Vol.11.Issue.3.2023 (July-Sept.)
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BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

http://www.bomsr.com

Email:editorbomsr@gmail.com

INTERNATIONAL STANDARD SERIAL NUMBER 2348-0580

RESEARCH ARTICLE

A Modish Glimpse on Mag-Shanthi Integer Sequence

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DOI:10.33329/bomsr.11.3.38

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ABSTRACT

A new integer sequence called Mag-Shanthi integer sequence generated from the recurrence relation $MS_{n+2} = 8MS_{n+1} - 7MS_n, n \geq 0$ with the initial conditions $MS_0 = 6, MS_1 = 48$ is analyzed for varieties of interesting properties.

Keywords: integer sequence, Mag-Shanthi numbers

Introduction:

Number is the essence of mathematical calculations. Numbers have varieties of patterns and have varieties of range and richness. Many numbers exhibit fascinating properties, they form sequences, they form patterns and so on. In this context one may refer [1-12]. This communication presents varieties of fascinating properties on Mag-Shanthi numbers.

Method of analysis :

The Mag-Shanthi sequence denoted by $\{MS_n\}$ is defined by the recurrence relation

$$MS_{n+2} = 8MS_{n+1} - 7MS_n \tag{1}$$

with the initial conditions

$$MS_0 = 6, MS_1 = 48 \tag{2}$$

The auxiliary equation associated with the recurrence relation (1) is given by

$$m^2 - 8m + 7 = 0 \tag{3}$$

whose roots are

$m_1 = 1, m_2 = 7$

Thus,the general solution of (1) is

$MS_n = A\,7^n + B$

From the initial conditions ,we infer that

$A = 7, B = -1$

Hence ,the n^{th} integer in the sequence $\{MS_n\}$ is given by

$MS_n = 7^{n+1} - 1$

A few interesting properties among Mag-Shanthi numbers are presented below :

Property 1: $MS_{n+1} - 7\,MS_n = 6$

Property 2: $288(MS_{n+1} - 7\,MS_n) + 1 = 1729, \text{Taxicab number}$

Property 3: $6\,(MS_{2n+1} - 2\,MS_{2n} + MS_{2n-1}) = \text{area of pythagorean triangle } (18*7^n, 24*7^n, 30*7^n)$

Proof:

$$\begin{aligned} \text{L.H.S.} &= 6[(7^{2n+2} - 1) - 2\,(7^{2n+1} - 1) + (7^{2n} - 1)] \\ &= 6\,(6*7^n)^2 \\ &= \text{area of Pythagorean triangle } (18*7^n, 24*7^n, 30*7^n) \end{aligned}$$

Property **4:** $MS_{kn+k-1} = 7\,MS_{kn+k-2} + 6$

Proof:

$\text{L.H.S,} = 7^{kn+k} - 1 = 7\,(7^{kn+k-1} - 1 + 1) - 1$

$= 7\,(7^{kn+k-1} - 1) + 6 = \text{R.H.S.}$

Property 5: $MS_{3n+2} = MS_n * MS_{2n+1} + MS_n + MS_{2n+1}$

Proof :

$$\text{L.H.S.} = 7^{3n+3} - 1 = 7^{n+1} * 7^{2n+2} - 1$$
$$= (7^{n+1} - 1)(7^{2n+2} - 1) + 7^{n+1} + 7^{2n+2} - 2$$
$$= (7^{n+1} - 1)(7^{2n+2} - 1) + (7^{n+1} - 1) + (7^{2n+2} - 1)$$
$$= \text{R.H.S.}$$

Property 6: $\sum_{i=1}^{n} MS_i = \frac{7\,MS_n - 6(7+n)}{6}$

Proof:

$$\text{L.H.S.} = \sum_{i=1}^{n} (7^{i+1} - 1) = 7\sum_{i=1}^{n} 7^i - n = \frac{7(7^{n+1} - 7)}{6} - n$$
$$= \frac{7(7^{n+1} - 1 - 6)}{6} - n = \frac{7\,MS_n}{6} - 7 - n = \text{R.H.S.}$$

Prpperty 7: $\sum_{i=1}^{n} (MS_i)^2 = \frac{49\,MS_{2n+1} - 112\,MS_n + 48(n-35)}{48}$

Proof:

$$\text{L.H.S.} = 7^2 \sum_{i=1}^{n} 7^{2i} - 14\sum_{i=1}^{n} 7^i + n$$
$$= \frac{7^2(7^{2n+2} - 49)}{48} - \frac{14(7^{n+1} - 7)}{6} + n$$
$$= \frac{7^2(MS_{2n+1} - 48)}{48} - \frac{7(MS_n - 6)}{3} + n$$
$$= \frac{49\,MS_{2n+1}}{48} - \frac{7\,MS_n}{3} - 35 + n$$
$$= \text{R.H.S.}$$

Property 8: $MS_p * MS_q = 7\,MS_{p+q} - MS_p - MS_q + 6$

Proof:

$$\text{L.H.S.} = (7^{p+1} - 1) * (7^{q+1} - 1) = 7^{p+q+2} - 7^{p+1} - 7^{q+1} + 1$$
$$= 7(7^{p+q+1} - 1 + 1) - (7^{p+1} - 1 + 1) - (7^{q+1} - 1 + 1) + 1$$
$$= 7\,MS_{p+q} - MS_p - MS_q + 6 = \text{R.H.S.}$$

Property 9: $\sum_{i=0}^{n} (MS_{i+1} - MS_i) = 7\,MS_n$

Proof:

$$\text{L.H.S.} = (MS_1 - MS_0) + (MS_2 - MS_1) + ... + (MS_n - MS_{n-1}) + (MS_{n+1} - MS_n)$$
$$= MS_{n+1} - MS_0 = (7^{n+2} - 1) - 6$$
$$= 7^{n+2} - 7 = 7\,(7^{n+1} - 1) = 7\,MS_n = \text{R.H.S.}$$

Property 10: $MS_n\,(\sum_{i=1}^{k} 7^i\,(MS_{(i+1)n} + 1)) = (MS_n + 1)^2\,((MS_n + 1)^k - 1)$

Proof:

By definition ,one has

$$MS_n + 1 = 7^{n+1} = 7 * 7^n \qquad (4)$$

Replacing n by 2 n in (4) , we have

$$MS_{2n} + 1 = 7 * 7^{2n} \qquad (5)$$

From (4) & (5) ,one obtains after some algebra that

$$7\,(MS_{2n} + 1) = (MS_n + 1)^2 \qquad (6)$$

Replacing n by 3 n ,4 n ,5 n ,....in (4) , it is seen in general that

$7^i\,(MS_{(i+1)n} + 1) = (MS_n + 1)^{i+1}\,, i = 1,2,3,...$

Therefore ,

$$\sum_{i=1}^{k} 7^i\,(MS_{(i+1)n} + 1) = \sum_{i=1}^{k} (MS_n + 1)^{i+1} \qquad (7)$$

Observe that

$$\sum_{i=1}^{k} (MS_n + 1)^{i+!} = \frac{(MS_n + 1)^2\,((MS_n + 1)^k - 1)}{MS_n} \qquad (8)$$

Substituting (8) in (7) ,the required result is obtained.

Property 11: $48\sum_{i=1}^{k} (MS_i + 1)^2 = 7^4\,MS_{2k-1}$

Proof:

Consider (6) of Property 10. We have

$$\sum_{n=1}^{k}(MS_n+1)^2 = 7\sum_{n=1}^{k}(MS_{2n}+1)$$
$$= 7\sum_{n=1}^{k}MS_{2n}+7k$$
$$= 7\sum_{n=1}^{k}(7^{2n+1}-1)+7k$$
$$= 7^2\sum_{n=1}^{k}7^{2n}$$
$$= 7^4\frac{(7^{2k}-1)}{(7^2-1)}$$
$$= 7^4\frac{MS_{2k-1}}{48}$$

Hence the property.

Property 12: $MS_{kn+1} = 7^2\,MS_{kn-1}+48$

Proof:

$$MS_{kn+1} = 7^{kn+2}-1 = 7^2(7^{kn-1+1}-1+1)-1$$
$$= 7^2\,MS_{kn-1}+7^2-1 = 7^2\,MS_{kn-1}+48$$

Property 13: $7(MS_{2n+1}+1)-(MS_n+1)^2$ is a square multiple of 6

Proof:

$$7(MS_{2n+1}+1)-(MS_n+1)^2 = 7*7^{2n+2}-(7^{n+1})^2 = 6*(7^{n+1})^2$$

Property 14: $\sum_{i=1}^{n}MS_i*MS_{i+1} = \frac{7^3\,MS_{2n+1}}{48}-\frac{28\,MS_n}{3}+n-287$

Proof:

$$L.H.S. = \sum_{i=1}^{n}(7^{i+1}-1)(7^{i+2}-1)$$
$$= 7^3\sum_{i=1}^{n}7^{2i}-56\sum_{i=1}^{n}7^i+n$$
$$= 7^3[\frac{7^2(7^{2n}-1)}{48}]-56[\frac{7(7^n-1)}{6}]+n$$
$$= \frac{7^3}{48}(MS_{2n+1}-48)-\frac{28}{3}(MS_n-6)+n$$
$$= \frac{7^3}{48}MS_{2n+1}-\frac{28}{3}MS_n+n-287 = R.H.S.$$

For simplicity and brevity ,some more properties satisfied by Mag-Shanthi numbers are exhibited below:

Property 15: $\sum_{i=0}^{n-1}(MS_i+1)(MS_i+2)=\frac{49*7^{2n}+56*7^n-105}{48}$

Property 16 : $(MS_n+1)\,(MS_n+2)$ is twice the Triangular number of rank 7^{n+1}

Property 17 : $(MS_n+1)\,(MS_n+2)\,(MS_n+3)$ is six times the Triangular pyramidal number of rank 7^{n+1}

Property 18 : $(MS_n+1)^2\,(MS_n+2)$ is twice the Pentagonal pyramidal number of rank 7^{n+1}

Property 19 : Powers of Mag-Shanthi numbers are presented :

$$(MS_n)^2=MS_{2n+1}-2MS_n$$
$$(MS_n)^3=MS_{3n+2}-3MS_{2n+1}+3MS_n$$
$$(MS_n)^4=MS_{4n+3}-4MS_{3n+2}+6MS_{2n+1}-4MS_n$$
$$(MS_n)^5=MS_{5n+4}-5MS_{4n+3}+10MS_{3n+2}-10MS_{2n+1}+5MS_n$$
$$(MS_n)^6=MS_{6n+5}-6MS_{5n+4}+15MS_{4n+3}-20MS_{3n+2}+15MS_{2n+1}-6MS_n$$

and so on. Observe the beautiful pattern of coefficients in the above representations.

Property 20 : $14*\text{Pentagonal pyramidal number of rank } 7^n-MS_{n-1}+MS_{2n-1}(1-MS_n)+2$ is a perfect square.

Conclusion

In this paper , a new sequence of integers named as Mag-Shanthi numbers have been introduced along with interesting relations among Mag-Shanthi numbers. One may search for other connections between Mag-Shanthi numbers.

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