1.3)$$

with the property, $\dots, q_n)$ $Q = (q_1, q_2, \dots$ $Q = (q_1, q_2, \dots \dots \dots, q_n)$

$$g'(1) = -\sum_{i=1}^{n} p_i \ln p_{i/} q_i \quad (1.4)$$

$$g^{r}(1) = \sum_{i=1}^{n} p_i (\ln p_{i/} q_i)^r \ , \quad r = 1,2,3, \dots \dots \dots \qquad (1.5)$$

In 1997, Kapur [4] defined the generating function

$$f_\alpha(t) = \frac{1}{1-\alpha}\left(\sum_{i=1}^{n}(p_i)^t - 1\right), \ \alpha \neq 1 \qquad (1.6)$$

with the property,

$$f_\alpha(1) = \frac{1}{1-\alpha}\left(\sum_{i=1}^{n} p_i^\alpha - 1\right), \ \alpha \neq 1 \qquad (1.7)$$

and $$f'_\alpha(0) = \frac{1}{1-\alpha} \ln \sum_{i=1}^{n} p_i^\alpha \ , \ \alpha \neq 1 \qquad (1.8)$$

Kapur [3] also defined the generating function for relative information or cross-entropy or directed divergence of $P = (p_1, p_2, \dots \dots \dots, p_n)$ from another probability distribution $Q = (q_1, q_2, \dots \dots \dots, q_n)$ by

$$g_\alpha(t) = \frac{1}{\alpha-1}\left[\left(\sum_{i=1}^{n} p_i^\alpha q_i^{1-\alpha}\right)^t - 1\right], \ \alpha \neq 1 \qquad (1.9)$$

with the property that,

$$g_\alpha(1) = \frac{1}{\alpha-1}\left[\sum_{i=1}^{n} p_i^\alpha q_i^{1-\alpha} - 1\right], \ \alpha \neq 1 \qquad (1.10)$$

And $$g'_\alpha(1) = \frac{1}{\alpha-1} \ln \sum_{i=1}^{n} p_i^\alpha q_i^{1-\alpha} \ , \ \alpha \neq 1 \qquad (1.11)$$

Later in 2012, Verma, R. K. [7] introduce the new measures of information

$$V_a(P) = -\sum_{i=1}^{n} \ln(1 + a p_i) + \sum_{i=1}^{n} \ln p_i + \ln(1 + a), \qquad a > 0 \qquad (1.12)$$

and its fuzzified form $i.e.$

$$V_a(A) = -\sum_{i=1}^{n} \ln\left(1 + a\mu_A(x_i)\right) - \sum_{i=1}^{n} \ln\left(1 + a - \mu_A(x_i)\right) + \sum_{i=1}^{n} \ln \mu_A(x_i) + \sum_{i=1}^{n} \ln\left(1 - \mu_A(x_i)\right) + \ln(1 + a), \qquad a > 0. \qquad (1.13)$$

Verma [7] also introduce the corresponding directed divergence, using Kullback-Liebler [5] concept, is given by

$$D_a(P{:}\,Q) = \sum_{i=1}^{n} q_i \ln\left(\frac{q_i + a p_i}{p_i}\right) - \ln(1 + a)\,, \qquad a > 0 \qquad (1.14)$$

whose measures in fuzzy set is given by,

$$D_a(A{:}\,B) = \sum_{i=1}^{n} \mu_B(x_i) \ln\left(\frac{a\mu_A(x_i) + \mu_B(x_i)}{\mu_A(x_i)}\right) + \sum_{i=1}^{n}\left(1 - \mu_B(x_i)\right) \ln\left(\frac{1 + a - a\mu_A(x_i) - \mu_B(x_i)}{1 - \mu_A(x_i)}\right) - \ln(1 + a)\,, \qquad a > 0 \qquad (1.15)$$

and the directed divergence deduced by general method, is given by

$$D_a(P{:}\,Q) = \sum_{i=1}^{n} q_i (1 + a q_i) \ln \frac{q_i(1 + a p_i)}{p_i(1 + a q_i)} \ , \qquad a > 0 \qquad (1.16)$$

whose measures in fuzzy set is given by,

$$D_a(A{:}\,B) = \sum_{i=1}^{n}\left(\mu_B(x_i) + a\mu_B^2(x_i)\right) \ln \frac{\mu_B(x_i)\left(1 + a\mu_A(x_i)\right)}{\mu_A(x_i)\left(1 + a\mu_B(x_i)\right)} + \sum_{i=1}^{n}\left(\left(1 - \mu_B(x_i)\right) + a\left(1 - \mu_B(x_i)\right)^2\right) \ln \frac{\left(1 - \mu_B(x_i)\right)\left(1 + a - a\mu_A(x_i)\right)}{\left(1 - \mu_A(x_i)\right)\left(1 + a - a\mu_B(x_i)\right)} \ , \qquad a > 0 \qquad (1.17)$$

Verma [8] also defined,

$$f_{a,b}(t) = -\sum_{i=1}^{n}\left(\frac{b+ap_i}{bp_i}\right)^t + \sum_{i=1}^{n}(b+a)^t p_i, \qquad a,b>0 \qquad (1.18)$$

as generating function for, Verma [8] measures of information, with the property that,

$$f'_{a,b}(t) = -\sum_{i=1}^{n}\left(\frac{b+ap_i}{bp_i}\right)^t \ln\left(\frac{b+ap_i}{bp_i}\right) + \sum_{i=1}^{n}(b+a)^t \ln(b+a)\, p_i, \quad a,b>0 \qquad (1.19)$$

so that, $$f'_{a,1}(0) = -\sum_{i=1}^{n}\ln\left(\frac{1+ap_i}{p_i}\right) + \ln(1+a), \qquad a>0 \qquad (1.20)$$

which is the same as (1.12).

again Verma [8] defined the generating function, for Verma [8] measures of information, in fuzzy set

$$\sum_{i=1}^{n}\left(\frac{b^2\mu_A(x_i)-b^2\mu_A^2(x_i)}{b^2+ab^2+a^2b^2\mu_A(x_i)-a^2b^2\mu_A^2(x_i)}\right)^t + (b+a)^t$$

$$= \sum_{i=1}^{n}\left(\frac{b\mu_A(x_i)(b-b\mu_A(x_i))}{(b+ab\mu_A(x_i))(b+ab-ab\mu_A(x_i))}\right)^t + (b+a)^t, \qquad a,b>0. \qquad (1.21)$$

on differentiating with respect to t and taking $t=0$, $b=1$ we get the following result

$$\sum_{i=1}^{n}\ln\left(\frac{\mu_A(x_i)(1-\mu_A(x_i))}{(1+a\mu_A(x_i))(1+a-a\mu_A(x_i))}\right) + \ln(1+a)$$

$$= -\sum_{i=1}^{n}\ln\big(1+a\mu_A(x_i)\big) - \sum_{i=1}^{n}\ln\big(1+a-a\mu_A(x_i)\big) +$$

$$\sum_{i=1}^{n}\ln\mu_A(x_i) + \sum_{i=1}^{n}\ln\big(1-\mu_A(x_i)\big) + \ln(1+a), \qquad a>0 \qquad (1.22)$$

which is the same as (1.13).

again Verma [7] defined the generating function for Verma [7] measures of corresponding directed divergence (effected by Kullback-Leibler [5])

$$g_a(t) = \sum_{i=1}^{n} q_i\left(\frac{q_i+ap_i}{p_i}\right)^t - (1+a)^t, \qquad a>0 \qquad (1.23)$$

so that, $$g'_a(0) = \sum_{i=1}^{n} q_i \ln\left(\frac{q_i+ap_i}{p_i}\right) - \ln(1+a) \qquad a>0 \qquad (1.24)$$

which is the same as (1.14).

Verma [7] also defined the generating function for Verma [7] measures of corresponding directed divergence (using general method)

$$g_a(t) = \sum_{i=1}^{n} q_i(1+aq_i)\left(\frac{(q_i+ap_iq_i)}{(p_i+ap_iq_i)}\right)^t, \qquad a>0 \qquad (1.25)$$

on differentiating with respect to t and taking $t=0$ we get the following result

$$g'_a(0) = \sum_{i=1}^{n} q_i(1+aq_i)\ln\left(\frac{1+ap_i/1+aq_i}{p_i/q_i}\right), \qquad a>0 \qquad (1.26)$$

which is same as (1.16)

Verma [9] defined the generating function for Verma [7] measures of corresponding directed divergence (effected by Kullback-Leibler [5]) in fuzzy set

$$\sum_{i=1}^{n}\mu_B(x_i)\left(\frac{a\mu_A(x_i)-a\mu_A^2(x_i)+\mu_B(x_i)-\mu_A(x_i)\mu_B(x_i)}{\mu_A(x_i)+a\mu_A(x_i)-a\mu_A^2(x_i)-\mu_A(x_i)\mu_B(x_i)}\right)^t +$$

$$\sum_{i=1}^{n}\left(\frac{1+a-a\mu_A(x_i)-\mu_B(x_i)}{1-\mu_A(x_i)}\right)^t-(1+a)^t, \qquad a>0 \qquad (1.27)$$

on differentiating with respect to t and taking limit $t \to 0$ we get the following result

$$\sum_{i=1}^{n}\mu_B(x_i)\ln\left(\frac{a\mu_A(x_i)+\mu_B(x_i)}{\mu_A(x_i)}\right)+\sum_{i=1}^{n}\big(1-\mu_B(x_i)\big)\ln\left(\frac{1+a-a\mu_A(x_i)-\mu_B(x_i)}{1-\mu_A(x_i)}\right)-\ln(1+a)\ , \qquad a>0 \qquad (1.28)$$

which is same as (1.15).

Again Verma [9] defined the generating function for Verma [7] measures of corresponding directed divergence (using general method) in fuzzy set

$$\sum_{i=1}^{n}\ \mu_B(x_i)\left(\frac{\mu_B(x_i)(1+a\mu_A(x_i))}{\mu_A(x_i)(1+a\mu_B(x_i))}\cdot\frac{(1-\mu_A(x_i))(1+a-a\mu_B(x_i))}{(1-\mu_B(x_i))(1+a-a\mu_A(x_i))}\right)^t+\sum_{i=1}^{n}a\mu_B^2(x_i)\left(\frac{\mu_B(x_i)(1+a\mu_A(x_i))}{\mu_A(x_i)(1+a\mu_B(x_i))}\cdot\frac{(1-\mu_B(x_i))(1+a-a\mu_A(x_i))}{(1-\mu_A(x_i))(1+a-a\mu_B(x_i))}\right)^t+\sum_{i=1}^{n}\big(1+a-2a\mu_B(x_i)\big)\left(\frac{(1-\mu_B(x_i))(1+a-a\mu_A(x_i))}{(1-\mu_A(x_i))(1+a-a\mu_B(x_i))}\right)^t,\quad a>0 \qquad (1.29)$$

on differentiating with respect to t and taking limit $t \to 0$ we get the following result we get, the measure in fuzzy set,

$$\sum_{i=1}^{n}\left(\mu_B(x_i)+a\mu_B^2(x_i)\right)\ln\frac{\mu_B(x_i)(1+a\mu_A(x_i))}{\mu_A(x_i)(1+a\mu_B(x_i))}+\sum_{i=1}^{n}\left(\big(1-\mu_B(x_i)\big)+a\big(1-\mu_B(x_i)\big)^2\right)\ln\frac{(1-\mu_B(x_i))(1+a-a\mu_A(x_i))}{(1-\mu_A(x_i))(1+a-a\mu_B(x_i))}\ ,\quad a>0 \qquad (1.30)$$

which is same as (1.17).

In the present paper we investigate some generating functions for modified version of Verma [7] measures of information

$$V_a(P)=\sum_{i=1}^{n}\ln(1+ap_i)-\sum_{i=1}^{n}p_i\ln p_i-\sum_{i=1}^{n}\ln(1+a)p_i\ ,\quad a>0 \qquad (1.31)$$

and their corresponding information measures in fuzzy set under discrete probability distribution. All these are shown in the following sections **2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7** respectively.

2. NEW RESULTS

2.1 GENERATING FUNCTIONS FOR THE MODIFIED VERSION OF VERMA MEASURES OF INFORMATION

We define,

$$f_a(t)=\sum_{i=1}^{n}(1+ap_i)^{t-1}-\sum_{i=1}^{n}p_i^t-\sum_{i=1}^{n}(1+a)^{t-1}p_i, \qquad a>0 \qquad (2.1.1)$$

as generating function for modified version of Verma [7] measures of information with the property that,

$$f_a'(t)=\sum_{i=1}^{n}(1+ap_i)^{t-1}\ln(1+ap_i)-\sum_{i=1}^{n}p_i^t\ln p_i-\sum_{i=1}^{n}(1+a)^{t-1}\ln(1+a)\,p_i\ , \qquad a>0 \qquad (2.1.2)$$

so that, $$f_a'(1)=\sum_{i=1}^{n}\ln(1+ap_i)-\sum_{i=1}^{n}p_i\ln p_i-\sum_{i=1}^{n}\ln(1+a)p_i\ , \qquad a>0 \qquad (2.1.3)$$

which is the same as (1.1.31). Again, we define

$$f_{a,b}(t)=\sum_{i=1}^{n}(b+ap_i)^{t-1}-\sum_{i=1}^{n}bp_i^t-\sum_{i=1}^{n}(b+a)^{t-1}p_i, \qquad a,b>0 \qquad (2.1.4)$$

as generating function for modified version of Verma [7] measures of information with the property that,

$$f'_{a,b}(t)=\sum_{i=1}^{n}(b+ap_i)^{t-1}\ln(b+ap_i)-\sum_{i=1}^{n}bp_i^t\ln p_i-\sum_{i=1}^{n}(b+a)^{t-1}\ln(b+a)\,p_i \qquad a,b>0 \qquad (2.1.5)$$

so that, $f'_{a,1}(1)=\sum_{i=1}^{n}\ln(1+ap_i)-\sum_{i=1}^{n}p_i\ln p_i-\sum_{i=1}^{n}\ln(1+a)p_i, \qquad a>0 \qquad (2.1.6)$

which is the same as (1.1.31).

2.2 MODIFIED VERSION OF VERMA MEASURES OF INFORMATION IN A FUZZY SET

Modified version of Verma [7] measures of information $i.e.$

$$V_a(P)=\sum_{i=1}^{n}\ln(1+ap_i)-\sum_{i=1}^{n}p_i\ln p_i-\sum_{i=1}^{n}\ln(1+a)p_i\,, \qquad a>0$$

we get the corresponding measures in fuzzy set,

$$V_a(A)=\sum_{i=1}^{n}\ln\big(1+a\mu_A(x_i)\big)+\sum_{i=1}^{n}\ln\big(1+a-a\mu_A(x_i)\big)-\sum_{i=1}^{n}\mu_A(x_i)\ln\mu_A(x_i)-\sum_{i=1}^{n}\big(1-\mu_A(x_i)\big)\ln\big(1-\mu_A(x_i)\big)-\ln(1+a)\,, \qquad a>0 \qquad (2.2.1)$$

2.3 GENERATING FUNCTIONS FOR THE MODIFIED VERSION OF VERMA MEASURES OF INFORMATION IN A FUZZY SET

We define,

$$\sum_{i=1}^{n}\left(\frac{1+a+a^2\mu_A(x_i)-a^2\mu_A^2(x_i)}{1-\mu_A(x_i)}\right)^{t-1}+\sum_{i=1}^{n}\mu_A(x_i)\left(\frac{1-\mu_A(x_i)}{\mu_A(x_i)}\right)^{t-1}-(1+a)^{t-1}$$

$$=\sum_{i=1}^{n}\left(\frac{(1+a\mu_A(x_i))(1+a-a\mu_A(x_i))}{(1-\mu_A(x_i))}\right)^{t}+\sum_{i=1}^{n}\mu_A(x_i)\left(\frac{1-\mu_A(x_i)}{\mu_A(x_i)}\right)^{t-1}-(1+a)^{t-1}, \qquad a>0 \qquad (2.3.1)$$

Now, differentiating (2.3.1) with respect to t, we get

$$\sum_{i=1}^{n}\left(\frac{(1+a\mu_A(x_i))(1+a-a\mu_A(x_i))}{(1-\mu_A(x_i))}\right)^{t-1}\ln\left(\frac{(1+a\mu_A(x_i))(1+a-a\mu_A(x_i))}{(1-\mu_A(x_i))}\right)+\sum_{i=1}^{n}\mu_A(x_i)\left(\frac{1-\mu_A(x_i)}{\mu_A(x_i)}\right)^{t-1}\ln\left(\frac{1-\mu_A(x_i)}{\mu_A(x_i)}\right)-(1+a)^{t-1}\ln(1+a)\,, \qquad a>0 \qquad (2.3.2)$$

On taking $t=1$ we get,

$$\sum_{i=1}^{n}\ln\left(\frac{(1+a\mu_A(x_i))(1+a-a\mu_A(x_i))}{(1-\mu_A(x_i))}\right)+\sum_{i=1}^{n}\mu_A(x_i)\ln\left(\frac{1-\mu_A(x_i)}{\mu_A(x_i)}\right)-\ln(1+a)$$

$$=\sum_{i=1}^{n}\ln\big(1+a\mu_A(x_i)\big)+\sum_{i=1}^{n}\ln\big(1+a-a\mu_A(x_i)\big)-\sum_{i=1}^{n}\mu_A(x_i)\ln\mu_A(x_i)-\sum_{i=1}^{n}\big(1-\mu_A(x_i)\big)\ln\big(1-\mu_A(x_i)\big)-\ln(1+a)\,, \qquad a>0 \qquad (2.3.3)$$

which is the same as (2.2.1).

Again, we define

$$\sum_{i=1}^{n}\left(\frac{b^2+ab^2+a^2b^2\mu_A(x_i)-a^2b^2\mu_A^2(x_i)}{b^2-b^2\mu_A(x_i)}\right)^{t-1}+\sum_{i=1}^{n}\mu_A(x_i)\left(\frac{b-b\mu_A(x_i)}{b\mu_A(x_i)}\right)^{t-1}-(b+a)^{t-1}$$

$$=\sum_{i=1}^{n}\left(\frac{(b+ab\mu_A(x_i))(b+ab-ab\mu_A(x_i))}{b^2-b^2\mu_A(x_i)}\right)^{t-1}+\sum_{i=1}^{n}\mu_A(x_i)\left(\frac{b-b\mu_A(x_i)}{b\mu_A(x_i)}\right)^{t-1}-(b+a)^{t-1},$$

$$a,b>0. \qquad (2.3.4)$$

Now, differentiating (2.3.4) with respect to t we get,

$$\sum_{i=1}^{n}\left(\frac{(b+ab\mu_A(x_i))(b+ab-ab\mu_A(x_i))}{b^2-b^2\mu_A(x_i)}\right)^{t-1}\ln\left(\frac{(b+ab\mu_A(x_i))(b+ab-ab\mu_A(x_i))}{b^2-b^2\mu_A(x_i)}\right)+$$

$$\sum_{i=1}^{n}\mu_A(x_i)\left(\frac{b-b\mu_A(x_i)}{b\mu_A(x_i)}\right)^{t-1}\ln\left(\frac{b-b\mu_A(x_i)}{b\mu_A(x_i)}\right)-$$

$$(b+a)^{t-1}\ln(b+a)\,, \qquad a,b>0. \qquad (2.3.5)$$

On taking $t=1$ and $b=1$ we get,

$$\sum_{i=1}^{n}\ln\left(\frac{(1+a\mu_A(x_i))(1+a-a\mu_A(x_i))}{(1-\mu_A(x_i))}\right)+\sum_{i=1}^{n}\mu_A(x_i)\ln\left(\frac{1-\mu_A(x_i)}{\mu_A(x_i)}\right)-\ln(1+a)$$

$$=\sum_{i=1}^{n}\ln\big(1+a\mu_A(x_i)\big)+\sum_{i=1}^{n}\ln\big(1+a-a\mu_A(x_i)\big)-$$

$$\sum_{i=1}^{n}\mu_A(x_i)\ln\mu_A(x_i)-\sum_{i=1}^{n}\big(1-\mu_A(x_i)\big)\ln\big(1-\mu_A(x_i)\big)-\ln(1+a)\,,$$

$$a>0$$

which is the same as (2.2.1).

2.4 CORRESPONDING MEASURE OF DIRECTED-DIVERGENCE

Motivated by Kullback and Liebler [5] we get the corresponding measure of directed- divergence

$$D_a(P:Q)=-\sum_{i=1}^{n}q_i\ln\left(1+a\frac{p_i}{q_i}\right)+\sum_{i=1}^{n}p_i\ln\frac{p_i}{q_i}+\ln(1+a), \qquad a>0 \qquad (2.4.1)$$

and the directed divergence, due to general rule, is given by

$$D_a(P:Q)=\sum_{i=1}^{n}\left(\ln\left(\frac{1+ap_i}{1+aq_i}\right)-p_i\ln p_i+q_i\ln q_i\right).\frac{1+aq_i}{a-aq_i-\ln q_i-aq_i\ln q_i-1},\ a>0. \qquad (2.4.2)$$

Here, $D_a(P:Q)$ satisfies the properties $D_a(P:Q)\geq 0$, vanishes iff $Q=P$ and is a convex function of both $p_1,p_2,\dots\dots\dots,p_n$ and $q_1,q_2,\dots\dots\dots,q_n$.

2.5 GENERATING FUNCTIONS FOR THE CORRESPONDING MEASURE OF DIRECTED-DIVERGENCE

Let,

$$g_a(t)=\sum_{i=1}^{n}q_i\left(\frac{q_i}{q_i+ap_i}\right)^t+\sum_{i=1}^{n}p_i\left(\frac{p_i}{q_i}\right)^t+(1+a)^t, \qquad a>0 \qquad (2.5.1)$$

so that,

$$g_a'(t)=\sum_{i=1}^{n}q_i\left(\frac{q_i}{q_i+ap_i}\right)^t\ln\left(\frac{q_i}{q_i+ap_i}\right)+\sum_{i=1}^{n}p_i\left(\frac{p_i}{q_i}\right)^t\ln\frac{p_i}{q_i}+(1+a)^t\ln(1+a),\ a>0 \quad (2.5.2)$$

then,

$$g_a'(0) = \sum_{i=1}^n q_i \ln\left(\frac{q_i}{q_i+ap_i}\right) + \sum_{i=1}^n p_i \ln\frac{p_i}{q_i} + \ln(1+a), \qquad a > 0 \qquad (2.5.3)$$

or,

$$g_a'(0) = -\sum_{i=1}^n q_i \ln\left(\frac{q_i+ap_i}{q_i}\right) + \sum_{i=1}^n p_i \ln\frac{p_i}{q_i} + \ln(1+a), \qquad a > 0$$

which is the same as (2.4.1).

Again, we define

$$g_{a,b}(t) = \sum_{i=1}^n q_i \left(\frac{bq_i}{bq_i+ap_i}\right)^t + \sum_{i=1}^n bq_i \left(\frac{bp_i}{bq_i}\right)^t - (b+a)^t, \qquad a,b > 0 \qquad (2.5.4)$$

so that,

$$g_{a,b}'(t) = \sum_{i=1}^n q_i \left(\frac{bq_i}{bq_i+ap_i}\right)^t \ln\left(\frac{bq_i}{bq_i+ap_i}\right) + \sum_{i=1}^n bq_i \left(\frac{bp_i}{bq_i}\right)^t \ln\frac{bp_i}{bq_i} + (b+a)^t \ln(b+a),$$

$$a,b > 0 \qquad (2.5.5)$$

then,

$$g_{a,1}'(0) = -\sum_{i=1}^n q_i \ln\left(\frac{q_i+ap_i}{q_i}\right) + \sum_{i=1}^n p_i \ln\frac{p_i}{q_i} + \ln(1+a), \qquad a > 0$$

which is the same as (2.4.1).

Now, we define

$$g_a(t) = \sum_{i=1}^n \left(\frac{{q_i}^{q_i}+ap_i{q_i}^{q_i}}{{p_i}^{p_i}+aq_i{p_i}^{p_i}}\right)^t \frac{1+aq_i}{a-aq_i-\ln q_i-1}, \qquad a > 0 \qquad (2.5.6)$$

so that,

$$g_a'(t) = \sum_{i=1}^n \left(\frac{{q_i}^{q_i}+ap_i{q_i}^{q_i}}{{p_i}^{p_i}+aq_i{p_i}^{p_i}}\right)^t \ln\left(\frac{{q_i}^{q_i}+ap_i{q_i}^{q_i}}{{p_i}^{p_i}+aq_i{p_i}^{p_i}}\right) \frac{1+aq_i}{a-aq_i-\ln q_i-1}, \qquad a > 0 \qquad (2.5.7)$$

then,

$$g_a'(0) = \sum_{i=1}^n \ln\left(\frac{{q_i}^{q_i}+ap_i{q_i}^{q_i}}{{p_i}^{p_i}+aq_i{p_i}^{p_i}}\right) \frac{1+aq_i}{a-aq_i-\ln q_i-1}, \qquad a > 0 \qquad (2.5.8)$$

or,

$$g_a'(0) = \sum_{i=1}^n \left(\ln\left(\frac{1+ap_i}{1+aq_i}\right) - p_i \ln p_i + q_i \ln q_i\right).\frac{1+aq_i}{a-aq_i-\ln q_i-aq_i\ln q_i-1}, \quad a > 0$$

which is the same as (2.4.2).

2.6 CORRESPONDING MEASURE OF FUZZY DIRECTED-DIVERGENCE

Corresponding directed-divergence for the modified Verma measures of information, motivated by Kullback-Liebler [5] concept, is given by

$$D_a(P:Q) = -\sum_{i=1}^n q_i \ln\left(1 + a\frac{p_i}{q_i}\right) + \sum_{i=1}^n p_i \ln\frac{p_i}{q_i} + \ln(1+a), \qquad a > 0$$

we get, the corresponding measures in fuzzy set,

$$D_a(A:B) = \sum_{i=1}^n \mu_B(x_i) \ln\left(\frac{\mu_B(x_i)}{a\mu_A(x_i)+\mu_B(x_i)}\right) + \sum_{i=1}^n \left(1-\mu_B(x_i)\right) \ln\left(\frac{1-\mu_B(x_i)}{1+a-a\mu_A(x_i)-\mu_B(x_i)}\right)$$

$$\sum_{i=1}^n \mu_A(x_i) \ln\frac{\mu_A(x_i)}{\mu_B(x_i)} + \sum_{i=1}^n \left(1-\mu_A(x_i)\right) \ln\left(\frac{1-\mu_A(x_i)}{1-\mu_B(x_i)}\right) +$$

$$\ln(1+a)\,, \qquad a>0 \qquad (2.6.1)$$

and the directed-divergence (2.7.2), deduced by general method, is given by

$$D_a(P:Q)=\sum_{i=1}^{n} q_i(1+aq_i)\ln\frac{q_i(1+ap_i)}{p_i(1+aq_i)}\,, \qquad a>0$$

we get, the measure in fuzzy set,

$$D_a(A:B)= \sum_{i=1}^{n}\left(\ln\left(\frac{1+\mu_A(x_i)}{1+\mu_B(x_i)}\right)-\mu_A(x_i)\ln\mu_A(x_i)+\mu_B(x_i)\ln\mu_B(x_i)\right)\frac{1+a\mu_B(x_i)}{a-a\mu_B(x_i)-\ln\mu_B(x_i)-1}+\sum_{i=1}^{n}\left(\ln\left(\frac{1+a-a\mu_A(x_i)}{1+a-a\mu_B(x_i)}\right)+\left(1-\mu_A(x_i)\right)\ln\left(1-\mu_A(x_i)\right)+\left(1-\mu_B(x_i)\right)\ln\left(1-\mu_B(x_i)\right)\right)\frac{1+a-a\mu_B(x_i)}{a-a\left(1-\mu_B(x_i)\right)-\ln\left(1-\mu_B(x_i)\right)-1}\,,$$

$$a>0. \qquad (2.6.2)$$

2.7 GENERATING FUNCTIONS FOR THE CORRESPONDING MEASURE OF FUZZY DIRECTED-DIVERGENCE

Let,

$$\sum_{i=1}^{n}\mu_A(x_i)\left(\frac{\mu_A(x_i)-\mu_A(x_i)\mu_B(x_i)}{\mu_B(x_i)-\mu_A(x_i)\mu_B(x_i)}\right)^t+\sum_{i=1}^{n}\mu_B(x_i)\left(\frac{\mu_B(x_i)+a\mu_B(x_i)-a\mu_A(x_i)\mu_B(x_i)-\mu_B^2(x_i)}{a\mu_A(x_i)+\mu_B(x_i)-a\mu_A(x_i)\mu_B(x_i)-\mu_B^2(x_i)}\right)^t+\sum_{i=1}^{n}\left(\frac{1-\mu_A(x_i)-\mu_B(x_i)+\mu_A(x_i)\mu_B(x_i)}{1+a-a\mu_A(x_i)-2\mu_B(x_i)-a\mu_B(x_i)+a\mu_A(x_i)\mu_B(x_i)+\mu_B^2(x_i)}\right)^t+(1+a)^t$$

$$=\sum_{i=1}^{n}\mu_A(x_i)\left(\frac{\mu_A(x_i)\left(1-\mu_B(x_i)\right)}{\mu_B(x_i)\left(1-\mu_A(x_i)\right)}\right)^t+\sum_{i=1}^{n}\mu_B(x_i)\left(\frac{\mu_B(x_i)\left(1+a-a\mu_A(x_i)-\mu_B(x_i)\right)}{\left(a\mu_A(x_i)+\mu_B(x_i)\right)\left(1-\mu_B(x_i)\right)}\right)^t+\sum_{i=1}^{n}\left(\frac{\left(1-\mu_A(x_i)\right)\left(1-\mu_B(x_i)\right)}{\left(1-\mu_B(x_i)\right)\left(1+a-a\mu_A(x_i)-\mu_B(x_i)\right)}\right)^t+(1+a)^t\,, \qquad a>0. \qquad (2.7.1)$$

Now, differentiating (2.7.1) with respect to t we get,

$$\sum_{i=1}^{n}\mu_A(x_i)\left(\frac{\mu_A(x_i)\left(1-\mu_B(x_i)\right)}{\mu_B(x_i)\left(1-\mu_A(x_i)\right)}\right)^t\ln\left(\frac{\mu_A(x_i)\left(1-\mu_B(x_i)\right)}{\mu_B(x_i)\left(1-\mu_A(x_i)\right)}\right)+\sum_{i=1}^{n}\mu_B(x_i)\left(\frac{\mu_B(x_i)\left(1+a-a\mu_A(x_i)-\mu_B(x_i)\right)}{\left(a\mu_A(x_i)+\mu_B(x_i)\right)\left(1-\mu_B(x_i)\right)}\right)^t\ln\left(\frac{\mu_B(x_i)\left(1+a-a\mu_A(x_i)-\mu_B(x_i)\right)}{\left(a\mu_A(x_i)+\mu_B(x_i)\right)\left(1-\mu_B(x_i)\right)}\right)+\sum_{i=1}^{n}\left(\frac{\left(1-\mu_A(x_i)\right)\left(1-\mu_B(x_i)\right)}{\left(1-\mu_B(x_i)\right)\left(1+a-a\mu_A(x_i)-\mu_B(x_i)\right)}\right)^t\ln\left(\frac{\left(1-\mu_A(x_i)\right)\left(1-\mu_B(x_i)\right)}{\left(1-\mu_B(x_i)\right)\left(1+a-a\mu_A(x_i)-\mu_B(x_i)\right)}\right)+(1+a)^t\ln(1+a)\,, \qquad a>0.$$

On taking $t=0$ we get,

$$\sum_{i=1}^{n}\mu_A(x_i)\ln\left(\frac{\mu_A(x_i)\left(1-\mu_B(x_i)\right)}{\mu_B(x_i)\left(1-\mu_A(x_i)\right)}\right)+\sum_{i=1}^{n}\mu_B(x_i)\ln\left(\frac{\mu_B(x_i)\left(1+a-a\mu_A(x_i)-\mu_B(x_i)\right)}{\left(a\mu_A(x_i)+\mu_B(x_i)\right)\left(1-\mu_B(x_i)\right)}\right)+$$

$$\sum_{i=1}^{n} \ln\left(\frac{(1-\mu_A(x_i))(1-\mu_B(x_i))}{(1-\mu_B(x_i))(1+a-a\mu_A(x_i)-\mu_B(x_i))}\right) + \ln(1+a)$$

$$= \sum_{i=1}^{n} \mu_B(x_i) \ln\left(\frac{\mu_B(x_i)}{a\mu_A(x_i)+\mu_B(x_i)}\right) + \sum_{i=1}^{n}(1-\mu_B(x_i)) \ln\left(\frac{1-\mu_B(x_i)}{1+a-a\mu_A(x_i)-\mu_B(x_i)}\right) +$$

$$\sum_{i=1}^{n} \mu_A(x_i) \ln\frac{\mu_A(x_i)}{\mu_B(x_i)} + \sum_{i=1}^{n}(1-\mu_A(x_i)) \ln\left(\frac{1-\mu_A(x_i)}{1-\mu_B(x_i)}\right) + \ln(1+a)\,, \quad a > 0.$$

which is the same as (2.6.1). Again, we define

$$\sum_{i=1}^{n} \mu_A(x_i) \left(\frac{b\mu_A(x_i)-\mu_A(x_i)\mu_B(x_i)}{b\mu_B(x_i)-\mu_A(x_i)\mu_B(x_i)}\right)^t +$$

$$\sum_{i=1}^{n} \mu_B(x_i) \left(\frac{b\mu_B(x_i)+ab\mu_B(x_i)-ab\mu_A(x_i)\mu_B(x_i)-b\mu_B^2(x_i)}{ab\mu_A(x_i)+b\mu_B(x_i)-a\mu_A(x_i)\mu_B(x_i)-\mu_B^2(x_i)}\right)^t +$$

$$\sum_{i=1}^{n} \left(\frac{b^2-b\mu_A(x_i)-b\mu_B(x_i)+\mu_A(x_i)\mu_B(x_i)}{b^2+ab-ab\mu_A(x_i)-2b\mu_B(x_i)-a\mu_B(x_i)+a\mu_A(x_i)\mu_B(x_i)+\mu_B^2(x_i)}\right)^t + (b+a)^t$$

$$= \sum_{i=1}^{n} \mu_A(x_i) \left(\frac{\mu_A(x_i)(b-\mu_B(x_i))}{\mu_B(x_i)(b-\mu_A(x_i))}\right)^t + \sum_{i=1}^{n} \mu_B(x_i) \left(\frac{\mu_B(x_i)(b+ab-ab\mu_A(x_i)-b\mu_B(x_i))}{(a\mu_A(x_i)+\mu_B(x_i))(b-\mu_B(x_i))}\right)^t +$$

$$\sum_{i=1}^{n} \left(\frac{(b-\mu_A(x_i))(b-\mu_B(x_i))}{(b-\mu_B(x_i))(b+a-a\mu_A(x_i)-\mu_B(x_i))}\right)^t + (b+a)^t\,, \quad a, b > 0. \tag{2.7.2}$$

Now, differentiating (2.7.2) with respect to t we get,

$$\sum_{i=1}^{n} \mu_A(x_i) \left(\frac{\mu_A(x_i)(b-\mu_B(x_i))}{\mu_B(x_i)(b-\mu_A(x_i))}\right)^t \ln\left(\frac{\mu_A(x_i)(b-\mu_B(x_i))}{\mu_B(x_i)(b-\mu_A(x_i))}\right) +$$

$$\sum_{i=1}^{n} \mu_B(x_i) \left(\frac{\mu_B(x_i)(b+ab-ab\mu_A(x_i)-b\mu_B(x_i))}{(a\mu_A(x_i)+\mu_B(x_i))(b-\mu_B(x_i))}\right)^t \ln\left(\frac{\mu_B(x_i)(b+ab-ab\mu_A(x_i)-b\mu_B(x_i))}{(a\mu_A(x_i)+\mu_B(x_i))(b-\mu_B(x_i))}\right) +$$

$$\sum_{i=1}^{n} \left(\frac{(b-\mu_A(x_i))(b-\mu_B(x_i))}{(b-\mu_B(x_i))(b+a-a\mu_A(x_i)-\mu_B(x_i))}\right)^t \ln\left(\frac{(b-\mu_A(x_i))(b-\mu_B(x_i))}{(b-\mu_B(x_i))(b+a-a\mu_A(x_i)-\mu_B(x_i))}\right) +$$

$$(b+a)^t \ln(b+a)\,, \quad a, b > 0.$$

On taking $t = 0$ and $b = 1$ we get,

$$\sum_{i=1}^{n} \mu_A(x_i) \ln\left(\frac{\mu_A(x_i)(1-\mu_B(x_i))}{\mu_B(x_i)(1-\mu_A(x_i))}\right) +$$

$$\sum_{i=1}^{n} \mu_B(x_i) \ln\left(\frac{\mu_B(x_i)(1+a-a\mu_A(x_i)-\mu_B(x_i))}{(a\mu_A(x_i)+\mu_B(x_i))(1-\mu_B(x_i))}\right) +$$

$$\sum_{i=1}^{n} \ln\left(\frac{(1-\mu_A(x_i))(1-\mu_B(x_i))}{(1-\mu_B(x_i))(1+a-a\mu_A(x_i)-\mu_B(x_i))}\right) + \ln(1+a)$$

$$= \sum_{i=1}^{n} \mu_B(x_i) \ln\left(\frac{\mu_B(x_i)}{a\mu_A(x_i)+\mu_B(x_i)}\right) + \sum_{i=1}^{n}(1-\mu_B(x_i)) \ln\left(\frac{1-\mu_B(x_i)}{1+a-a\mu_A(x_i)-\mu_B(x_i)}\right) +$$

$$\sum_{i=1}^{n} \mu_A(x_i) \ln\frac{\mu_A(x_i)}{\mu_B(x_i)} + \sum_{i=1}^{n}(1-\mu_A(x_i)) \ln\left(\frac{1-\mu_A(x_i)}{1-\mu_B(x_i)}\right) + \ln(1+a)\,, \quad a > 0.$$

which is the same as (2.6.1)

Now, we define

$$\sum_{i=1}^{n} \left(\left(\frac{1+\mu_A(x_i)}{1+\mu_B(x_i)}\right)^t - (\mu_A(x_i))^{t+1} + (\mu_B(x_i))^{t+1}\right) \frac{1+a\mu_B(x_i)}{a-a\mu_B(x_i)-\ln\mu_B(x_i)-1} +$$

$$\sum_{i=1}^{n}\left(\left(\frac{1+a-a\mu_A(x_i)-\mu_B(x_i)-a\mu_B(x_i)+a\mu_A(x_i)\mu_B(x_i)}{1+a-a\mu_B(x_i)-\mu_A(x_i)-a\mu_A(x_i)+a\mu_A(x_i)\mu_B(x_i)}\right)^t+\mu_A(x_i)\big(1-\mu_A(x_i)\big)^t+\mu_B(x_i)\big(1-\mu_B(x_i)\big)^t\right)\frac{1+a-a\mu_B(x_i)}{a-a(1-\mu_B(x_i))-\ln(1-\mu_B(x_i))-1}$$

$$=\sum_{i=1}^{n}\left(\left(\frac{1+\mu_A(x_i)}{1+\mu_B(x_i)}\right)^t-\big(\mu_A(x_i)\big)^{t+1}+\big(\mu_B(x_i)\big)^{t+1}\right)\frac{1+a\mu_B(x_i)}{a-a\mu_B(x_i)-\ln\mu_B(x_i)-1}+\sum_{i=1}^{n}\left(\left(\frac{(1+a-a\mu_A(x_i))(1-\mu_B(x_i))}{(1+a-a\mu_B(x_i))(1-\mu_A(x_i))}\right)^t+\mu_A(x_i)\big(1-\mu_A(x_i)\big)^t+\mu_B(x_i)\big(1-\mu_B(x_i)\big)^t\right)\frac{1+a-a\mu_B(x_i)}{a-a(1-\mu_B(x_i))-\ln(1-\mu_B(x_i))-1},\quad a>0. \tag{2.7.3}$$

Now, differentiating (2.7.3) with respect to t we get,

$$\sum_{i=1}^{n}\left(\left(\frac{1+\mu_A(x_i)}{1+\mu_B(x_i)}\right)^t\ln\left(\frac{1+\mu_A(x_i)}{1+\mu_B(x_i)}\right)-\big(\mu_A(x_i)\big)^{t+1}\ln\mu_A(x_i)+\big(\mu_B(x_i)\big)^{t+1}\ln\mu_B(x_i)\right)\frac{1+a\mu_B(x_i)}{a-a\mu_B(x_i)-\ln\mu_B(x_i)-1}+\sum_{i=1}^{n}\left(\left(\frac{(1+a-a\mu_A(x_i))(1-\mu_B(x_i))}{(1+a-a\mu_B(x_i))(1-\mu_A(x_i))}\right)^t\ln\left(\frac{(1+a-a\mu_A(x_i))(1-\mu_B(x_i))}{(1+a-a\mu_B(x_i))(1-\mu_A(x_i))}\right)+\mu_A(x_i)\big(1-\mu_A(x_i)\big)^t\ln\big(1-\mu_A(x_i)\big)+\mu_B(x_i)\big(1-\mu_B(x_i)\big)^t\ln\big(1-\mu_B(x_i)\big)\right)\frac{1+a-a\mu_B(x_i)}{a-a(1-\mu_B(x_i))-\ln(1-\mu_B(x_i))-1}$$

when $t=0$, then

$$\sum_{i=1}^{n}\left(\ln\left(\frac{1+\mu_A(x_i)}{1+\mu_B(x_i)}\right)-\mu_A(x_i)\ln\mu_A(x_i)+\mu_B(x_i)\ln\mu_B(x_i)\right)\frac{1+a\mu_B(x_i)}{a-a\mu_B(x_i)-\ln\mu_B(x_i)-1}+\sum_{i=1}^{n}\left(\ln\left(\frac{(1+a-a\mu_A(x_i))(1-\mu_B(x_i))}{(1+a-a\mu_B(x_i))(1-\mu_A(x_i))}\right)+\mu_A(x_i)\ln\big(1-\mu_A(x_i)\big)+\mu_B(x_i)\ln\big(1-\mu_B(x_i)\big)\right)\frac{1+a-a\mu_B(x_i)}{a-a(1-\mu_B(x_i))-\ln(1-\mu_B(x_i))-1}$$

$$=\sum_{i=1}^{n}\left(\ln\left(\frac{1+\mu_A(x_i)}{1+\mu_B(x_i)}\right)-\mu_A(x_i)\ln\mu_A(x_i)+\mu_B(x_i)\ln\mu_B(x_i)\right)\frac{1+a\mu_B(x_i)}{a-a\mu_B(x_i)-\ln\mu_B(x_i)-1}+\sum_{i=1}^{n}\left(\ln\left(\frac{1+a-a\mu_A(x_i)}{1+a-a\mu_B(x_i)}\right)+\big(1-\mu_A(x_i)\big)\ln\big(1-\mu_A(x_i)\big)+\big(1-\mu_B(x_i)\big)\ln\big(1-\mu_B(x_i)\big)\right)\frac{1+a-a\mu_B(x_i)}{a-a(1-\mu_B(x_i))-\ln(1-\mu_B(x_i))-1},\quad a>0$$

which is the same as (2.6.2).

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