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RESEARCH ARTICLE

On search for a new type of Recursive Function[*I*]

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ABSTRACT

A new type of function that comprises both a periodic addition and a periodic increment and compounding with an initial value has been searched for, and a function as such is found out finally.

Keywords: Function, Recursive Relation, Periodic Increment, Induction

Introduction

A function that starts from an initial value (called, say, Principal value) at an instant undergoes i) periodic increment that is proportional to the instantaneous principal value (proportionality constant being a fixed number), ii) periodic compounding of the total increment after each period with the then instantaneous principal value, period being a fixed interval; and iii) mandatorily a fixed amount of supplementary addition to instantaneous principal value also periodically, period may or may not be same as that of compounding, is proposed to be called a Hypercoditional Recursion Function(HRF) and is to be found out in this article.

Suppose a function $f(x_l)$ starts from an initial value of $x = a$, going on increasing periodically and proportionally at a constant rate becomes $f(x_l) = f(a + a\sigma\nu)$after ν unit of periodic interval σ being the constant of proportionality. Here of course periodic supplementary addition is not considered. Now let us consider the periodic supplementary addition/deposition(PSA) only with the period of a unit interval considered above and then the function will be written as $f(x_l) = f(a + nc)$, c being the constant amount of supplementary addition to be added with the instantaneous principal value each period. Here the periodic increment is not considered.

But when both the periodic compounding of net increment with instant-principal and the periodic supplementary addition of a fixed amount with the same are considered to run simultaneously, how will the value of the function change, and what will be the explicit form of the generalised function then? Achieving this is the ultimate aim to be addressed in this article.

Method and Movement

The usual procedure to the quest for obtaining any unique form and formula for a sequential variation is to apply the *Method of Induction* on a set of raw data obtained either directly from real-life activities or indirectly from manipulation with a variational statistical formula and, thereafter, successive modifications of the same. The method of induction has been applied here in the following naïve way; After the first interval, the function will be $f_1(x_l) = f(a + a\sigma)$; After second interval $f_2(x_l) = f(2a + a\sigma + 2a\sigma)$.In this way after *i'*th interval it will be $f_i(x_l) = f\left(ia + \sum_{j=1}^{j=i} ja\sigma\right)$,$i<m$, *m'*being number of basic interval representing the constant period after which the net increment gets compounded with instantaneous principal value. Here of course the initial principal value and the periodic supplementary addition has been considered to be the same and equal to *a*. The period of supplementary addition has been assumed to be equal to the basic unit of interval. Instead if the amount of supplementary addition had been *'b'* formula would have become $f_i(x_l) = f\left(ia + \sum_{j=1}^{j=i-1}(a + jb)\sigma\right)$, $i<m$.......(1)

For simplicity we consider here $b=a$. Then just at the end of *m'*th interval, it will be

$f_m(x_l) = f\left((ma + \sigma ma) + \sum_{j=1}^{j=m-1} ja\sigma\right) = f(ma + \sum_{j=1}^{m}(ja\sigma))$....(2)

After *(m+1)'*th interval $f_{m+1}(x_l) = f\big((m + 1)a + (m + 1)a\sigma\big) + \sigma a(1 + \sigma)\sum_{j=1}^{j=m} j$). Similarly after*(m+2)'*th interval

$f_{m+2}(x_l) = f\big((m + 2)a + (m + 2)a\sigma\big) + (m + 1)a\sigma + \sigma a(1 + 2\sigma)\sum_{j=1}^{j=m} j$).

Likewise, after *2m'*th interval

$f_{2m}(x_l) = f((2ma + 2ma\sigma) + a\sigma\sum_{m+1}^{2m-1} j + \sigma a(1 + m\sigma)\sum_{j=1}^{j=m} j$ ......(3)

In this way, the functional value will go on increasing in a complex manner. This functional value represents the instantaneous total value of the function after a certain number of intervals, which actually has two components: a) The instantaneous principal value and b) The instantaneous total increment. After deriving many more steps in this naïve way, as is usually done to achieve a complete method of induction and thus arrive at a final formula for exact manipulation the following recursive functions[RC Recursion Relations] have been found out;

$\boldsymbol{f_N(x_l) = f(P_{iN}(x_l) + I_{iN}(x_l))}$**...(4) where ,** $\boldsymbol{P_N = (P_{N-1} + a) + \delta_N I_{N-1}}$ ....(5)

and

$\boldsymbol{I_N = P_N\sigma + \delta^N I_{N-1}}$**.....(6) with the dual conditions that** $\boldsymbol{\delta_N = 1\ for N = T\tau + 1}$

while $\delta_N = 0\ for N \neq T\tau + 1$ ..... (7), and similarly $\delta^N = 1$ for $for N \neq T\tau + 1$ and $\delta^N = 0\ for N = T\tau + 1$.....(8) τ→ Serial number of period of compounding,

$T \rightarrow$ Constant periods of compounding counted in terms of a number of basic intervals.

Now let us find out, for example, the explicit form of function for $N = 5$ and $N = 7$

The above recursive relations for $T = 3$.

$$f_5(x_l) = f(P_{i5}(x_l) + I_{i5}(x_l))$$

$P_{i5} = P_{i4} + a + \delta_5 I_4 = P_{i4} + a = P_{i3} + 2a + I_{i3} = P_{i2} + a + 2a + P_{i3}\sigma + I_{i2}$

$= 5a + 3a\sigma + 2a\sigma + I_{i1} = 5a + 6a\sigma$.

$I_{i5} = P_{i5}\sigma + I_{i4} = (P_{i4} + a)\sigma + P_{i4}\sigma = (P_{i3} + 2a + I_{i3})\sigma + (P_{i3} + a + I_{i3})\sigma$

$= (5a + P_{i3}\sigma + I_{i2})\sigma + (4a + P_{i3}\sigma + I_{i2})\sigma = (5a + 3a\sigma + P_{i2}\sigma + a\sigma)\sigma +$

$(4a + 3a\sigma + P_{i2}\sigma + a\sigma)\sigma = (5a + 6a\sigma)\sigma + (4a + 6a\sigma)\sigma$

$= 9a\sigma + 12a\sigma^2$.

Therefore $f_5(x_l) = 5a + 15a\sigma + 12a\sigma^2$. [for **T=3** and $\boldsymbol{x_1 = a, x_2 = \sigma}$ **]....(9)**

Similarly $f_7(x_l) = f(P_{i7}(x_l) + I_{i7}(x_l))$ and

$P_{i7} = (P_{i6} + a) + \delta_7 I_{i6} = P_{i6} + a + I_{i6} = P_{i5} + 2a + P_{i6}\sigma + I_{i5}$

$= P_{i5} + 2a + \sigma(P_{i5} + a) + I_{i5} = P_{i5}(1 + \sigma) + a(2 + \sigma) + I_{i5}$

$= (5a + 6a\sigma)(1 + \sigma) + a\,(2 + \sigma) + 9a\sigma + 12a\sigma^2 = 7a + 21a\sigma + 18a\sigma^2$.

$I_{i7} = P_{i7}\sigma = 7a\sigma + 21a\sigma^2 + 18a\sigma^3$.

Hence $f_7(x_l) = 7a + 28a\sigma + 39a\sigma^2 + 18a\sigma^3$. [for **T=3** and $\boldsymbol{x_1 = a,\ x_2 = \sigma}$**]....(10)**

For an example of application , say for $a = 10000$ and σ = 0.01

$f_5(x_l)$ = 51512.00 and $f_7(x_l) = 72839.18$

If *b≠a* which , in general is the actual case in reality then the succeeding equations will change in form. Then for $i \nless$ m the relevant equations onward will be like the following[Certainly following the method of induction];

$f_1(x_l) = f(a + a\sigma)$,$f_2(x_l) = f(a + b + 2a\sigma + b\sigma)$,

$f_3(x_l) = f((a + 2b) + (3a\sigma + 3b\sigma))\ldots$

..$f_m(x_l) = f((a + (m-1)b) + ma\sigma + \sum_{j=1}^{m-1} jb\sigma)$.........(11)

$$f_{m+1}(x_l) = f((a + mb)(1 + \sigma) + ma\sigma(1 + \sigma) + \sum_{j=1}^{m-1} jb\sigma(1 + \sigma))$$

$$f_{m+2}(x_l) = f((a + (m+1)b)(1 + \sigma) + (a + mb)\sigma +$$

$(ma + \sum_{j=1}^{m-1} jb)\sigma(1 + 2\sigma))$

$f_{2m}(x_l) = f((a + (2m-1)b)(1 + \sigma) + \sigma(a(2m-1) + b\sum_{j=m+1}^{2m-2} j) + \sigma b(1 + m\sigma)\sum_{j=1}^{m-1} j\,)$......(12)

Now the general form of the function for application will be likewise given by $f_N\ (x_l)$ = $f(P_N(x_l) + I_N(x_l))$…..(13) with dual binary delta function

(as has been defined here) $\boldsymbol{P_N = P_{N-1} + b + \delta_N I_{N-1}}$ and

$\boldsymbol{I_N = P_N\sigma + \delta^N I_{N-1}}$ **……(14)**

Now for an exemplary application of this general form of equation, let us choose to evaluate same two functional form as above f_5 and $f_7[\mathbf{T} = \mathbf{3}\ ]$.

$P_5 = P_4 + b = P_3 + b + b + I_3 = (a + 4b) + (P_3\sigma + I_2)$

$= (a + 4b) + (a + 2b)\sigma + P_2\sigma + I_1 = (a + 4b) + (a + 2b)\sigma + (a + b)\sigma + a\sigma = (a + 4b) + (3a + 3b)\sigma$.

$I_5 = P_5\sigma + I_4 = [(a + 4b) + (3a + 3b)\sigma]\sigma + P_4\sigma$

$= [(a + 4b) + (3a + 3b)\sigma]\sigma + (P_3 + b + I_3)\sigma = [(a + 4b) + (3a + 3b)\sigma]\sigma$

$+ (a + 3b)\sigma + (a + 2b)\sigma^2 + (a + b)\sigma^2 + a\sigma^2$

$= (2a + 7b)\sigma + (6a + 6b)\sigma^2$

Then $f_5 = (a + 4b) + (5a + 10b)\sigma + (6a + 6b)\sigma^2 = f_5(a, b, \sigma)$….(15)

Similarly for f_7

$P_7 = P_6 + b + I_6 = P_5 + 2b + P_6\sigma + I_5 = P_5 + 2b + (P_5 + b)\sigma + I_5$

$= (a + 6b) + (6a + 15b)\sigma + (9a + 9b)\sigma^2$

$I_7 = P_7\sigma = (a + 6b)\sigma + (6a + 15b)\sigma^2 + (9a + 9b)\sigma^3$ and

$f_7 = (a + 6b) + (7a + 21b)\sigma + (15a + 24b)\sigma^2 + (9a + 9b)\sigma^3$…..(16)

Now one can check by putting $a = b$ in eqns.(15) and (16) one gets equations (9) and (10). Now some more successive calculations are given below to show that explicit form of the function get substantially changed when the value of T gets changed.

For $\mathbf{T} = \mathbf{1}$ $P_1 = a, I_1 = a\sigma, P_2 = P_1 + b + I_1 = a + b + a\sigma, P_3 = P_2 + b + I_2$

$= a + b + a\sigma + b + P_2\sigma = (a + 2b) + (2a + b)\sigma + a\sigma^2$ and so on. Similarly

$I_2 = P_2\sigma = (a + b)\sigma + a\sigma^2$,$I_3 = P_3\sigma = (a + 2b)\sigma + (2a + b)\sigma^2 + a\sigma^3$ and so on.

For $\mathbf{T} = \mathbf{2}$ $P_1 = a, I_1 = a\sigma, P_2 = P_1 + b = a + b, P_3 = P_2 + b + I_2 = a + 2b + I_2$

$= (a + 2b) + (2a + b)\sigma$ and it goes like this. $I_2 = P_2\sigma + I_1 = (2a + b)\sigma$,

$I_3 = P_3\sigma = (a + 2b)\sigma + (2a + b)\sigma^2$ .For $a = b$ in the above two cases

$P_1 = a, I_1 = a\sigma, P_2 = 2a + a\sigma, P_3 = 3a + 3a\sigma + a\sigma^2, I_2 = 2a\sigma + a\sigma^2$

$I_3 = 3a\sigma + 3a\sigma^2 + a\sigma^3$ and $P_1 = a, I_1 = a\sigma, P_2 = 2a, P_3 = 3a + 3a\sigma$

$I_2 = 3a\sigma, I_3 = 3a\sigma + 3a\sigma^2$etc.

Again putting b=0 in (Eq.13) and (14) subsequently one gets the result for no supplementary addition to the instantaneous principal amount.

Conclusion

The function that is derived here is constituted of two mutually interlinked recursive types of relations[RC Recursion Relations], which are typical in nature by their specific property. There are some immediate applications for this in the field of various commercial statistics, including, in general, banking systems where deposits with compound interest are involved as in a savings scheme like recurring deposits and so on. But this is not the only field of application of all findings related to this functional form, and all those findings have presumably greater potential in a more consolidated form regarding successful application to many other scientific fields beyond just the commercial and banking systems, as mentioned above. It is proposed here those newer avenues will be explored and will possibly get published in another article in future as a supplement to this one.

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About the author.

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After schooling in a remote rural area of West Bengal at Paschim Gopinathpur and Dwarhatta, Haripal, Hooghly, the author completed his H.S. and graduated with Honours in Physics from Ramakrishna Mission Ashrama Residential College, Narendrapur. He then studied M.Sc. and B.Ed. successively in the University of Calcutta. Finally, he obtained his Ph.D. degree in Atmospheric Sciences from Jadavpur University. Dr. Chattopadhyay is a life member of IAPT, IPS, ICSP, IACS, ASI, ISEC and MTA(India). He has published over fifty papers in national and international journals in India and abroad and about fifty-five science–popularizing articles to his credit. He has published eight books to his credit. Dr Chattopadhyay retired as an assistant teacher of Physics at Haripal G.D. Institution, Haripal, Hooghly, West Bengal, India.