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On the Exposition of Hyper-conditional Recursive(RC)-Function(II)

Rabindranath Chattopadhyay
Email: rchattopadhyay03@yahoo.co.in

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Rabindranath hattopadhyay

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ABSTRACT

In the previous paper[1] on 'Hyper-conditional Recursive(RC) Function the author has proposed a new type of Recursive Relation along with a new pair of Delta-Function with the help of which the problem of finding instantaneous compounded total value of a function comprising an initial principal value and a periodic increment can be solved. In earlier paper the result has been successfully applied to banking calculations. In this paper the author present a preliminary study on various other typical mathematical properties of the 'Newly Devised Functions' so that they can be effectively used and exploited for application to other important fields.

Keywords: Recursion-Relation, Periodicity, Principal Amount, Increment, Delta-Function
.

Introduction

In the previous paper of the author it has been proposed a new type of recursive relation and its corresponding function which is as follows;

$f_{hcr}(x_i) = f(P_N(x_i) + \ I_N(x_i))\ldots(1)$

where $\boldsymbol{P_N(x_i) = P_{N-1}(x_i) + b + \delta_N I_{N-1}}$ ..(2)

and

$\boldsymbol{I_N(x_i)} = \boldsymbol{\sigma P_N(x_i) + \delta^N I_{N-1}}$….(3),

b' is the fixed amount of supplementary increment,

a' is the initial principal amount and all these with stipulated condition mandatorily to be satisfied by a pair of specific delta function as follows;

$\boldsymbol{\delta_N} = 1$ for $\boldsymbol{N = T\tau + 1}$ and $\boldsymbol{\delta_N} = 0$ for $\boldsymbol{N \neq T\tau + 1}$..(4) and

$\boldsymbol{\delta^N = 0}$ for $\boldsymbol{N = T\tau + 1}$ and $\boldsymbol{\delta^N = 1}$ for $\boldsymbol{N \neq T\tau + 1}$ …(5),

where $\boldsymbol{T'}$ is a case-wise fixed period in terms of unit of a chosen interval, $\boldsymbol{\tau'}$ is an integer(natural number).

This typical function (1) have been formed and established with the help of the 'Method of Induction' which has been related to a function f_{hcr} undergoing following type of variation;

i) Function has an initial principal value(a ,say)

ii) Function undergoes increment at a fixed rate periodically with a fixed period and that is proportional to instantaneous principal value, and

iii) Function undergoes also a supplementary addition/increment of a fixed (case-wise) amount with the principal value at each unit of interval as a unit period,

iv) Function also undergoes periodic compounding of all increment with the instantaneous principal value(Transformed/Increased) with a fixed period , period may or may not be same as periodic proportional increment.

In the earlier paper it has been shown to be successfully applied to the case of Banking/Saving Scheme such as recurring deposit in particular mainly based on a specific type of Recursion-Relation as has been found out. In this paper the properties of all the mathematical accessories utilized there such as the different components of the expression for the new function have been studied to explore for finding more dimensions to which the function may represent and help solving similar problems in other fields.

Properties of the involved mathematical accessories:

Definitions of Terms:

1) $P_N = P_{N-1} + b + \delta_N I_{N-1}$

2) $I_N = \sigma P_N + \delta^N I_{N-1}$

with case-wise constant value of parameters b and σ.

3) $\delta_N = 1$ for $N = T\tau + 1$
$= 0$ for $N \neq T\tau + 1$

4) $\delta^N = 0$ for $N = T\tau + 1$
$= 1$ for $N \neq T\tau + 1$

T' is a period in terms of any appropriate number of units of chosen interval , interval in space, in time or in anything else , τ′ being an integer(Natural Number).

Each δ_N value has its corresponding δ^N value and their values form a finite set with only two elements '0' and '1'.

Simple Algebraic Properties:

5) $\delta_N + \delta^N = 1$,

6) $\delta_N . \delta^N = 0$,

7) $\delta_N (\delta^N + \delta_N) = \delta_N . \delta^N + {\delta_N}^2 = 0 + {\delta_N}^2 = {\delta_N}^2 = \delta_N$,

8) $\delta_N . \delta^N = \delta^N . \delta_N$,

9) $\delta^N + \delta_N = \delta_N + \delta^N$

10) $\delta^N (\delta^N + \delta_N) = {\delta^N}^2 + \delta^N . \delta_N = {\delta^N}^2 + 0 = {\delta^N}^2 = \delta^N$,

11) $(\delta^N + \delta_N)^m = (\delta^N)^m + (\delta_N)^m$,

12) $\delta^N - \delta_N = \pm 1$[respectively for$N \neq T\tau + 1$ and $N = T\tau + 1$]

13) $i = \sqrt{(}\delta^N - \delta_N)$ $for\ N = T\tau + 1$,

14)$\delta_i \delta^{i \pm \tau T} = 0 = \delta_{i \pm \tau T} \delta^i$,

15)$\delta_i + \delta^{i \pm \tau T} = 1 = \delta_{i \pm \tau T} + \delta^i$

Hence ,having essential properties of a group in general these two δ's form a finite group .

16) $(\delta^N)^m = \delta^N$,

17) $(\delta_N)^m = \delta_N$,

18) $P_N + I_N = P_M = P_{N-1} + \sigma P_N + b + \delta^N I_{N-1} + \delta_N I_{N-1} = (\sigma + 1)a + \sigma \sum_{i=1}^{N} P_i + Nb$

19) $f_N(P_N + I_N) = f_N(\sigma, N, a, b) \rightarrow$ A parametric function with the only independent variable 'N' , the integer (Natural number).

19(a) $f(N + 1) = f(N) + b + \sigma P(N + 1)$

Various Plausible Representation-Trials along with Related Significances:

A)Graphical Representation of Deltas: Deltas conceived here are of two types and as per their adopted representation they are named 'Up-delta' and 'Down-delta' that are symbolically expressed as respective delta with specific subscript and superscript separately for integer '*N*'.

Their values are either 1 or 0 depending on the value of integer '*N*' being either equal or not to value of $(T\tau + 1)$ that depends intrinsically on some periodicity(T) and its serial number as integer(τ).

Down-delta resembles to some extent Krönecker Delta while Up-delta has some similarity with Dirac- Delta function in regard of being continuous within certain range in a periodic interval. But these two delta-functions are not at all identical with Kroenecker- or Dirac-Deltas.

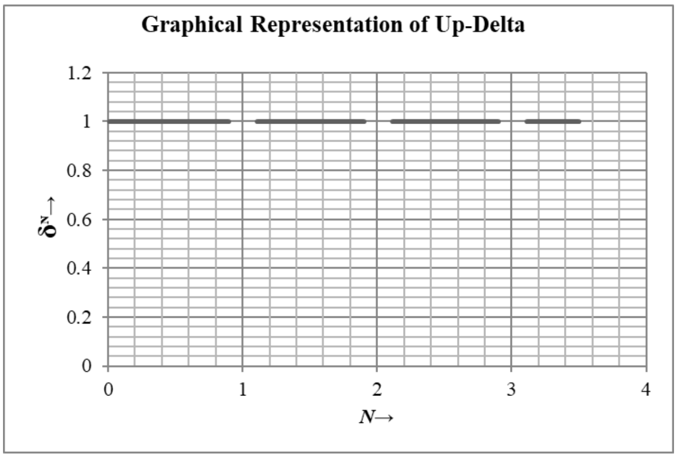


Figure1: Graphical Representation of Up-Delta

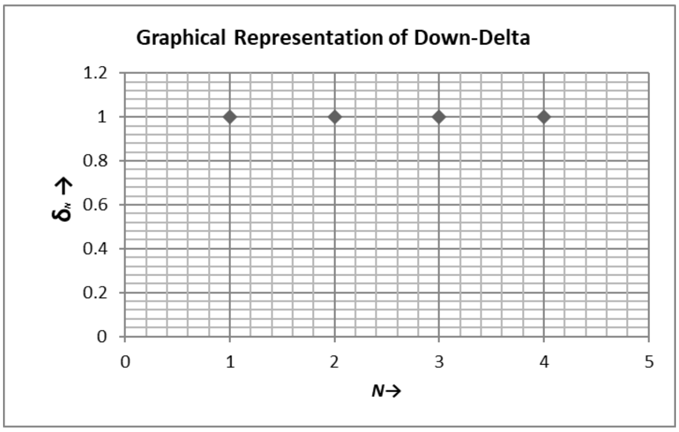


Figure2: Graphical Representation of Down-Delta

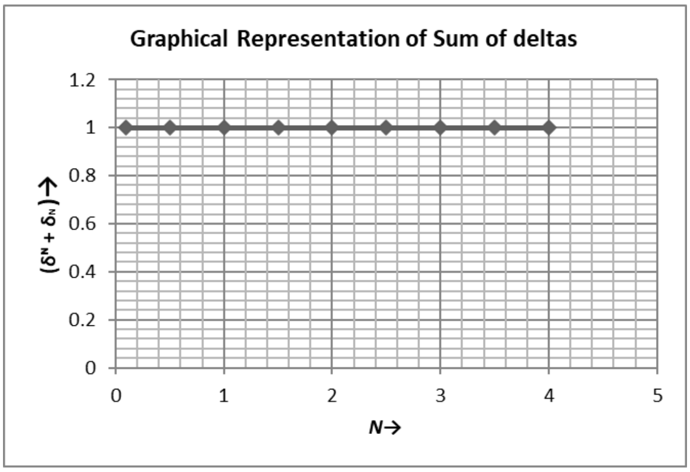


Figure 3:Graphical Representation of Sum of deltas

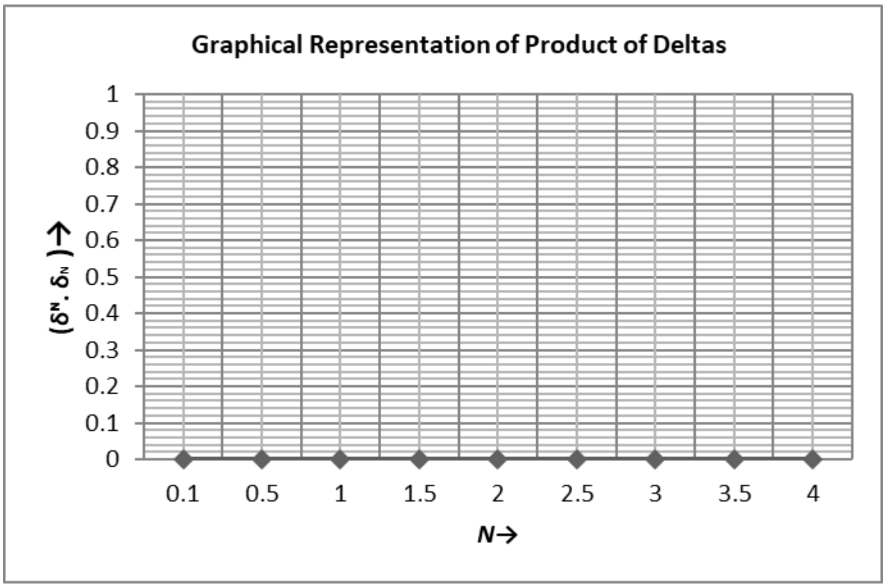


Figure 4: Graphical Representation of Product of Deltas

B)***Trigonometric Representation of Deltas:*** If one tries to represent these Deltas trigonometrically one can simply write a composite function as following;

$(\delta^N + \delta_N)^m = (\delta^N)^m + (\delta_N)^m = \delta^N + \delta_N = (\sin\theta)^2 + (\cos\theta)^2$

with precondition $\theta = (2n \pm \frac{1}{2})\pi$ and in that case equality between δ^N and δ_N poses a phase-lag of '$(\frac{\pi}{2})$' and thus refrain from simultaneity. If rotation is introduced and let θ' be expressed as 'ωt' with the rotational period being the same as the period of the periodic interval the following results are obtained;

For $T = 1\, unit$ N = 2,3,4,5,6…….and correspondingly t= 0.75, 1.25, 1.75, 2.25 …unit of time as valid values. Similarly for $T = 2\, units$ N = 3,5,7,9,11……and correspondingly t= 1.50, 2.50, 3.50, 4.5 …unit of time as valid values and so on and then

t $=\frac{(N-1)(n\pm 0.25)}{\tau}$

unit where interestingly valid time-points depend on three arbitrary integers all of which may take any integer-value from natural numbers. Either any two of these or all the three may or may not be equal to each other.

Now let us express a function like the following: $\varphi_N\ (t) = \delta^N\ \cos N\omega t + \delta_N\ \sin N\omega t$

For $N = T\tau + 1$, $\varphi_N\ (t) = \sin 2\pi\left(\tau + \frac{1}{T}\right)t$, for $N \neq T\tau + 1$, $\varphi_N\ (t) = \cos 2\pi(\frac{N}{T})t$.

For periodic interval of space (one dimensional ,say a distance $N = n\lambda + 1$ and the values of function $\varphi_N\ (x)$ will be expressed accordingly. From the very starting condition τ or n has to be an integer. But if one considers to go beyond that saying these may be any real fraction also ,which is feasible because of arbitrariness of values of T or λ so as to adjust so that N remains an integer when equal then for such specific points of values one might have written $N =$ t or $N = x$ for some integral values of either τ or n respectively. In the starting preconditions it is seen that the period or the periodic interval is important and not period being an integer always in terms of units of entity relating to interval or period.

Hence there are scopes for further investigation in this regard.

C) Matrix Representation of Deltas: δ_N and δ^N can be expressed as δ_{ij} and δ^{ij} respectively where $i = N$ and $j = T\tau + 1$

where in, purely mathematical n – dimension both i and j can take any integral value from 1 to n . For $n = 3$ one can write the matrix-form as the following'

$\delta_{ij} = \begin{pmatrix} x & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $\delta^{ij} = \begin{pmatrix} x^* & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$;$x = 1$ and x^* =0 for no periodicity or zero periodicity (or otherwise say ' just at the initiation').

Here δ_{ij} and δ^{ij} are both always nonsingular . One must not confuses these matrices with co-variant or contra-variant tensors. These are simple matrices with the used suffix and super-fix having specific meaning in accord with proposed definition of the delta functions considered in this article. No periodicity signifies either continuously increasing or decreasing ,or randomly varying or invariant function. For all nonzero periodicity x and x^* are both undeterminable. Otherwise beyond our common conception about nonzero periodicity and only as per simple mathematical perception x and x^* may take value equal to 1 and 0 respectively for negative periodicity which is not yet a practical proposition. But the basic properties of these up-delta and down-delta such as [$\delta_{ij}.\delta^{ij} = 0$]and [$\delta_{ij} + \delta^{ij} = 1$] compels that $x.x^* = 0$ and $x + x^* = 1$ if not eventually happens to be exceptionally otherwise. Depending on the periodicity all matrix-elements take value among '0' , '1', 'x' and 'x^*'. 'x' and 'x^*' are not exactly determinable. While expressed through matrix representation the matrix should normally obey the basic 'rules of matrix'. But here in this very representation the matrix considered have elements which depend intrinsically on certain periodicity in accord of the primary definition of delta-functions and therefore does not necessarily satisfy all the general rules of matrix in results. For example

For $T = 1$ $\delta_{22} = \delta_{33} = \ldots = 1 = \delta^{23} = \delta^{32} = \delta^{42} = \delta^{24} = \cdots$ while $\delta_{11} = x$

$\delta^{11} = x^*$and for $T = 2$ $\delta_{jj} = 1$ for $j > 2$ and $\delta^{jj} = 0$ for $j > 2$ and $\delta_{ij} = x$,

$\delta^{ij} = x^*$for $i \neq j$ and $j \neq T\tau + 1$. x and x^* may be conditionally equal to *0* and *1* respectively.

Functional Properties:

$f_N(x) = f\big(P_N(x) + I_N(x)\big) = f((\sigma + 1)a + Nb + \sigma \sum_{i=1}^{N} P_i) = f(\sigma, N, a, b)$.

Primarily σ, a, b are all parameters while N is the only variable.

Again $\sum_{i=1}^{N} P_i = P_1 + P_2 + P_3 + P_4 + .. = \sum_{i=1}^{N-m} P_i + mP_{N-m} + b\sum_{i=1}^{m} i + \sum_{i=1}^{m} i\delta_{N-i+1} I_{N-i}$.

The first and the last summation terms are continuously feeding to the total effective value when expanded which depends on the value of '*N*' and therefore the value of '*N*'may be called its '*Stretch Value*'.While expressed as a function of four entities one can examine the function's differential properties keeping in mind that those four entities are implicitly independent of each other.

As these entities are all mutually independent from their sources of definition they may be called the dimensions of the function.

$f(x_i) = f(x_1, x_2, x_3, x_4)$ and $\frac{df}{dx_i} = \sum_{i=1}^{N=4}(\frac{\partial y}{\partial x_i})dx_i$ and in the present context $x_1 = \sigma\,, x_2 = a\,, x_3 = b$ *and* $x_4 = N$.

$\frac{df}{d\sigma} = a + \sum_{i=1}^{N} P_i + \sigma \frac{d\sum_{i=1}^{N} P_i}{d\sigma} = a + \sum_{i=1}^{N} P_i + \sigma \sum_{i=1}^{N}(\frac{dP_i}{d\sigma})$,

$\frac{df}{da} = \sigma + 1 + \sigma\sum_{i=1}^{N}(\frac{dP_i}{da})$, $\frac{df}{db} = N + \sigma \sum_{i=1}^{N}(\frac{dP_i}{db})$, $\frac{df}{dN} = b + \sigma \sum_{i=1}^{N}(\frac{dP_i}{dN})$ with a consideration of continuous value of $'N'$ $(Beyond\ its\ usual\ property\ of\ being\ integer)$.

Following above one gets $\frac{d^2 f}{d\sigma^2} - 2\frac{d\sum_{i=1}^{i=N} P_i}{d\sigma} - \sigma\frac{d^2 \sum_{i=1}^{N} P_i}{d\sigma^2} = 0$, $\frac{d^2 f}{da^2} = \sigma\frac{d^2 \sum_{i=1}^{N} P_i}{da^2}$, $\frac{d^2 f}{db^2} = \sigma\frac{d^2 \sum_{i=1}^{N} P_i}{db^2}$, $\frac{d^2 f}{dN^2} = \sigma\frac{d^2 \sum_{i=1}^{N} P_i}{dN^2}$.

Evaluating function for a few values of parameters and a brief study on associated pattern:

1) For T=1 ,a = 100 , σ = 0.005 b = 5

Table 1

N / P_i, I_i, f_i	1	2	3	4	5	6	7	8
P_i	100	105.50	111.03	116.58	122.17	127.78	133.41	139.08
I_i	0.5	0.53	0.555	0.583	0.611	0.64	0.67	0.7
f_i	100.5	106.03	111.583	117.17	122.78	128.41	134.08	139.78

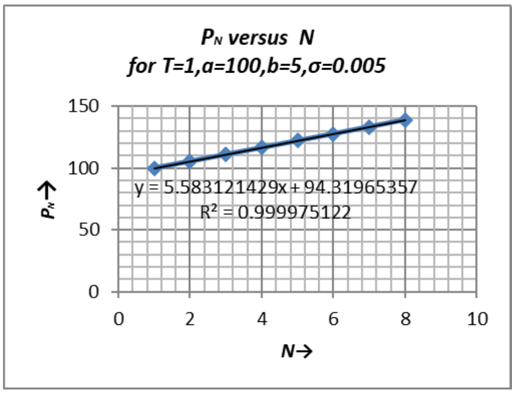


Figure 5

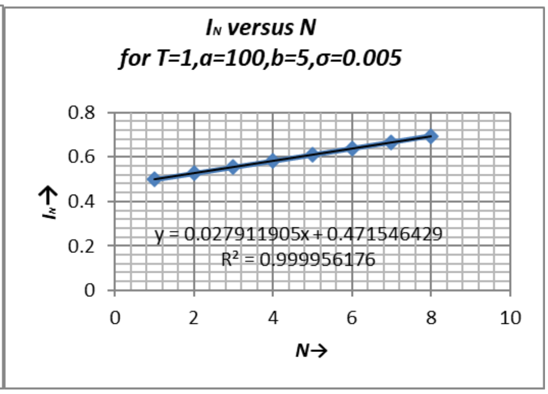


Figure 6

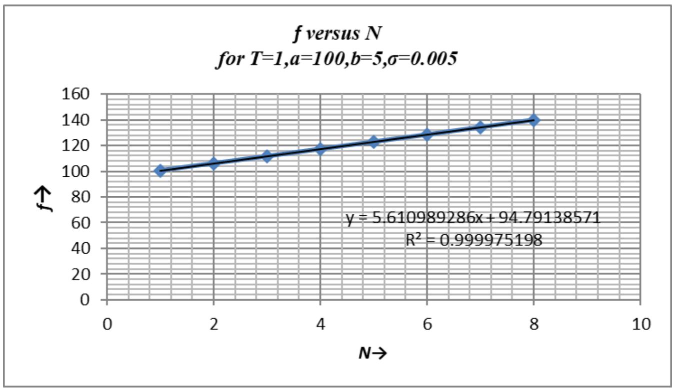


Figure 7

2) For T=4 ,a = 100 , σ = 0.005 b = 5

Table – II

N / P_i I_i ,f_i	1	2	3	4	5	6	7	8
P_i	100	105	110	115	122.62	127.62	132.62	137.53
I_i	0.5	1.025	1.575	2.15	0.613	1.25	1.91	2.6
f_i	100.5	106.02	111.57	117.15	123.22	128.87	134.53	140.22

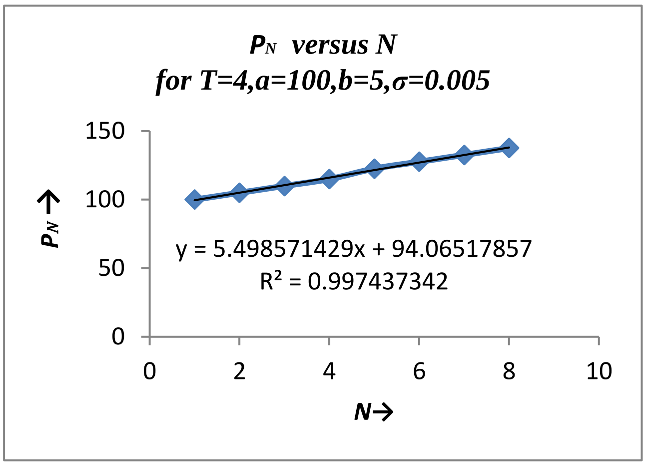


Figure 8

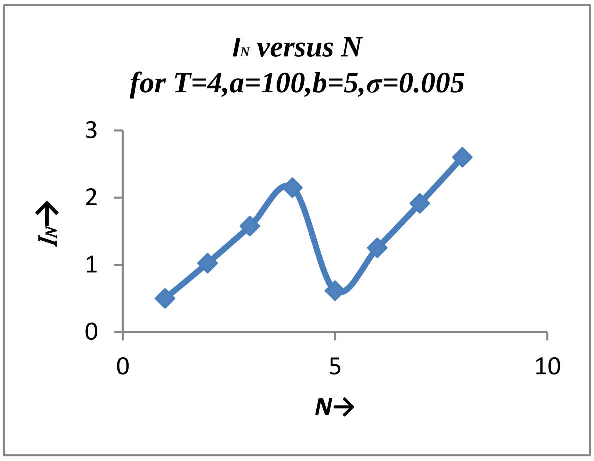


Figure 9

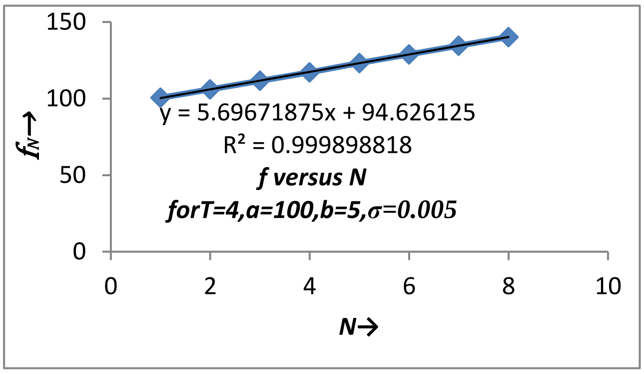


Figure 10

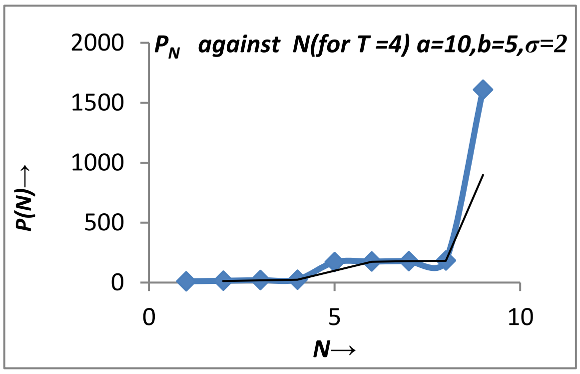


Figure 11 (a)

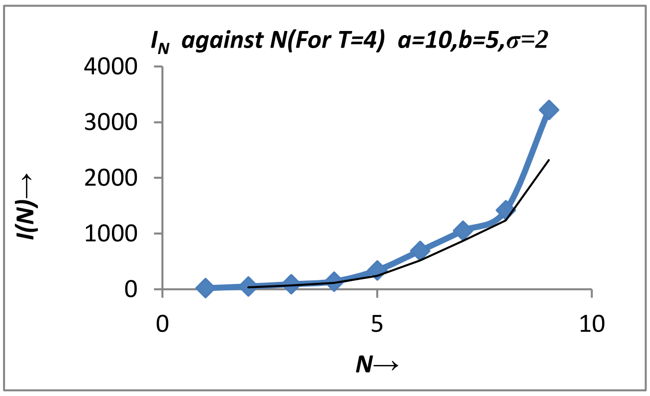


Figure 11 (b)

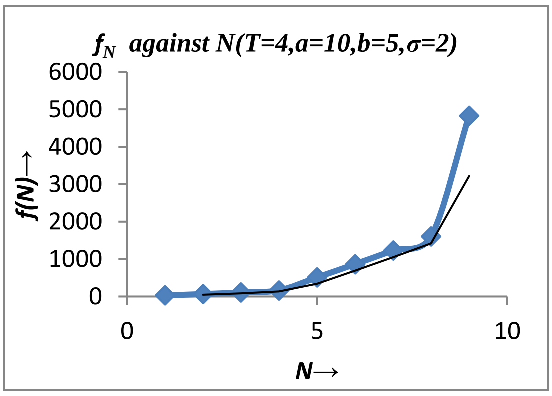


Figure 11 (c)

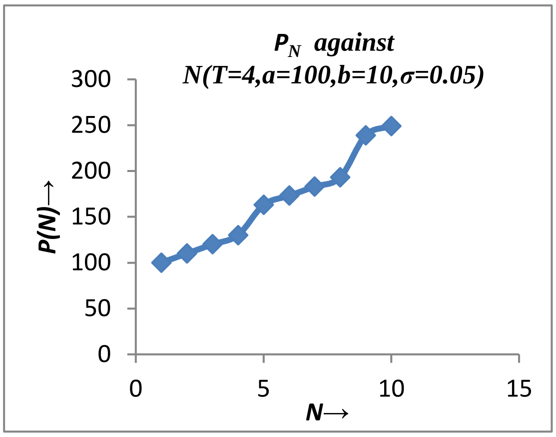


Figure 12 (a)

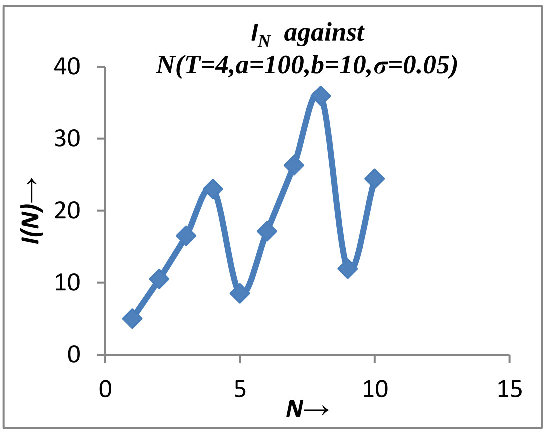


Figure 12 (b)

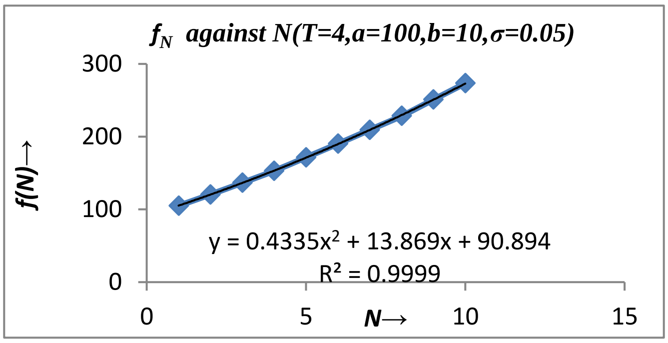


Figure 12 (c)

3) For T=1 ,a = 100 ,b = 10

Table -III

σ \ p	0.01	0.02	0.03	0.04	0.05
P1	100	100	100	100	100
P2	111	112	113	114	115
P3	122.11	124.24	126.39	128.56	130.75
P4	133.33	136.725	140.18	143.14	147.29
P5	144.66	149.452	154.39	158.92	164.65

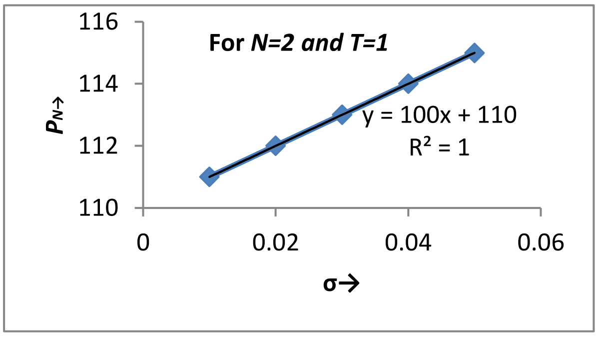


Figure 13

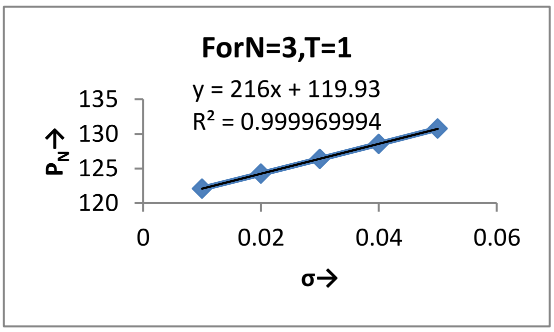


Figure 14

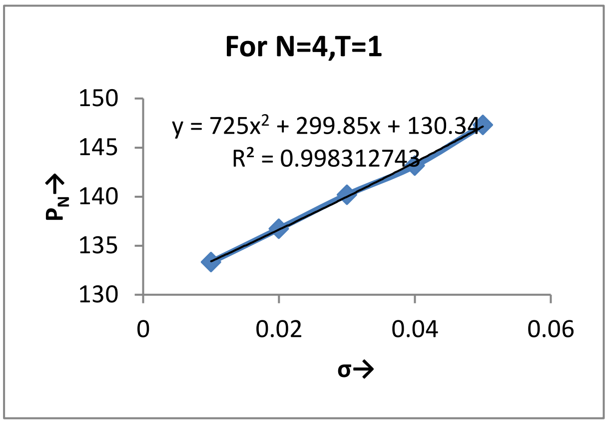


Figure 15

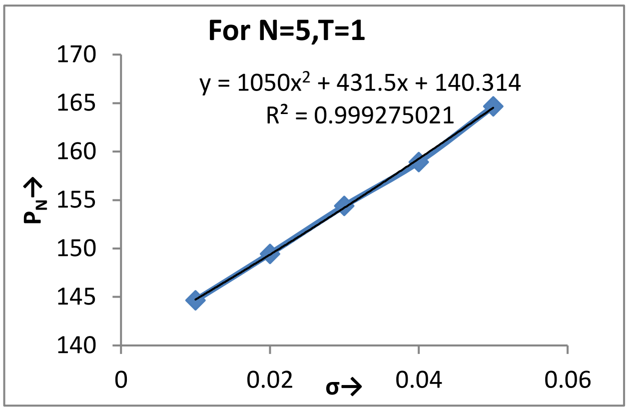
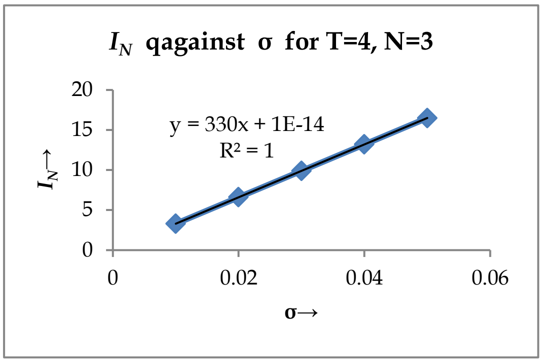


Figure 16

4) For T=4 ,a = 100 ,b = 10

Table-IV

I \ σ	0.01	0.02	0.03	0.04	0.05
I_3	3.30	6.60	09.90	13.20	16.50
I_4	4.60	9.20	13.80	18.40	23.00
I_5	1.45	2.98	04.61	06.34	08.15
I_6	2.99	6.17	09.53	13.10	16.80



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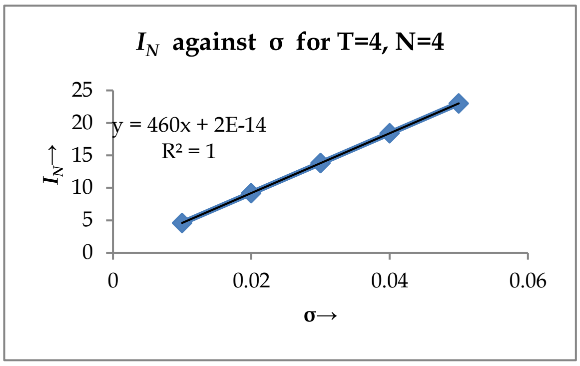

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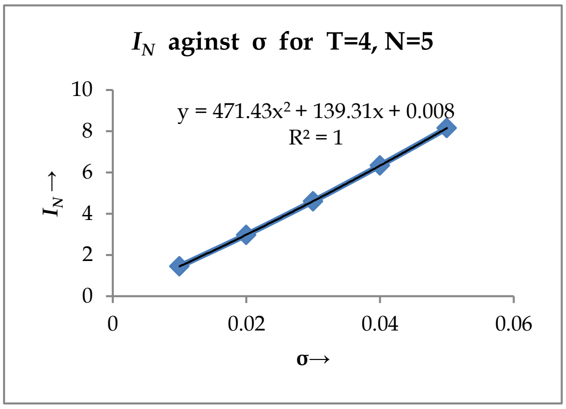

Figure 19

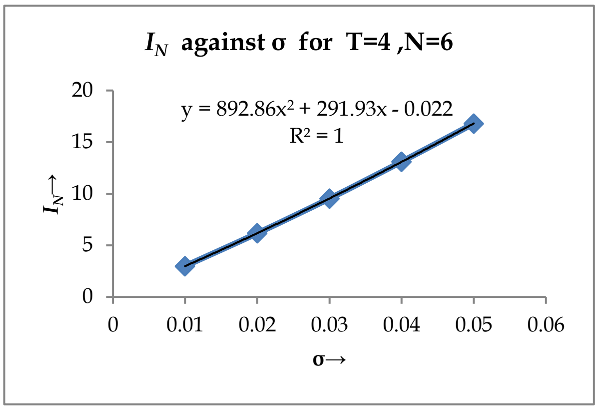

Figure 20

5)For T=4 ,a = 100 ,b = 10

Table V

P/f σ	0.01	0.02	0.03	0.04	0.05
P/f (N=5)	**144.6/146**	**149.2/152.2**	**153.8.6/158.4**	**158.4/164.7**	**163.0/171.1**
P/f (N=6)	**154.6/157.6**	**159.2.6/165.4**	**163.8/173.3**	**168.4/181.5**	**173.0/189.8**

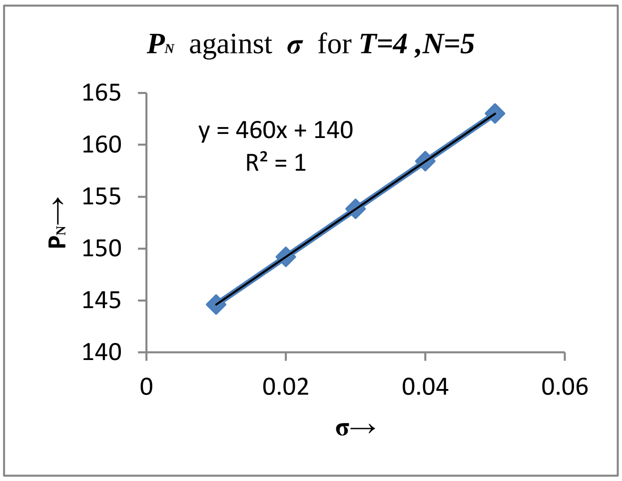

Figure 21

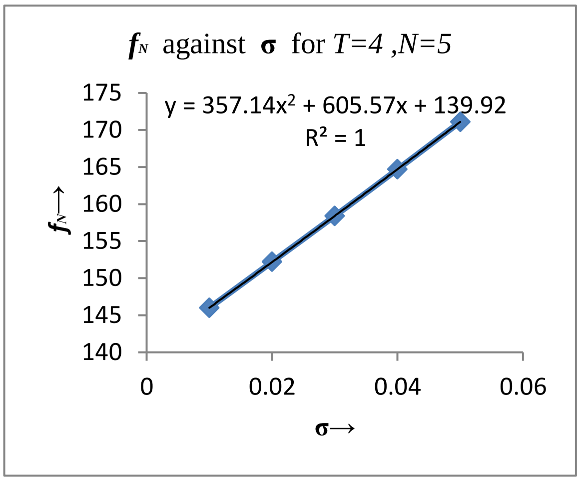

Figure 22

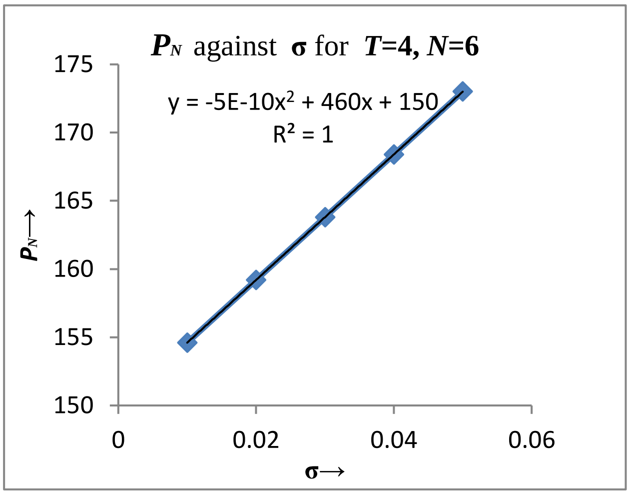

Figure 23

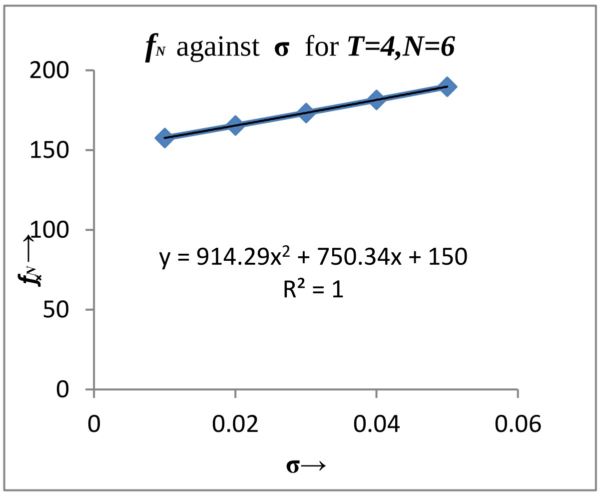

Figure 24

***Some observations on the change in the behavior of the function in changed form*:**

In simple linear form the function f_N may be written as ,

f_N= $(\sigma + 1)a + Nb + \sigma \sum_{i=1}^{N} P_i$

Now it is sought to examine how does the explicit form of the function transform with changes in the nature of the parameters. Let one consider 'b' as follows;

$$b = \pm\, \alpha P_N$$

for considering b in the $N'th$ instant of addition and one operational pre-symbol at a time i,e 'either + or' - .

Let us consider here $b = -\,\alpha P_N$.

For $T = 1 P_i = \left(\frac{1}{1+\alpha}\right)(\delta_i\sigma + 1)P_{i-1} =$ ----- = $\prod_{i=2}^{j} \frac{1}{(1+\alpha)^{i-1}} (\delta_i\, \sigma+1)$

Following from the basic definitions of the terms [Eqns.1-4] and in the way of elementary induction method one obtains

$f_N = a[(\sigma + 1) + (\sigma - \alpha)\sum_{j=2}^{N} \frac{1}{(1+\alpha)^{i-1}} \prod_{i=2}^{j}(\delta_i\sigma+1)]$

$\sum_{j=2}^{N} P_j = P_2 + P_3 + P_4 + \cdots + P_N = \frac{a}{(1+\alpha)}(\delta_2\sigma+1) + \frac{a}{(1+\alpha)^2}(\delta_2\sigma+1)(\delta_3\sigma + 1)$

$+ \frac{a}{(1+\alpha)^3}(\delta_2\sigma+1)(\delta_3\sigma + 1)(\delta_4\sigma + 1) + \ldots + \frac{a}{(1+\alpha)^{N-1}}(\delta_2\sigma+1)\ldots.(\delta_{N-1}\sigma + 1).$

Thus for α=0 , σ =1 $f_N = a[2^N]${ And of course for T=1}; $f_1 = 2a$, $f_2 = 4a$ and so on.

Similarlyfor α=0 , σ =2 and α=0 , σ =3 for$T$=1 $f_N = a[3^N]$ and $f_N = a[4^N]$respectively. For α=1 , σ =1 $f_N = (N + 1)a$; For α=1 , σ =2 $f_N = (2^N + 1)a$ andfor α=1 , σ =3 $f_N = (2^{N+1})a$. Choosing this way different plausible values of α, σ and N for T=1 one can generate different number-sequence and that is why the function may be called' a sequence-generating function' too. But for T= m ,m > 1 f_N' explicitly is a bit too complex.

$$f_N = (\sigma + 1)a + (\sigma - \alpha)\sum_{j=2}^{N} P_j$$

where $P_j = \left\{\frac{1}{1+\alpha}\right\}[P_{j-1} + \delta_j I_{i-1}]\ldots\ldots = \left\{\frac{1}{1+\alpha}\right\}^3 (\delta_j\sigma + 1)(\delta_{j-1}\sigma + 1).$

$(P_{j-3} + \delta_{j-2}I_{j-3}) + \left\{\frac{1}{1+\alpha}\right\}^2 (\delta_{j-1}\delta^{j-2}\sigma P_{j-3}) + \left\{\frac{1}{1+\alpha}\right\}\delta_j\delta^{j-1}\sigma(P_{j-3}+\delta_{j-2}I_{j-3})\ \ldots\ldots$

and so on.

For T >1 the functional value of P_j is not closed in itself like in the case of T=1 and instead evolves continuously until terminated by choosing a particular value of N.

For example, for $T = 2$ and $N = 3$ (say)

$f_N = f_3 = (\sigma + 1)a + (\sigma - \alpha)[\, a + \frac{a}{1+\alpha} + \frac{a}{(1+\alpha)^2} + \frac{\sigma a}{(1+\alpha)^3} + \sigma a]\,.$

Now for $\alpha = 0$ and $\sigma = 1$ $f_3 = 7a$ and for α=1 , σ =1 $f_3 = 2a$.

Discussion

Depending on the fundamental properties of the two delta functions proposed, defined, and derived in this article—as well as the functions constructed around them—multiple types of representation have been explored. This is part of an ongoing effort to discover potential new fields of application.

Simple graphical representations of delta functions are, of course, quite obvious. However, trigonometric representations offer additional interest, as they allow the transformation of variables—along with rotational angle (if considered as such)—from continuously varying values into quantized ones. This quantization is determined by the value of *T*, the inherent periodicity.

A matrix representation has also been attempted, expanding the original single-number form into a logically viable dual-indexed representation. This approach facilitates the study of delta-matrices in terms of the total number of determinable elements in rows, columns, and the entire matrix, which depend on both the inherent periodicity and the chosen serial number *N*.

The implicit form of the HCRC-function has been moderately revealed by applying continuous expansion, consistent with its own definition. Various characteristics of this function have been traced to their preliminary states. The differential properties have also been preliminarily examined, revealing that the differential coefficient for any order remains recursive, due to the inherent recursive nature of the function's components. As a result, the function and its derivatives are clearly nonlinear.

Following this foundational study, a few evaluations of the associated functions *P*, *I*, and *f* have been carried out. These assessments, conducted through graphical plots, aim to examine straightforward variation patterns with respect to relevant variables and parameters.

Numerical evaluations of both the individual components and the function as a whole have been conducted by varying *N* and the parameter *σ*, keeping one fixed at a time to demonstrate their dependency patterns. Since *N* is directly tied to the periodicity of compounding, I—the effective periodic increment with respect to N—exhibits a wavy variation. This occurs because the purely incremental component periodically reduces to zero at the end of each period, with its immediate prior value being added to the instantaneous principal value through compounding.

From the examination of graphs (Figure 5 to Figure 10), it is observed that the graphical representations align approximately with the following equations, with the exception of the *I* versus *N* graph for $T \neq 1$:

- $P_N \cong (b + \sigma a)N + [\{a(1-\sigma) - b\} + \frac{2\sigma b}{a}]$
- $I_N \cong \sigma P_N = (b + \sigma a)N\sigma + [\{a(1-\sigma) - b\} + \frac{2\sigma b}{a}]\sigma$
- $f_N \cong P_N + I_N = (1+\sigma)[(b+\sigma a)N + \{a(1-\sigma) - b + \frac{2\sigma b}{a}\}]$

All other functions, such as *P* and *f*, are observed to increase continuously—primarily in a manner equivalent to linear variation with both. In some cases, however, they exhibit polynomial variation of degree 2. Figures 11 and 12 highlight some interesting features in the variation of the function and its components.

It is seen that for the same inherent periodicity ($T = 4$) and a specific set of parameter values (a, b, σ), the I_N-N curve and the f_N-N curve are nearly identical. Both show steep slopes increasing successively at each period. Meanwhile, the P_N-N curve, though distinct, displays a successive plateau-step pattern in each period. Despite these structural differences, all three curves increase with N (as seen in Figure 11 with a moving average fit for a period of 2 in the series).

For the same period ($T = 4$) but with a different set of identical parameters, the P_N-N and I_N-N curves (Figure 12) show similar variation—somewhat sinusoidal with an increasing bias. This bias is less pronounced for P_N-N, but highly evident for I_N-N. The resulting curve-fitting formula for this trend may be termed *"Sinunomial"* (a presumably new term introduced here). In contrast, the f_N-N curve in this case displays polynomial-like growth with respect to N, differing from the other two.

It is interesting to observe that within each period, the periodic increases and decreases of P_N-N and I_N-N appear to be somewhat opposite in nature. As a result, the f_N-N curve—after exhibiting a quasi-balance between the two—remains largely free of periodicity, though it continues to increase overall.

This exemplifies the fact that even with one invariant parameter, a multi-variant function can undergo abrupt changes in its overall variation pattern—shifting, for example, from stepwise increase to a "Sinunomial" form—depending on changes in the values of other associated parameters.

Regarding the variation of P_N, I_N, and $f_N$with respect to σ, the following observations are made:

- Both P_N–σ and I_N–σ graphs show approximately straight-linear variation with σ for $T = 1$.
- For $T > 1$, they display linear trends only for smaller values of N, while becoming nonlinear for higher N.
- Under the condition $T > 1$, the f_N–σ graph shows increasing nonlinearity with respect to σ, the degree of nonlinearity being a function of N.

Conclusion

HCRC-function is doubtlessly not a primitive recursive function (eq. 19(a)) [2,3]. It is, on one hand, constituted of an additive combination of an instantaneously compounded principal value and a periodic increment—both of which are, again, constituted of, along with other ordinary algebraic numbers, two different types of delta-functions, all of which are newly proposed and defined.

A recursion relation essentially implies some repetition in an order, which may be recognized as a periodic order in terms of a certain number-scale. The period refers to a certain interval in numbers, which may either vary or remain invariant. The recursive relation (function) proposed and discussed here is of a non-primitive type and may be regarded as a mutually inclusive periodic recursion-combination [3].

In this paper, the author has sincerely attempted—based on the basic characteristic properties of the function and its constituent parts—to explore the possibilities of various types of representations of the function that may help unlock its potential for application in various fields.

Nutrition and growth together in almost all terrestrial organisms seem to follow a type of synergism, where appropriate modeling could probably be supported by the HCRC-function (eq. 18). Within the basic frame of this function—including the newly proposed pair of delta-functions—all the parameters concerned have, as such, no fixed identity with uniqueness in explicit form, and may necessarily or optionally be transformed according to one's need or consideration for examining its consequences.

Like many other recursion relations, this too may undergo various computational algorithms that could prove to be extremely useful. Unlike other recursive functions, this function has newer constituents—namely, a pair of mutually complementary delta-functions—and also a new feature of comprising two mutually inclusive recursive sub-functions. These are its distinguishing features, which can be further explored and exploited as a multifaceted mathematical tool across various fields.

In nuclear transmutation or any similar nuclear interaction process, where growth and decay of any nuclear species occur simultaneously—starting from an initial mass through mutual transformation of energy to mass and mass to energy—this function may, presumably, be helpful for appropriate modeling. In many other interactions where variations of several fields influence and escalate one another, this function may also serve as a tool to represent their collective growth dynamics.

Furthermore, any entity—especially in the microcosmic world—that is represented as a consummation of two states, particle and wave, may perhaps be conceptualized as a combination of wave functions associated with the two delta-functions defined here as complementary eigenstates. Moreover, in various modeling processes involving interaction-energy functions [4], particularly under perturbative methods where the coupling is treated as the main energy and the resulting kinetic energy of emitted particles as a perturbation, this function may possibly aid in resolving certain aspects through proper representational analysis.

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