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RESEARCH ARTICLE

ANALOGOUS COMPATIBLE M-DISTANCE METRIC ASSOCIATED WITH KL-DISTANCE METRIC IN IF-SETTINGS

ROHIT KUMAR VERMA
Associate Professor, Department of Mathematics,
Bharti Vishwavidyalaya, Durg, C.G., India.
E-mail- rohitkverma73@rediffmail.com
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ROHIT KUMAR VERMA

ABSTRACT

Here, a method for creating new metrics for intuitionistic fuzzy M-distance (divergence) is suggested. A probability distribution's distance from $P = p_1, \ldots, p_n$ to another probability distribution $Q = q_1, \ldots, q_n$ is measured using the M-distance (divergence) metric when the probabilities in both distributions are monotonically increasing or monotonically decreasing. In the field of image segmentation, the intuitionistic fuzzy M-distance (divergence) metric has a variety of uses. The suggested solution additionally separates and minimizes the imperfect and perfect threshold pictures.

Keywords: Intuitionistic fuzzy set, M-distance (divergence), Image segmentation, Convex function, Monotonic function etc.

1. INTRODUCTION

Information theory (IT) was developed by Shannon [15] in 1948 as a new area of mathematics and an effective tool for comprehending the complexities of communication. Renyi [13] took the initiative and generalized the Shannon measure as a result of the Shannon measure's restrictions in some circumstances. Following Renyi, numerous generalized metrics for various circumstances were developed. The measure of discrimination between two probability distributions—one ideal and the other observed—was developed by Kullback and Leibler [10]. In the final two decades of the 20th century, there has been a significant expansion of the body of literature on divergence measures.

Generalized information and divergence measurements have been developed, according to Besseville [4], Esteban, and Morales [9]. Research and progress in the field were revolutionized by Zadeh's [20] introduction of the idea of fuzziness. The measure of fuzzy entropy that corresponds to Shannon's [1] measure of entropy was established by De-Luca and Termini [8].

1.1 DIVERGENCES FOR FUZZY SETS

To quantify the difference between two fuzzy sets, several measures have been developed [4], [6], and [20] apart from that, in 2023 Verma [18, 19] also developed some new concepts regarding this.While a specific situation was extensively investigated in [20], where an axiomatic formulation of a divergence [14] measure for fuzzy sets was introduced, a full study on the comparison of fuzzy sets was presented in [6]. It was based on the following characteristics of nature.

(i) It is a symmetric, nonnegative function of the two fuzzy sets (i).

(ii) A fuzzy set has zero divergence with itself.

(iii) The divergence between two fuzzy sets decreases the "more similar" they are. The following formal description applies to these characteristics.

DEFINITION 1.2 (found in [20]): Consider the universe X. If each pair of fuzzy sets A and B meets the requirements, then the map $D: FS(X) \times FS(X) \to R$ is a divergence measure.

Div.1: $D(A, B) = D(B, A)$.

Div.2: $D(A, A) = 0$.

Div.3: $D(A \cap C, B \cap C) \leq D(A, B)$, for every $C \in FS(X)$.

Div.4: $D(A \cup C, B \cup C) \leq D(A, B)$, for every $C \in FS(X)$.

The preceding axioms do not demand that the divergence be non-negative. The axioms Div.2 and Div.3 (or Div.2 and Div.4) can be used to easily deduce it. Measurements of fuzzy entropy equivalent to Renyi [13] entropy and measurements of fuzzy directed divergence equivalent to Kullback Leibler [10] divergence measure were defined by De-Luca and Termini [8]. The body of knowledge about the creation of divergence metrics has grown significantly in recent years. Fuzzy information and divergence measurements were surveyed by De-Luca and Termini [8]. Here, we use threshold, a well-liked image segmentation method, to extract the items from a picture. The threshold values for segmentation can be selected at the multimodal histogram's valley points if the objects can be easily distinguished from the background. To maximize the class separatability, which was based on within-class variation, between-class variance, and total variance of grey levels, Otsu [12] chose the threshold. The literature reports a lot of great investigations on various thresholding strategies. Information-theoretic metrics were utilized by Verma [17] and Kapur et al. [11], Brink and Pendcock [5] to threshold a picture.

1.3 INTUITIONISTIC FUZZY SETS

IFSs model scenarios in which each point in the universe is given a level of membership and a level of non-participation. Accordingly, Atanassov provided the following description of an IFS (see [1]):

$$A = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$$

where μ_A and ν_A signify the degree of membership and non-membership of the element to the set, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ is a function of $\mu_A, \nu_A: X \to [0, 1]$. The function $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$, also known as the intuitionistic fuzzy index or the hesitant index, denotes ignorance regarding membership in A. We may occasionally refer to $A = (\mu_A, \nu_A)$ as just A when there is no possibility of a mistake.

The comparable representation of IF-sets [2, 3] is an interval-valued set, where each element's $x \in X$ corresponding interval is $[\mu_A(x), 1 - \nu_A(x)]$. It implies that the interval includes the element's real degree of set membership as a result. The breadth of the interval matches the hesitancy index.

We can think of a fuzzy set A on X as an IFS with non-membership degree $1 - A$ and $\pi_A = 0$. Therefore, if $FS(X) =$ set of all fuzzy sets on X and $IFSs(X) =$ set of all IFSs on X, then $FS(X) \subset IFSs(X)$.

For $A, B \in IFSs(X)$, the union, intersection, complement, inclusion, and inclusion relations are defined.

(i) Union of A and B:

$A \cup B = \{(x, \mu_{A\cup B}(x), \nu_{A\cup B}(x)) | \, x \in X\}$ where $\mu_{A\cup B}(x) = \max\{\mu_A(x), \nu_A(x)\}$ and $\mu_{A\cup B}(x) = \min\{\nu_A(x), \nu_B(x)\}$.

(ii) Intersection of A and B:

$A \cap B = \{(x, \mu_{A\cap B}(x), \nu_{A\cap B}(x)) | \, x \in X\}$where $\mu_{A\cap B}(x) = \min\{\mu_A(x), \nu_A(x)\}$ and $\mu_{A\cap B}(x) = \max\{\nu_A(x), \nu_B(x)\}$.

(iii) Complement of A: $A^c = \{(x, \nu_A(x), \mu_A(x)) | \, x \in X\}$.

(iv) A is a subset of B (denoted by $A \subseteq B$) if and only if for every $x \in X$ it holds that $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$.

1.4 DIVERGENCE MEASURES FOR INTUITIONISTIC FUZZY SETS

We define a measure of comparison between two IFSs axiomatically. We first present the axioms and look at how distances, IF-divergences, and IF-dissimilarities relate to one another. Then, we give various IF-divergences and IF-dissimilarities instances, as well as some fundamental properties and construction techniques for IF-divergences.

To measure the differences between IFSs, numerous functions have been published in the literature [6, 7] apart from that, in 2023 Verma [16] also developed some new concepts regarding this. The most frequent ones are differences. Remember that an IFSs dissimilarity measure, or IF-dissimilarity for short, is a function D from $IFSs(X) \times IFSs(X)$ to R that satisfies the following criteria for each $A, B, C \in IFSs(X)$:

IF-Diss.1: $D(A, B) = D(B, A)$.

IF-Diss.2: $D(A, A) = 0$.

IF-Diss.3: $A \subseteq B \subseteq C$, then $D(A, C) \geq \max(D(A, B), D(B, C))$.

The literature has a few instances of dissimilarity metrics. In reality, [6, 7] provides an outline. Some of these comparisons have limitations because there are cases in which such differences lead to paradoxical metrics for IFSs. Consider, for instance, Chen's definition of the dissimilarity [6, 7] and the universe $= \{x_1, \ldots, x_n\}$:

$$D_C(A,B) = \frac{1}{2n}\sum_{i=1}^{n}|S_A(x_i) - S_B(x_i)|$$

$S_A(x_i) = |\mu_A(x_i) - \nu_A(x_i)|$ and $S_B(x_i) = |\mu_B(x_i) - \nu_B(x_i)|$ respectively. $D_C(A,B) = 0$ for this dissimilarity measure whenever $S_A(x_i) = S_B(x_i)$ for all $i = 1, \ldots, n$. In reality, the dissimilarity between them is zero if $\mu_A(x_i) = \nu_A(x_i) = 0$ and $\mu_B(x_i) = \nu_B(x_i) = 0.5$ for all $i = 1, \ldots, n$. The two sets, however, are distinctly different.

To avoid such absurd circumstances, a measure of comparison must be introduced that has stronger properties than dissimilarities. An IF-divergence can be described as a measure of difference that must satisfy the following rational properties, which is the same concept as fuzzy divergences.

(i) The two IF-sets are measured by this nonnegative, symmetric quantity.

(ii) An IF-set has zero IF-divergence with itself.

(iii) The IF-divergence between two IF-sets decreases as they become "more" similar to one another.

(iv) The IF-divergence turns into a divergence for fuzzy sets.

Formally, the following axiomatic definition describes the idea of a divergence measure for IFSs.

DEFINITION 1.5 Assuming X is a finite universe, $IFSs(X)$ is the collection of all $IFSs$ on X. If a map $D_{IF}: IFSs(X) \times IFSs(X) \to R$ has the properties listed below for any $A, B \in IFSs(X)$, it is an $IFSs$ divergence measure (also known as an IF-divergence).

IF-Diss.1: $D_{IF}(A,B) = D_{IF}(B,A)$.

IF-Diss.2: $D_{IF}(A,A) = 0$.

IF-Div.3: $D_{IF}(A \cap C, B \cap C) \le D_{IF}(A,B)$, for every $C \in IFSs(X)$.

IF-Div.4: $D_{IF}(A \cup C, B \cup C) \le D_{IF}(A,B)$, for every $C \in IFSs(X)$.

2. OUR RESULTS

2.1 THE FIRST MEASURE OF M-DIVERGENCE METRIC IN INTUITIONISTIC FUZZY SETTING

The first such measure is defined by

$$D_1(A,B) = \mu_A(x_1)\ln\frac{\mu_A(x_1)}{\mu_B(x_1)} + \nu_A(x_1)\ln\frac{\nu_A(x_1)}{\nu_B(x_1)} + \big(\mu_A(x_2) - \mu_A(x_1)\big)\ln\frac{\mu_A(x_2)-\mu_A(x_1)}{\mu_B(x_2)-\mu_B(x_1)} +$$
$$\big(\nu_A(x_2) - \nu_A(x_1)\big)\ln\frac{\nu_A(x_2)-\nu_A(x_1)}{\nu_B(x_2)-\nu_B(x_1)} + \ldots\ldots\ldots + \big(\mu_A(x_n) - \mu_A(x_{n-1})\big)\ln\frac{\mu_A(x_n)-\mu_A(x_{n-1})}{\mu_B(x_n)-\mu_B(x_{n-1})} +$$
$$\big(\nu_A(x_n) - \nu_A(x_{n-1})\big)\ln\frac{\nu_A(x_n)-\nu_A(x_{n-1})}{\nu_B(x_n)-\nu_B(x_{n-1})} + \big(1 - \mu_A(x_n)\big)\ln\frac{1-\mu_A(x_n)}{1-\mu_B(x_n)} +$$
$$\big(1 - \nu_A(x_n)\big)\ln\frac{1-\nu_A(x_n)}{1-\nu_B(x_n)}$$

subject to $\mu_B(x_1) < \mu_B(x_2) < \cdots < \mu_B(x_n)$ and $\nu_B(x_1) < \nu_B(x_2) < \cdots < \nu_B(x_n)$.

Also, $\mu_A(x_1) < \mu_A(x_2) < \cdots < \mu_A(x_n)$ and $\nu_A(x_1) < \nu_A(x_2) < \cdots < \nu_A(x_n)$.

Now, $\frac{\partial D_1}{\partial \mu_A(x_1)} = \ln\frac{\mu_A(x_1)}{\mu_B(x_1)} + \ln\frac{\nu_A(x_1)}{\nu_B(x_1)} - \ln\frac{\mu_A(x_2)-\mu_A(x_1)}{\mu_B(x_2)-\mu_B(x_1)} - \ln\frac{\nu_A(x_2)-\nu_A(x_1)}{\nu_B(x_2)-\nu_B(x_1)}$

and

$$\frac{\partial}{\partial \mu_A(x_1)}\left(\frac{\partial D_1}{\partial \mu_A(x_1)}\right) = \frac{1}{\mu_A(x_1)} + \frac{1}{\mu_A(x_2)-\mu_A(x_1)} + \frac{1}{\nu_A(x_1)} + \frac{1}{\nu_A(x_2)-\nu_A(x_1)} > 0$$

similarly

$$\frac{\partial D_1}{\partial \mu_A(x_2)} = \ln\frac{\mu_A(x_2)-\mu_A(x_1)}{\mu_B(x_2)-\mu_B(x_1)} + \ln\frac{\nu_A(x_2)-\nu_A(x_1)}{\nu_B(x_2)-\nu_B(x_1)} - \ln\frac{\mu_A(x_3)-\mu_A(x_2)}{\mu_B(x_3)-\mu_B(x_2)} - \ln\frac{\nu_A(x_3)-\nu_A(x_2)}{\nu_B(x_3)-\nu_B(x_2)}$$

and

$$\frac{\partial}{\partial \mu_A(x_2)}\left(\frac{\partial D_1}{\partial \mu_A(x_2)}\right) = \frac{1}{\mu_A(x_2)-\mu_A(x_1)} + \frac{1}{\nu_A(x_2)-\nu_A(x_1)} + \frac{1}{\mu_A(x_3)-\mu_A(x_2)} + \frac{1}{\nu_A(x_3)-\nu_A(x_2)} > 0$$

$$\ldots \quad \ldots \quad \frac{\partial D_1}{\partial \mu_A(x_n)} = \ln\frac{\mu_A(x_n)-\mu_A(x_{n-1})}{\mu_B(x_n)-\mu_B(x_{n-1})} + \ln\frac{\nu_A(x_n)-\nu_A(x_{n-1})}{\nu_B(x_n)-\nu_B(x_{n-1})} - \ln\frac{1-\mu_A(x_n)}{1-\mu_B(x_n)} - \ln\frac{1-\nu_A(x_n)}{1-\nu_B(x_n)}$$

$$\frac{\partial}{\partial \mu_A(x_2)}\left(\frac{\partial D_1}{\partial \mu_A(x_2)}\right) = \frac{1}{\mu_A(x_n)-\mu_A(x_{n-1})} + \frac{1}{\nu_A(x_n)-\nu_A(x_{n-1})} + \frac{1}{1-\mu_A(x_n)} + \frac{1}{1-\nu_A(x_n)} > 0$$

and $\frac{\partial^2 D_1}{\partial \mu_A(x_i)\partial \mu_A(x_{1+1})} = -\frac{1}{\mu_A(x_{i+1})-\mu_A(x_i)} - \frac{1}{\nu_A(x_{i+1})-\nu_A(x_i)}$.

Hence, $\frac{\partial}{\partial \mu_A(x_i)}\left(\frac{\partial D_1}{\partial \mu_A(x_i)}\right) \cdot \frac{\partial}{\partial \mu_A(x_{i+1})}\left(\frac{\partial D_1}{\partial \mu_A(x_{i+1})}\right) - \left(\frac{\partial^2 D_1}{\partial \mu_A(x_i)\partial \mu_A(x_{i+1})}\right)^2 > 0.$

Obviously, $D_1(A, B)$ is a convex function of $\mu_A(x_1), \mu_A(x_2), \ldots, \mu_A(x_n)$ and $\nu_A(x_1), \nu_A(x_2), \ldots$ $\ldots, \nu_A(x_n)$. Its minimum value subject to $\sum_{i=1}^{n} \mu_A(x_i) + \nu_A(x_i) = 1$ is given as follows

$$\frac{\mu_A(x_2)+\nu_A(x_2)-\mu_A(x_1)-\nu_A(x_1)}{\mu_A(x_1)+\nu_A(x_1)} = \frac{\mu_B(x_2)+\nu_B(x_2)-\mu_B(x_1)-\nu_B(x_1)}{\mu_B(x_1)+\nu_B(x_1)},$$

$$\frac{\mu_A(x_3)+\nu_A(x_3)-\mu_A(x_2)-\nu_A(x_2)}{\mu_A(x_2)+\nu_A(x_2)-\mu_A(x_1)-\nu_A(x_1)} = \frac{\mu_B(x_3)+\nu_B(x_3)-\mu_B(x_2)-\nu_B(x_2)}{\mu_B(x_2)+\nu_B(x_2)-\mu_B(x_1)-\nu_B(x_1)}, \ldots \ldots,$$

$$\frac{\mu_A(x_n)+\nu_A(x_n)-\mu_A(x_{n-1})-\nu_A(x_{n-1})}{\mu_A(x_{n-1})+\nu_A(x_{n-1})-\mu_A(x_{n-2})-\nu_A(x_{n-2})} = \frac{\mu_B(x_n)+\nu_B(x_n)-\mu_B(x_{n-1})-\nu_B(x_{n-1})}{\mu_B(x_{n-1})+\nu_B(x_{n-1})-\mu_B(x_{n-2})-\nu_B(x_{n-2})}$$

$$\frac{\mu_A(x_n)+\nu_A(x_n)-\mu_A(x_{n-1})-\nu_A(x_{n-1})}{1-\mu_A(x_n)-\nu_A(x_n)} = \frac{\mu_B(x_n)+\nu_B(x_n)-\mu_B(x_{n-1})-\nu_B(x_{n-1})}{1-\mu_B(x_n)-\nu_B(x_n)}.$$

This condition is met if $\mu_A(x_1) + \nu_A(x_1) = \mu_B(x_1) + \nu_B(x_1), \mu_A(x_2) + \nu_A(x_2) = \mu_B(x_2) + \nu_B(x_2), \ldots \ldots, \mu_A(x_n) + \nu_A(x_n) = \mu_B(x_n) + \nu_B(x_n)$ $i.e. A = B$. So that when $A = B$ and $D_1(A, B) \geq 0$, $D_1(A, B)$ has its minimal value. In intuitionistic fuzzy settings where both $\mu_A(x_i) + \nu_A(x_i)$ and $\mu_B(x_i) + \nu_B(x_i)$ are monotonically increasing, we can utilize this $D_1(A, B)$ as an M-distance metric. As a result, the minimal M-distance probability distribution is provided when there are no constraints other than the natural constraint $\sum_{i=1}^{n} \mu_A(x_i) + \nu_A(x_i) = 1$ and the inequality constraints $\mu_A(x_i) + \nu_A(x_i) \geq 0, 1 \geq \mu_A(x_i) + \nu_A(x_i) \geq \mu_A(x_{i-1}) + \nu_A(x_{i-1}), i = 1, \ldots, n,$ the minimum M-distance probability distribution is given by $\mu_A(x_1) + \nu_A(x_1) = \mu_B(x_1) + \nu_B(x_1), \mu_A(x_2) + \nu_A(x_2) = \mu_B(x_2) + \nu_B(x_2), \ldots \ldots \quad \ldots \ldots, \mu_A(x_n) + \nu_A(x_n) = \mu_B(x_n) + \nu_B(x_n)$ and is same as the apriori distribution.

2.2 THE SECOND MEASURE OF M-DIVERGENCE METRIC IN INTUITIONISTIC FUZZY SETTING

The second such measure is defined by

$$D_2(A, B) = \left(1 + \mu_A(x_1)\right)\ln\frac{1+a\mu_A(x_1)}{1+a\mu_B(x_1)} + \left(1 + \nu_A(x_1)\right)\ln\frac{1+a\nu_A(x_1)}{1+a\nu_B(x_1)} +$$

$$a\left(\mu_A(x_2) - \mu_A(x_1)\right)\ln\frac{\mu_A(x_2)-\mu_A(x_1)}{\mu_B(x_2)-\mu_B(x_1)} + a\left(\nu_A(x_2) - \nu_A(x_1)\right)\ln\frac{\nu_A(x_2)-\nu_A(x_1)}{\nu_B(x_2)-\nu_B(x_1)} + \ldots \ldots$$

subject to $\mu_B(x_1) < \mu_B(x_2) < \cdots < \mu_B(x_n)$ and $\nu_B(x_1) < \nu_B(x_2) < \cdots < \nu_B(x_n)$.

Also, $\mu_A(x_1) < \mu_A(x_2) < \cdots < \mu_A(x_n)$ and $\nu_A(x_1) < \nu_A(x_2) < \cdots < \nu_A(x_n)$.

Now, $\frac{\partial D_2}{\partial \mu_A(x_1)} = a\ln\frac{1+a\mu_A(x_1)}{1+a\mu_B(x_1)} + a\ln\frac{1+a\nu_A(x_1)}{1+a\nu_B(x_1)} - a\ln\frac{\mu_A(x_2)-\mu_A(x_1)}{\mu_B(x_2)-\mu_B(x_1)} - a\ln\frac{\nu_A(x_2)-\nu_A(x_1)}{\nu_B(x_2)-\nu_B(x_1)},$

so $\frac{\partial}{\partial\mu_A(x_1)}\left(\frac{\partial D_2}{\partial\mu_A(x_1)}\right)=\frac{a^2}{1+a(\mu_A(x_2)+\mu_A(x_1))}+\frac{a}{\mu_A(x_2)+\nu_A(x_2)-\mu_A(x_1)-\nu_A(x_1)}>0$

similarly

$$\frac{\partial D_2}{\partial\mu_A(x_2)}=a\ln\frac{\mu_A(x_2)-\mu_A(x_1)}{\mu_B(x_2)-\mu_B(x_1)}+a\ln\frac{\nu_A(x_2)-\nu_A(x_1)}{\nu_B(x_2)-\nu_B(x_1)}-a\ln\frac{\mu_A(x_3)-\mu_A(x_2)}{\mu_B(x_3)-\mu_B(x_2)}-a\ln\frac{\nu_A(x_3)-\nu_A(x_2)}{\nu_B(x_3)-\nu_B(x_2)}$$

and

$$\frac{\partial}{\partial\mu_A(x_2)}\left(\frac{\partial D_2}{\partial\mu_A(x_2)}\right)=\frac{a}{\mu_A(x_2)-\mu_A(x_1)}+\frac{a}{\nu_A(x_2)-\nu_A(x_1)}+\frac{a}{\mu_A(x_3)-\mu_A(x_2)}+\frac{a}{\nu_A(x_3)-\nu_A(x_2)}>0$$

$$\ldots\ \frac{\partial D_2}{\partial\mu_A(x_n)}=a\ln\frac{\mu_A(x_n)-\mu_A(x_{n-1})}{\mu_B(x_n)-\mu_B(x_{n-1})}+a\ln\frac{\nu_A(x_n)-\nu_A(x_{n-1})}{\nu_B(x_n)-\nu_B(x_{n-1})}-a\ln\frac{\mu_A(x_{n+1})-\mu_A(x_n)}{\mu_B(x_{n+1})-\mu_B(x_n)}-a\ln\frac{\nu_A(x_{n+1})-\nu_A(x_2)}{\nu_B(x_{n+1})-\nu_B(x_2)}$$

$$\frac{\partial}{\partial\mu_A(x_2)}\left(\frac{\partial D_1}{\partial\mu_A(x_2)}\right)=\frac{a}{\mu_A(x_n)-\mu_A(x_{n-1})}+\frac{a}{\nu_A(x_n)-\nu_A(x_{n-1})}+\frac{a}{\mu_A(x_{n+1})-\mu_A(x_n)}+\frac{a}{\nu_A(x_{n+1})-\nu_A(x_n)}>0$$

and $\frac{\partial^2 D_1}{\partial\mu_A(x_i)\partial\mu_A(x_{1+1})}=-\frac{a}{\mu_A(x_{i+1})-\mu_A(x_i)}-\frac{a}{\nu_A(x_{i+1})-\nu_A(x_i)}$.

Hence, $\frac{\partial}{\partial\mu_A(x_i)}\left(\frac{\partial D_1}{\partial\mu_A(x_i)}\right).\frac{\partial}{\partial\mu_A(x_{i+1})}\left(\frac{\partial D_1}{\partial\mu_A(x_{i+1})}\right)-\left(\frac{\partial^2 D_1}{\partial\mu_A(x_i)\partial\mu_A(x_{i+1})}\right)^2>0.$

Obviously, $D_2(A,B)$ is a convex function of $\mu_A(x_1),\mu_A(x_2),\ldots,\mu_A(x_n)$ and $\nu_A(x_1),\nu_A(x_2),\ldots$ $\ldots,\nu_A(x_n)$. Its minimum value subject to $\sum_{i=1}^{n}\mu_A(x_i)+\nu_A(x_i)=1$ is given as follows

$$\frac{\mu_A(x_2)+\nu_A(x_2)-\mu_A(x_1)-\nu_A(x_1)}{1+a(\mu_A(x_1)+\nu_A(x_1))}=\frac{\mu_B(x_2)+\nu_B(x_2)-\mu_B(x_1)-\nu_B(x_1)}{1+a(\mu_B(x_1)+\nu_B(x_1))},$$

$$\frac{\mu_A(x_3)+\nu_A(x_3)-\mu_A(x_2)-\nu_A(x_2)}{\mu_A(x_2)+\nu_A(x_2)-\mu_A(x_1)-\nu_A(x_1)}=\frac{\mu_B(x_3)+\nu_B(x_3)-\mu_B(x_2)-\nu_B(x_2)}{\mu_B(x_2)+\nu_B(x_2)-\mu_B(x_1)-\nu_B(x_1)},\ldots\ldots,$$

$$\frac{\mu_A(x_{n+1})+\nu_A(x_{n+1})-\mu_A(x_n)-\nu_A(x_n)}{\mu_A(x_n)+\nu_A(x_n)-\mu_A(x_{n-1})-\nu_A(x_{n-1})}=\frac{\mu_B(x_{n+1})+\nu_B(x_{n+1})-\mu_B(x_n)-\nu_B(x_n)}{\mu_B(x_n)+\nu_B(x_n)-\mu_B(x_{n-1})-\nu_B(x_{n-1})}.$$

This condition is met if $\mu_A(x_1)+\nu_A(x_1)=\mu_B(x_1)+\nu_B(x_1),\mu_A(x_2)+\nu_A(x_2)=\mu_B(x_2)+\nu_B(x_2),\ldots\ldots,\mu_A(x_n)+\nu_A(x_n)=\mu_B(x_n)+\nu_B(x_n)\ i.e. A=B$. So that when $A=B$ and $D_2(A,B)\geq 0$, $D_2(A,B)$ has its minimal value. In intuitionistic fuzzy settings where both $\mu_A(x_i)+\nu_A(x_i)$ and $\mu_B(x_i)+\nu_B(x_i)$ are monotonically increasing, we can utilize this $D_2(A,B)$ as an M-distance metric. As a result, the minimal M-distance probability distribution is provided when there are no constraints other than the natural constraint $\sum_{i=1}^{n}\mu_A(x_i)+\nu_A(x_i)=1$ and the inequality constraints $\mu_A(x_i)+\nu_A(x_i)\geq 0, 1\geq\mu_A(x_i)+\nu_A(x_i)\geq\mu_A(x_{i-1})+\nu_A(x_{i-1}), i=1,\ldots,n,$ the minimum M-distance probability distribution is given by $\mu_A(x_1)+\nu_A(x_1)=\mu_B(x_1)+\nu_B(x_1),\mu_A(x_2)+\nu_A(x_2)=\mu_B(x_2)+\nu_B(x_2),\ldots\ldots\ \ldots\ldots,\mu_A(x_n)+\nu_A(x_n)=\mu_B(x_n)+\nu_B(x_n)$ and is same as the apriori distribution.

CONCLUSION

In this communication an approach to develop measures of intuitionistic fuzzy M-distance metric using aggregation operators is proposed. The proposed measure is a distance measure. To add flexibility in applications the divergence (distance) measures may be generalized by using a parameter. In the literature related to image segmentation is not done, but this is a measure to its own right and can be used for thresholding in some situations because different measures have their suitability in different situations. Finally, we have studied the most usual measures of IF-sets, concluding that they are IF-M-distance metric.

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Biography of author

Dr. Rohit Kumar is a well known author in the field of the journal scope. He obtained his highest degree from RSU, Raipur (C.G.) and worked in engineering institution for over a decade. Currently he is working in a capacity of Associate Professor and HOD, Department of Mathematics, Bharti Vishwavidyalaya, Durg (C.G.). In addition to 35 original research publications in the best journals, he also published two research book by LAP in 2013 and 2023 in the areas of information theory and channel capacity that fascinate him. He is the Chairperson, the Board of Studies Department of Mathematics at Bharti Vishwavidyalaya in Durg, C.G., India. He has published two patents in a variety of fields of research. In addition to numerous other international journals, he reviews for the American Journal of Applied Mathematics (AJAM). In addition to the Indian Mathematical Society (IMS), he is a member of the Indian Society for Technical Education (ISTE).