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RESEARCH ARTICLE

A Hybrid Ratio–Type Estimator Under Systematic Sampling

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Abstract

In this study, we employ both square root and exponential transformations to construct a ratio-type estimator for determining the population mean of the variable under systematic sampling. The bias and mean square error (MSE) are derived up to the first-order approximation, and a theoretical comparison is carried out using a numerical illustration.

Keywords: Systematic Sampling, Chain Ratio- Type, Bias, Mean Square Error (M.S.E), Efficiency.

1. Introduction

The use of auxiliary information in sample surveys is a well-established approach for improving the precision of estimators of population parameters. When the auxiliary variable and the study variable are highly correlated, ratio, product, and exponential-type estimators are frequently used to reduce sampling variance. Due to their practical applicability and improved efficiency, these methods have received a lot of attention in the literature on survey sampling. Due to its simplicity of operation and ease of implementation, systematic sampling is widely used in practice. When the population units exhibit a natural ordering, systematic sampling frequently yields more precise estimates than simple random sampling. When population units are arranged sequentially, the method is especially useful in agricultural, industrial, and socioeconomic surveys.

Estimators for systematic sampling have received significant contributions from a number of researchers. Swain (1964) introduced the ratio estimator in systematic sampling, while Shukla (1971) proposed the product estimator. Later, exponential-type estimators developed by Bahl and Tuteja (1991) influenced numerous subsequent modifications. Contributions by Kushwaha and Singh (1989), Singh and Singh (1998), Singh et al. (2011), Singh and Solanki (2012), and other researchers further extended these estimators using auxiliary information.

The current study proposes a hybrid ratio–type square root transformed exponential estimator for systematic sampling to estimate the population mean. To increase efficiency, the proposed estimator combines ratio and exponential transformation methods.

Consider a finite population U = (U_1,U_2.... U_N) of size N numbered from I to N. Assume that N = nk, where n and k are positive integers. The population is divided into k systematic samples each containing n units. One sample is selected randomly, and observations on the study variable y and auxiliary variable x are recorded. Let y_{ij} and x_{ij} denote the values of the study and auxiliary variables, respectively, for the j-th unit in the i-th systematic sample. Let $\bar{y}_{sys}$ and $\bar{x}_{sys}$ denote the systematic sample means of y and x, respectively. The usual systematic mean estimator is unbiased for the population mean. The proposed estimator incorporates auxiliary information through a chain ratio and exponential transformation framework.

$$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} x_{ij}, \bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_{ij}$$

$$\bar{y}_{\text{sys}} = \bar{y}_{\text{i.}} = \frac{1}{n}\sum_{j=1}^{n} y_{ij}, \ \bar{x}_{\text{sys}} = \bar{x}_{\text{i.}} = \frac{1}{n}\sum_{j=1}^{n} x_{ij}$$

It is to be noted that $\bar{y}_{\text{sys}}$ and $\bar{x}_{\text{sys}}$ are unbiased estimators of population mean $\bar{Y}$ and $\bar{X}$ respectively.

Let $S_y^2 = \frac{1}{N-1}\sum_{i=1}^{k}\sum_{j=1}^{n}(y_{ij} - \bar{Y})^2$

$$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{k}\sum_{j=1}^{k}(x_{ij} - \bar{X})^2$$

$$S_{yx} = \frac{1}{N-1}\sum_{i=1}^{k}\sum_{j=1}^{n}(x_{ij} - \bar{X})(y_{ij} - \bar{Y})$$

be the population variance and covariance of the study variable y and the auxiliary variable x respectively.

Also, $C_y^2 = \frac{S_y^2}{\bar{Y}^2}, \ C_x^2 = \frac{S_x^2}{\bar{X}^2}$

To obtain the Bias and MSE of the proposed estimators upto $o(1/\text{n})$, we define

$$e_0 = \frac{\bar{y}_{sys} - \bar{Y}}{\bar{Y}}, e_1 = \frac{\bar{x}_{sys} - \bar{X}}{\bar{X}}$$

$E(e_0) = E(e_1) = 0$

$E(e_0^2) = \theta C_y^2 \rho_y^*$

$E(e_1^2) = \theta C_x^2 \rho_x^*$

$E(e_0 e_1) = \theta K C_x^2 \sqrt{\rho_y^* \rho_x^*}$

Also,

$\rho_{yx} = \frac{S_{yx}}{S_y S_x}$, $K = \rho_{yx} \frac{C_y}{C_x}$

$\rho_y^* = \{1 + \rho_y (n-1)\}$

$\rho_x^* = \{1 + \rho_x (n-1)\}$

$\rho^{**} = \frac{\rho_y^*}{\rho_x^*}$

where ρ_y, ρ_x are intra class correlation among the pair of units for the variables x and y respectively.

Consider some estimators of the finite population mean in the sampling literature of systematic sampling.

The simple mean estimator under systematic random sampling is given by:

$t_0 = \frac{1}{n}\sum_{j=1}^{n} y_{ij} \; (i = 1,2,3,\ldots\ldots,k)$

$Var(t_0) = \theta\, \overline{Y}^2 \rho_y^* C_y^2$

Swain (1964) suggested the ratio estimator of population $\overline{Y}$ of the study variate based on the systematic sample as:

$t_1 = \overline{y}_{sys}\left[\frac{\overline{X}}{\overline{x}_{xys}}\right]$

Shukla (1971) proposed the product estimator of the finite population mean in systematic sampling as:

$t_2 = \overline{y}_{sys}\left[\frac{\overline{x}_{xys}}{\overline{X}}\right]$

Following Bahl and Tuteja (1991), Singh et al. (2013) utilising the known knowledge of the auxiliary variable suggested the following ratio and project type exponential estimators for estimating the population mean $\overline{Y}$ in systematic sampling.

$t_3 = \overline{y}_{sys} exp\left[\frac{\overline{X}-\overline{x}_{sys}}{\overline{X}+\overline{x}_{sys}}\right]$

$t_4 = \overline{y}_{syss} exp\left[\frac{\overline{x}_{sys}-\overline{X}}{\overline{x}_{sys}+\overline{X}}\right]$

2. Proposed Estimator

The novelty of the proposed estimator lies in the integration of chain ratio estimation with square root transformed exponential adjustment under systematic sampling. Unlike conventional ratio estimators, the proposed form improves stability and efficiency by utilizing transformed auxiliary information. This hybrid structure enhances estimation performance, particularly in situations where the auxiliary variable is strongly correlated with the study variable.

$$T = \overline{y}_{sys}\left(\frac{\overline{X}}{\overline{x}_{sys}}\right)^{\frac{1}{2}} e^{\frac{1}{2}}\left(\frac{\overline{X}-\overline{x}_{sys}}{\overline{X}}\right)$$

To obtain the Bias and MSE of the proposed Estimator are as follows.

$$T = \overline{Y}\ (1+e_0)\left(\frac{\overline{x}}{\overline{x}\,(1+e_1)}\right)^{\frac{1}{2}} e^{\frac{1}{2}}\left(\frac{\overline{x}-\overline{x}\,(1+e_1)}{\overline{x}}\right)$$

$$= \overline{Y}\ (1+e_0)\,(1+e_1)^{-\frac{1}{2}}\, e^{-e_{1/2}}$$

$$= \overline{Y}\ (1+e_0)\left(1-\frac{1}{2}\,e_1+\frac{3}{8}\,e^2\right)\left(1-\frac{e_1}{2}+\frac{e_1^2}{8}\right)$$

$$= \overline{Y}\ (1+e_0)\left(1-\frac{e_1}{2}+\frac{e_1^2}{8}-\frac{1}{2}e_1+\frac{1}{4}\,e_1^2+\frac{3}{8}\ e_1^2\right)$$

$$= \overline{Y}\ \left(1-\frac{e_1}{2}+\frac{e_1^2}{8}-\frac{1}{2}e_1+\frac{1}{4}\,e_1^2+\frac{3}{8}\,e_1^2+e_0-\frac{1}{2}\,e_0e_1-\frac{1}{2}\,e_0e_1\right)$$

$$B\,(T) = E\,(T) - \overline{y}$$

$$= \overline{Y}\left[\frac{3}{4}\,E\,(e_1^2)-E\,(e_0e_1)\right]$$

$$= \overline{Y}\left[\frac{3}{4}\,\theta\,c_x^2\,\rho_x^*-\theta\,kc_x^2\,\sqrt{\rho_y^*\ \rho_x^*}\right]$$

$$= \theta\,\overline{Y}\left[\frac{3}{4}\ c_x^2\,\rho_x^*-kc_x^2\ \sqrt{\rho_y^*\ \rho_x^*}\right]$$

$$MSE\,(T) \cong E\,(T-\overline{y})^2$$

$$= \theta\overline{Y}^2\left[c_y^2\,\rho_y^*+c_x^2\,\rho_x^*-2\,kc_x^2\,\sqrt{\rho_y^*\ \rho_x^*}\ \right]$$

3. Efficiency Comparison

In this section we have compared the MSE of the proposed estimator (T) with the MSE of estimator t_0 and t_1 and found some theoretical conditions under which the proposed estimator will always perform better.

(1) Comparison of t_0 with proposed estimator (T):

T is more efficient than t_0 if

$c_x^2\,\rho_x^*-2\,kc_x^2\,\sqrt{\rho_y^*\ \rho_x^*} < 0.$

(2) Comparison of t_1 with proposed estimator (T)

MSE (t_1) = MSE (T)

$$= \theta\overline{Y}^2\left[c_y^2\,\rho_y^*+c_x^2\,\rho_x^*-2\,kc_x^2\,\sqrt{\rho_y^*\ \rho_x^*}\ \right]$$

Thus to $o(1/n)$, MSE (t_1) = MSE (T) are equally efficient.

4. Empirical Study

To compare the proposed estimator (T) with other estimators t_0, t_1 and t_3 empirically, we are considering two natural population data sets. Descriptions of populations are given below.

Population I – [Source: Johnson and Wichard (2003), P – 275]

x : Male width y : Male height

$\overline{Y}$ = 38.80, S_y^2 = 4.89, C_y = 0.06, S_{xy} = 9.20

$\overline{X}$ = 84.27, S_x^2 = 23.79, C_x = 0.06, ρ_{xy} = 0.86

ρ_x = 0.77, ρ_y = 0.59, N = 15, n = 3, $\rho_x^* = 2.54$, $\rho_y^* = 2.18$, $k = 0.86$

Population II - [Source: Bhuyan (2005), P - 4]

x: Level of education of father (in Completed years)

y: Level of education of Mother (in Completed years)

$\overline{Y}$ = 10.93, S_y^2 = 26.50, C_y = 0.47, S_{xy} = 16.72

$\overline{X}$ = 5.13, S_x^2 = 10.55, C_x = 0.63, ρ_{xy} = 0.88

ρ_x = - 0.09, ρ_y = 0.10, N = 15, n = 3, $\rho_x^* = 0.82$, $\rho_y^* = 1.20$, $k = 0.651$

	Estimator	MSE
Population - I	t_o	3.6636
	t_1	1.1272
	t_3	1.3151
	T	1.1272
Population - II	t_o	9.8773
	t_1	2.9184
	t_3	3.3620
	T	2.9184

5. Conclusion

A hybrid ratio-type estimator has been proposed under systematic sampling for estimating $\bar{y}$, and its efficiency is compared with the simple mean estimator as well as the ratio-type estimators t_1 (Swain, 1964) and t_3 (Bahl & Tuteja, 1991). Through a numerical example, it is observed that the suggested estimator T performs with equal efficiency to t_1 and demonstrates greater efficiency than both t_0 and t_3. Theoretical analysis further shows that, under specific conditions, the proposed estimator exhibits less bias than t_1 when $k < \frac{1}{8}\frac{\rho_x^*}{\sqrt{\rho_y^*\rho_x^*}}$. Consequently, estimator T outperforms the alternatives t_1, t_0 and t_3 and is recommended for practical application in estimating the population mean.

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