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Characterization of Interior Γ-hyperideals of Γ-semihypergroups

J.J.Patil
Department of Mathematics, Indraraj Arts, Commerce, and Science College, Sillod,
Dist.: Aurangabad-431112.
Email:jjpatil.777@rediffmail.com.

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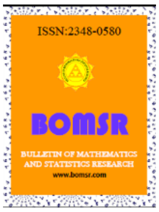

ABSTRACT

In this paper, characterization of interior Γ-hyperideals is made with respect to Γ-hyperideals of a Γ-semihypergroup. Some important results are proved for minimal interior Γ-hyperideals of a Γ-semihypergroup. Also interior-simple Γ-semihypergroup is introduced and studied basic results in this respect.

Keywords: Γ-semihypergroup, Interior-simple Γ-semihypergroup, Minimal interior Γ-hyperideal,

Mathematics Subject Classification (2010): 20N20, 16Y99

1. Introduction

In mathematics when we study the notions in classical algebraic structure where we take composition of two elements we get an element. We are all aware of classical algebraic structure approach in mathematics and different concepts of mathematics has been studied in classical algebraic structure and lot of study has been done in this respect. In 1934, when French mathematician Marty introduced the concept of hyperstructure theory while presenting the paper in conference another approach come before word in which the composition of two elements become a set. Initially hyperstructure theory was not popular amongst the mathematician in the word. But when it is found that the theory of hyperstructure has vast applications in various streams of science, hyperstructure theory has been widely studied. In 2003, Corsini and Leoreanu [1] have given application of theory of hyperstructures in various subjects like: geometry, cryptography, artificial intelligence, relation algebras, automata, median algebras, relation algebras, fuzzy sets and codes.

Anvariyeh et. al. [3] introduced and studied the notion of a Γ-semihypergroup as a generalization of semigroups, semihypergroups and Γ- semigroups. Also Pawar et al. [14], introduced and studied the notion of a regular Γ-semihyperrings. The prime and semiprime ideals in Γ-semihyperrings are studied by Patil and Pawar [11]. In [12], Patil and Pawar studied uniformly strongly prime Γ-semihyperrings briefly. In [10,13], Patil and Pawar studied Quasi-ideals and bi-ideals briefly in Γ-semihyperrings.

The main objective of this paper is to introduce and to study the concepts of classical algebraic theory to hyperstructure theory. We made analogous study of interior ideal from Γ-semiring on the line of [7]. In section-2, some preliminaries are given which are useful for us to take intensive idea about paper while reading. In section-3, we introduced the concept of interior Γ-hyperideals of Γ-semihypergroups and proved some properties regarding.

2. Preliminaries

Here we present some useful definitions further reader is requested to refer [1-3].

Definition 2.1. [1] Let H be a non-empty set and $o : H \times H \to \wp^*(H)$ be a hyperopertion, where $\wp^*(H)$ is the family of all non-empty subsets of H. The couple (H, o) is called a hypergroupoid.

For any two non-empty subsets A and B of H and $x \in H$,

$AoB = \cup_{a\in A, b\in B}\, aob$, $Ao\{x\} = Aox$ and $\{x\}oA = xoA$.

Definition 2.2. [1] A hypergroupoid (H, o) is called a semihypergroup if for all $a, b. c$ in H we have $(aob)oc = ao(boc)$.

In addition, if for every $a \in H$, $aoH = H = Hoa$, then (H, o) is called a hypergroup.

Definition 2.3. [1] Let S be a non-empty set and Γ be a non-empty set of binary operation on S. Then S is called a Γ-semigroup if:

(1) $s_1 \alpha s_2 \in S$.

(2) $(s_1 \alpha s_2)\beta s_3 = s_1 \alpha (s_2 \beta s_3)$

For all $s_1, s_2, s_3 \in S$ and all $\alpha, \beta \in \Gamma$.

In [8], Kehayopulu added the following property in the definition

(3) If $s_1, s_2, s_3, s_4 \in S, \gamma_1, \gamma_2 \in \Gamma$, are such that $s_1 = s_3, \gamma_1 = \gamma_2$ and $s_2 = s_4$ then $s_1 \alpha s_2 = s_3 \alpha s_4$.

Definition 2.4. [3] Let S and Γ be two non-empty sets. Then S is called a Γ-semihypergroup if every $\gamma \in \Gamma$ is a hyperoperation on S that is $x\gamma y \subseteq S$ for every $x, y \in S$ such that, for $a, b, c, d \in S, \gamma_1, \gamma_2 \in \Gamma$, $a = c, b = d, \gamma_1 = \gamma_2$ imply $a\gamma_1 b = c\gamma_2 d$, and for every $\alpha, \beta \in \Gamma$ and $x, y, z \in S$ we have the associative property

$$x\alpha(y\beta z) = (x\alpha y)\beta z,$$

Which means that, for all $\alpha, \beta \in \Gamma$ and $x, y, z \in S$ we have

$$\cup_{u\in(x\alpha y)}\, u\beta z = \cup_{v\in(y\beta z)}\, x\alpha v.$$

It is clear from above definition, if every $\gamma \in \Gamma$ is an ordinary operation, then S is a Γ-semigroup

Let A and B be two non-empty subsets of a Γ-semihypergroup S and $\gamma \in \Gamma$, we denote the following:

$$A\gamma B = \bigcup_{a\in A, b\in B} a\gamma b,$$

Also,

$$A\Gamma B = \{x \mid x \in a\alpha b, a \in A, b \in B, \alpha \in \Gamma\}.$$

Definition 2.5. [3] A non-empty subset A of Γ-semihypergroup S is said to be a Γ-sub semihypergroup if $A\Gamma A \subseteq A$.

Definition 2.6. [3] A non-empty subset A of Γ-semihypergroup S is said to be a right (left) Γ-hyperideal if $A\Gamma S \subseteq A$ ($S\Gamma A \subseteq A$).

If A is both right and left Γ-hyperideal of S, then we say that A is a two sided Γ-hyperideal or simply a Γ-hyperideal of S.

We can say Γ-semihypergroup S is said to be a simple Γ-semihypergroup if there is no Γ-hyperideal in other than S itself.

Definition 2.7. [5] A non-empty set B of a Γ-semihyperring S is called bi-Γ-hyperideal of S if B is a Γ-subsemihypergroup of S and $B\Gamma S\Gamma B \subseteq B$.

Example 2.8. [5] Let $S = \{a, b, c, d\}$ and $\Gamma = \{\alpha, \beta\}$. Define a hyperoperation *o* on S as follows.

o	*a*	*b*	*c*	*d*
a	*a*	*a*	*a*	*a*
b	*a*	*a*	*a*	*a*
c	*a*	*a*	*a*	*{a, b}*
d	*a*	*a*	*{a, b}*	*c*

Define $x\gamma y = xoy$ for all $x, y \in S$ and $\gamma \in \Gamma$. Then S is a Γ-semihypergroup.

Definition 2.9. [6] Let H be a Γ-semihypergroup S and Q a non-empty subset of H. Then Q is called a quasi Γ-hyperideal of S if $(H\Gamma Q)\cap(Q\Gamma H) \subseteq Q$.

Definition 2.10. [4] An element $x \in S$ is said be a regular element Γ-semihypergroup S if there exists $y \in S$ and $\alpha, \beta \in \Gamma$ such that $x \in x\alpha y\beta x$ or equivalently if $x \in x\Gamma S\Gamma x$.

A Γ-semihypergroup S is said to be a regular if every element of S is a regular.

Definition 2.11. [4] An element $x \in S$ is said be an intra-regular element Γ-semihypergroup S if there exists $y, z \in S$ and $\alpha, \beta, \gamma \in \Gamma$ such that $x \in y\alpha x\beta x\gamma z$ or equivalently if $x \in S\Gamma x\Gamma x\Gamma S$.

A Γ-semihypergroup S is said to be an intra-regular if every element of S is an intra-regular.

Theorem 2.12. [4] Let S be a Γ-semihypergroup. Then S is regular if and only if for any left Γ-hyperideal I and for any right Γ-hyperideal J of S. Then $I \cap J = J\Gamma I$

3. Interior Γ-hyperideals of a Γ-semihypergroup

The characterization of interior Γ-hyperideals of a Γ-semihypergroup has been made with respect to different types of hyperideals of a Γ-semihypergroup and different basic results proved in this respect on the line of [7,15]

Definition 3.1. A non-empty subset I of a Γ-semihypergroup S is an interior Γ-hyperideal of S if I is an Γ-subsemihypergroup of S and $S\Gamma I\Gamma S \subseteq I$.

Example 3.2. In Example 2.8. of a Γ-semihypergroup S. The subsets $I = \{a, b, c\}$ and $J = \{a, c\}$ of S are the interior Γ-hyperideals of a Γ-semihypergroup S.

Theorem 3.3. Let X be any non-empty subset of a Γ-semihypergroup S. Then $S\Gamma X\Gamma S$ is an interior Γ-hyperideal of S.

Corollary 3.4. If $a \in S$, then $S\Gamma a\Gamma S$ is an interior Γ-hyperideal of a Γ-semihypergroup S.

Theorem 3.5. Let X be any non-empty subset of a Γ-semihypergroup S. Then $S\Gamma X\Gamma X\Gamma S$ is an interior Γ-hyperideal of S.

Corollary 3.6. If $a \in S$, then $S\Gamma a\Gamma a\Gamma S$ is an interior Γ-hyperideal of a Γ-semihypergroup S.

Theorem 3.7. Arbitrary intersection of interior Γ-hyperideals of a Γ-semihypergroup S is an interior Γ-hyperideal provided that it is non-empty.

Theorem 3.8. If I is an interior Γ-hyperideal and T is a Γ-subsemihypergroup of S, then $I \cap T$ is an interior Γ-hyperideal of T.

Proof: - Let I be an interior Γ-hyperideal and T be a Γ-subsemihypergroup of a Γ-semihypergroup S. Then for any $a, b \in I \cap T$, we have $a\alpha b \subseteq I$, $a\alpha b \subseteq T$ for any $\alpha \in \Gamma$. Therefore we get , $a\alpha b \subseteq I \cap T$ i.e. $I \cap T$ is a Γ-subsemihypergroup of S.

Also, $T\Gamma(I \cap T)\Gamma T \subseteq T\Gamma I\Gamma T \subseteq I$ and $T\Gamma(I \cap T)\Gamma T \subseteq T\Gamma T\Gamma T \subseteq T$. Thus we get, $T\Gamma(I \cap T)\Gamma T \subseteq (I \cap T)$. Hence we get, $I \cap T$ is an interior Γ-hyperideal of T.

Remark 3.9. Let *X* be a non-empty subset of a Γ-semihypergroup *S*. Then $(X)_i$ be a smallest interior Γ-hyperideal containing *X.*

Theorem 3.10. If S is Γ-semihypergroup, then $(X)_i = (X \cup X\Gamma X \cup S\Gamma X\Gamma S)$ for any non-empty subset X of S.

Proof. Let X be a non-empty subset of a Γ-semihypergroup S and $I = (X \cup X\Gamma X \cup S\Gamma X\Gamma S)$. Then clearly $I\Gamma I = (X \cup X\Gamma X \cup S\Gamma X\Gamma S)\Gamma(X \cup X\Gamma X \cup S\Gamma X\Gamma S) \subseteq I$. Thus we get, *I* is a Γ-subsemihypergroup of *S.* Also, one can easily verify $S\Gamma I\Gamma S \subseteq I$. Therefore we get, I is an interior Γ-hyperideal of *S* containing *X.* Let M be an interior Γ-hyperideal of *S* containing *X.* Then clearly $I = X \cup X\Gamma X \cup S\Gamma X\Gamma S \subseteq M \cup M\Gamma M \cup S\Gamma M\Gamma S \subseteq M$. Thus we get, *I* is a smallest interior Γ-hyperideal of *S* containing *X* i.e. $I = (X)_i = (X \cup X\Gamma X \cup S\Gamma X\Gamma S)$.

Corollary 3.11. If S is Γ-semihypergroup and $a \in S$, then $(a)_i = (a \cup a\Gamma a \cup S\Gamma a\Gamma S)$.

Theorem 3.12. In intra-regular Γ-semihypergroup *S* a Γ-hyperideal and an interior Γ-hyperideal of *S* coincide.

Proof. Let *I* be a Γ-hyperideal of intra-regular Γ-semihypergroup *S*. Then cleary I is an interior Γ-hyperideal of *S.* Suppose that a non-empty subset *I* of a Γ-semihypergroup *S* is an interior Γ-hyperideal of *S*. Then for any $x \in I$, we have $x \in S\Gamma x\Gamma x\Gamma S \subseteq S\Gamma I\Gamma I\Gamma S$ since S is an intra-regular Γ-semihypergroup. Thus we get, $I \subseteq S\Gamma I\Gamma I\Gamma S$. Now, $S\Gamma I \subseteq S\Gamma(S\Gamma I\Gamma I\Gamma S) \subseteq S\Gamma I\Gamma S \subseteq I$ since *I* is an interior Γ-hyperideal of *S*. Similarly we can show that $I\Gamma S \subseteq I$. Therefore we get, *I* is a Γ-hyperideal of *S*.

Theorem 3.13. In regular Γ-semihypergroup *S* a Γ-hyperideal and an interior Γ-hyperideal of *S* coincide.

Proof. Let *I* be a Γ-hyperideal of regular Γ-semihypergroup S. Then cleary I is an interior Γ-hyperideal of *S.* Suppose that a non-empty subset *I* of a Γ-semihypergroup *S* is an interior Γ-hyperideal of *S*. Then for any $x \in I$, we have $x \in x\Gamma S\Gamma x \subseteq I\Gamma S\Gamma I$ since S is a regular Γ-semihypergroup. Thus we get, $I \subseteq I\Gamma S\Gamma I$. Now, $S\Gamma I \subseteq S\Gamma(I\Gamma S\Gamma I) \subseteq S\Gamma I\Gamma S \subseteq I$ since *I* is an interior Γ-hyperideal of *S*. Similarly we can show that $I\Gamma S \subseteq I$. Therefore we get, *I* is a Γ-hyperideal of *S*.

Theorem 3.14. If S is a regular Γ-semihypergroup, then $I = S\Gamma I\Gamma S$, for every interior Γ-hyperideal I of S.

Definition 3.15. An interior Γ-hyperideal *I* of a Γ-semihypergroup *S* is semiprime Γ-hyperideal if for any interior Γ-hyperideal *A* of *S*, $A\Gamma A \subseteq I$ implies that $A \subseteq I$.

Definition 3.16. An interior Γ-hyperideal *I* of a Γ-semihypergroup S is completetly semiprime Γ-hyperideal if for any $a \in S$, $a\Gamma a \subseteq I$ implies that $a \in I$.

Theorem 3.17. In intra-regular Γ-semihypergroup *S* a proper interior Γ-hyperideal *P* of *S* is a semiprime Γ-hyperideal of *S.*

Proof. Let S be a Γ-semihypergroup and P be an interior Γ-hyperideal of S and *I* be any interior Γ-hyperideal of *S* such that $I\Gamma I \subseteq P$. Then for any,$a \in I$, we have $a \in S\Gamma a\Gamma a\Gamma S \subseteq S\Gamma I\Gamma I\Gamma S \subseteq S\Gamma P\Gamma S \subseteq P$. Thus we get, $I \subseteq P$. Therefore *P* is a semiprime Γ-hyperideal of *S.*

Theorem 3.18. *S* is an intra-regular Γ-semihypergroup if and only if each interior Γ-hyperideal of *S* is a completely semiprime Γ-hyperideal of *S.*

Proof. Let *S* be a Γ-semihypergroup and *P* be a proper interior Γ-hyperideal of *S* and for any $a \in S$ such that $a\Gamma a \subseteq P$. Then we have $a \in S\Gamma a\Gamma a\Gamma S \subseteq S\Gamma P\Gamma S \subseteq P$. Thus we get, $a \in P$. Therefore P is a completely semiprime Γ-hyperideal of *S.*

Conversely, assume that each interior Γ-hyperideal of *S* is completely semiprime. Take any $a \in S$. We have, $S\Gamma a\Gamma a\Gamma S$ is an interior Γ-hyperideal of *S.* Therefore by assumption $S\Gamma a\Gamma a\Gamma S$ is completely semiprime Γ-hyperideal $(a\Gamma a)\Gamma(a\Gamma a) \subseteq S\Gamma a\Gamma a\Gamma S$. Then for any $x \in a\Gamma a$ we have, $x\Gamma x \subseteq S\Gamma a\Gamma a\Gamma S$ implies that $x \in S\Gamma a\Gamma a\Gamma S$ since $S\Gamma a\Gamma a\Gamma S$ is completely semiprime Γ-hyperideal. Thus we get, $a\Gamma a \subseteq S\Gamma a\Gamma a\Gamma S$. Therefore $a \in S\Gamma a\Gamma a\Gamma S$ i.e. we get, *S* is an intra-regular Γ-semihypergroup.

4. Interior-Simple Γ-semihypergroup

In this section interior-simple Γ-semihypergroup has been studied analogues with [7]

Definition 4.1. For a Γ-semihypergroup S if we assume zero element 0 has the property $0 \in 0\alpha s$, $0 \in s\alpha 0$ and $0 \notin a\alpha b$ for any $\alpha \in \Gamma, a(\neq 0), b(\neq 0), s \in$ S.

Definition 4.2. S is said to be an interior-simple Γ-semihypergroup S is without non-zero proper interior Γ-hyperideal.

Theorem 4.3. S is an interior-simple Γ-semihypergroup if and only if $S\Gamma a\Gamma S = S$ for any $a(\neq 0) \in$ S.

Proof. Suppose that S is an interior-simple Γ-semihypergroup. For any non-zero element a of S we have $S\Gamma a\Gamma S$ is an interior Γ-hyperideal of S. Since as S is an interior-simple Γ-semihypergroup we get, $S\Gamma a\Gamma S = S$.

Conversely, assume that $S\Gamma a\Gamma S = S$ for any $a(\neq 0) \in$ S. Let I be a non-zero interior Γ-hyperideal of S. Then for any $b(\neq 0) \in I$ we have $S = S\Gamma b\Gamma S \subseteq S\Gamma I\Gamma S \subseteq I$. Thus we get, $S \subseteq I$ but $I \subseteq S$ always holds. Therefore $S = I$ i.e. we get, S is an interior-simple Γ-semihypergroup.

Theorem 4.4. *S* is an interior-simple Γ-semihypergroup if and only if $S = (a)_i$ for any $a(\neq 0) \in S$.

Proof. Suppose that S is an interior-simple Γ-semihypergroup. For any non-zero element a of S we have $(a)_i$ is the smallest interior Γ-hyperideal of S. Since as S is an interior-simple Γ-semihypergroup we get, $(a)_i = S$.

Conversely, assume that $S = (a)_i$ for any $a(\neq 0) \in$ S. Let $(a)_i$ be smallest interior Γ-hyperideal of S containing a. Then for $a(\neq 0) \in (a)_i$ we have $S = S\Gamma a\Gamma S \subseteq S\Gamma(a)_i\Gamma S \subseteq (a)_i$. Thus we get, $S \subseteq (a)_i$ but $(a)_i \subseteq S$ always holds. Therefore $S = (a)_i$ i.e. we get, S is an interior-simple Γ-semihypergroup.

Theorem 4.5. Let I be an interior Γ-hyperideal and T be a Γ-subsemihypergroup of a Γ-semihypergroup *S.* If T is an interior-simple with $(T - \{0\}) \cap I \neq \varphi$, then $T \subseteq I$.

Proof. Let *T* be interior-simple Γ-subsemihypergroup and *I* be an interior Γ-hyperideal of a Γ-semihypergroup *S* with $(T - \{0\}) \cap I \neq \varphi$ and $a \in (T - \{0\}) \cap I \neq \varphi$. Then by Theorem 3.8. and 4.3. $T = T\Gamma a\Gamma T \subseteq T\Gamma I\Gamma T \subseteq S\Gamma I\Gamma S \subseteq I$, since I is an interior Γ-hyperideal of S. Hence we get, $T \subseteq I$.

5. Minimal Interior Γ-hyperideals of Γ-semihypergroup

Definition 5.1. An interior Γ-hyperideal I of a Γ-semihypergroup S is said to be a minimal interior Γ-hyperideal of S if I does not contain any other proper non-zero interior Γ-hyperideal of S.

Definition 5.2. [5] A Γ-semihypergroup S is said to be a duo Γ-semihypergroup if every left Γ-hyperideal of S is a right Γ-hyperideal and every right Γ-hyperideal of S is a left Γ-hyperideal.

Theorem 5.3. Let *I* be an interior Γ-hyperideal of S. Then *I* is a minimal interior Γ-hyperideal of S if and only if $S\Gamma a\Gamma S = I$ for any $a(\neq 0) \in I$.

Proof. Let *I* be a minimal interior Γ-hyperideal of S and $a(\neq 0) \in$ I. Then clearly $S\Gamma a\Gamma S$is an interior Γ-hyperideal of S and $S\Gamma a\Gamma S \subseteq S\Gamma I\Gamma S \subseteq I$. Since *I* is a minimal interior Γ-hyperideal of S we get, $S\Gamma a\Gamma S = I$.

Conversely, assume that $S\Gamma a\Gamma S = I$ for any $a(\neq 0) \in J$ where *J* is an interior Γ-hyperideal of S contained in an interior Γ-hyperideal *I* of S. Then for any $b(\neq 0) \in J$ we have $I = S\Gamma b\Gamma S \subseteq S\Gamma J\Gamma S \subseteq J$. Thus we get, $I \subseteq J$. Hence we have, *I=J.* Hence the converse.

Theorem 5.4. Let *I* be an interior Γ-hyperideal of S. Then *I* is a minimal interior Γ-hyperideal of S if and only if $I = (a)_i$ for any $a(\neq 0) \in I$.

Proof. Suppose that I is minimal interior Γ-hyperideal of a Γ-semihypergroup S. For any non-zero element a of I we have $(a)_i$ is the smallest interior Γ-hyperideal of S containing *a*. Since *I* is minimal interior Γ-hyperideal containing a. We get $I = (a)_i$.

Conversely, assume that $I = (a)_i$ for any $a(\neq 0) \in I$. Let I be a non-zero interior Γ-hyperideal of S. Let *J* be interior Γ-hyperideal of *S* contained in *I.* Then for any $a(\neq 0) \in J \subseteq I$ we have $I = S\Gamma a\Gamma S \subseteq S\Gamma J\Gamma S \subseteq J$. Thus we get, $I \subseteq J$. Therefore $J = I$ i.e. we get, I is minimal interior Γ-hyperideal of a Γ-semihypergroup S.

Theorem 5.5. A proper interior Γ-hyperideal of a Γ-semihypergroup S is a minimal if and only if the intersection of any two distinct proper interior Γ-hyperideal is empty.

Proof: - Assume that any proper interior Γ-hyperideal of a Γ-semihypergroup S is a minimal. Let A and B be any two distinct proper interior Γ-hyperideal of a S. Suppose that $A \cap B \neq \emptyset$. Therefore by Theorem 3.7., $A \cap B$ is an interior Γ-hyperideal of S. Thus we get, $A \cap B \subseteq A$ and $A \cap B \subseteq B$. Hence $A = B$ which is contradiction to the fact A and B are distinct proper interior Γ-hyperideals of a S. Therefore our supposition $A \cap B \neq \emptyset$ is a wrong. Therefore $A \cap B = \emptyset$.

Conversely, assume that the intersection of any two distinct proper interior Γ-hyperideal is empty. Then no any proper interior Γ-hyperideal of a Γ-semihypergroup S is contained in any other proper interior Γ-hyperideal. Therefore each proper interior Γ-hyperideal of S is a minimal interior Γ-hyperideal of S.

Theorem 5.6. Let B be a minimal right Γ-hyperideal and A be a minimal left Γ-hyperideal of a duo Γ-semihypergroup S. Then $A\Gamma B$ is a minimal interior Γ-hyperideal of S.

Proof: - Let B be a minimal right Γ-hyperideal and A be a minimal left Γ-hyperideal of a duo Γ-semihypergroup S and $I = A\Gamma B$. Then $S\Gamma(A\Gamma B)\Gamma S \subseteq A\Gamma B$. Hence $I = A\Gamma B$ is an interior Γ-hyperideal of S. Let J be an interior Γ-hyperideal of a Γ-semihypergroup S such that $J \subseteq I$. Since $S\Gamma J$ is a left Γ-hyperideal and $J\Gamma S$ is a right Γ-hyperideal of S .Then $S\Gamma J \subseteq S\Gamma I = S\Gamma(A\Gamma B) \subseteq A$ since S is duo Γ-semihypergroup. Similarly we can show that $J\Gamma S \subseteq B$. But B is a minimal right Γ-hyperideal and A is a minimal left Γ-hyperideal. Thus we get, $S\Gamma J = A$ and $J\Gamma S = B$. Hence $I = A\Gamma B = (S\Gamma J)\Gamma(J\Gamma S) \subseteq S\Gamma J\Gamma S \subseteq J$. Thus we get, $I = J$. Therefore $I = A\Gamma B$ is a minimal interior Γ-hyperideal of S.

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