Vol.11.Issue.3.2023 (July-Sept.)

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

http://www.bomsr.com

Email:editorbomsr@gmail.com

INTERNATIONAL STANDARD SERIAL NUMBER 2348-0580

SHORT NOTES

The Revised Ellipse Perimeter Approximation Formula P (a, b; 2, k).

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DOI:10.33329/bomsr.11.3.81

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Introduction

In this Note, the author informs the mathematical community that **a simpler formula for Ellipse Perimeter Approximation**, can be carved out of his formula published a few months ago [1]. In [1], the author introduced a new family of formulae P (a, b; p, k), containing two parameters p and k. By assigning 2.11 to p and a sixth-degree polynomial to k, the Relative Error in the approximation could be reduced to the order of 10^{-8}.

However, the school/college students may not appreciate the approximation formula due to the presence of the sixth-degree polynomial used as an index. Therefore, the author simplifies the approximation formula by putting **p = 2**, and, replacing the polynomial index **k** by a simple and easy to remember algebraic expression of the major and minor radii. These changes make the computation simpler; but the Relative Error increases to the order of 10^{-5}.

The Revised Ellipse Perimeter Approximation Formula P (a, b; 2, k).

P (a, b; 2, k): = 4* Q (a, b; 2, k), where the Quarter Perimeter Q (a, b; 2, k) is given by

Q (a, b; 2, k): = $\sqrt{a * a + b * b}$ + (π/2 - $\sqrt{2}$) * (GM/AM) ^{k} *(GM), where:

GM = $\sqrt{a * b}$, AM = (a + b)/2 and k = 2.633 + b / (a + b).

Obviously, P (a, a; 2, k) = 2 π ∗ a and P (a, 0; 2, k) = 4*a. Further, the Relative Error lies between -7.32894218*(10^{-5}) and 7.68006814*(10^{-5}). (Table)

Table for verification. (* QPM is abbreviation used for Quarter Perimeter)

a	b	Q (a, b; Sim): QPM* by Simpson's (1/3) Rule	Q (a, b; 2, r): QPM* by the Revised Formula	Rel. Error in Q (a, b; p, r) based on Q (a, b; Sim.) (Col. 4 - Col. 3) /Col. 3)
100	100	157.0796326795	157.0796326795	0.00000000E+00
100	99	156.2952211988	156.2951986495	-1.44273509E-07
100	98	155.5128030354	155.5127143670	-5.70167752E-07
100	97	154.7324086029	154.7322125824	-1.26683523E-06
100	96	153.9540689771	153.9537267656	-2.22281555E-06
100	95	153.1778159151	153.1772911227	-3.42603411E-06
100	94	152.4036818752	152.4029406137	-4.86380251E-06
100	93	151.6317000372	151.6307109708	-6.52282082E-06
100	92	150.8619043241	150.8606387162	-8.38918184E-06
100	91	150.0943294240	150.0927611818	-1.04483776E-05
100	90	149.3290108131	149.3271165286	-1.26853082E-05
Rows deleted				
100	70	134.5592245368	134.5501877033	-6.71587813E-05
100	69	133.8511777909	133.8419619879	-6.88511168E-05
100	68	133.1464274484	133.1370674395	-7.02986110E-05
100	67	132.4450361480	132.4355685671	-7.14830940E-05
100	66	131.7470682676	131.7375314676	-7.23871894E-05
100	65	131.0525899892	131.0430238791	-7.29944370E-05
100	64	130.3616693686	130.3521152372	**-7.32894218E-05**
100	63	129.6743764076	129.6648767340	-7.32579081E-05
100	62	128.9907831301	128.9813813816	-7.28869787E-05
100	61	128.3109636623	128.3017040787	-7.21651788E-05
100	60	127.6349943170	127.6259216815	-7.10826645E-05
Rows deleted				
100	30	109.6477517392	109.6556795520	7.23025562E-05
100	29	109.1470226588	109.1551663060	7.46117210E-05
100	28	108.6546463399	108.6629183532	7.61312433E-05
100	27	108.1708799880	108.1791875853	**7.68006814E-05**
100	26	107.6959938396	107.7042396084	7.65652322E-05
100	25	107.2302721895	107.2383549223	7.53773416E-05
100	24	106.7740145365	106.7818302346	7.31985037E-05
100	23	106.3275368684	106.3349799299	7.00012601E-05
100	22	105.8911731067	105.8981377190	6.57714146E-05
100	21	105.4652767431	105.4716584968	6.05104731E-05
100	20	105.0502226984	105.0559204459	5.42383183E-05
Rows deleted				
100	10	101.5993545025	101.5952794523	-4.01090165E-05
100	9	101.3375183618	101.3326907225	-4.76392100E-05
100	8	101.0940281651	101.0886387308	-5.33111046E-05
100	7	100.8699983194	100.8642887805	-5.66029443E-05
100	6	100.6667058367	100.6609697330	-5.69811405E-05
100	5	100.4856404786	100.4802196478	-5.39463231E-05
100	4	100.3285828267	100.3238550040	-4.71233872E-05
100	3	100.1977362407	100.1940846462	-3.64438818E-05
100	2	100.0959790450	100.0937221920	-2.25468901E-05
100	1	100.0274635978	100.0266776386	-7.85743371E-06
100	0	100.0000000000	100.0000000000	0.00000000E+00

Reference:

[1]. K. Idicula Koshy, "A New Ellipse Perimeter Approximation Formula that reduces the Relative Error to the order of 10-8", Bulletin of Math. and Stat. Research Vol. 11, Issue 1, 2023; pp 25-31.