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RESEARCH ARTICLE

Neutrosophic Contra αgs-Continuous Functions

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Abstract

In this paper, we introduce and study a new class of continuous functions, called Neutrosophic contra αgs-continuous functions and Neutrosophic almost contra αgs-continuous functions in Neutrosophic topological spaces are studied and their basic properties are discussed. Also, we define Neutrosophic contra αgs-irresolute functions and discussed their basic properties.

Keywords: Neutrosophic αgs-closed sets, Neutrosophic αgs-open sets, Neutrosophic αgs- continuous functions, Neutrosophic contra αgs-continuous functions, Neutrosophic almost contra αgs-continuous functions.

1. Introduction

Neutrality the degree of indeterminacy as an independent concept was introduced by Florentine Smarandache [5]. He also defined the Neutrosophic set on three components, namely Truth (membership), Indeterminacy, Falsehood (non-membership) from the fuzzy sets and intuitionistic fuzzy sets. Smarandache's Neutrosophic concepts have wide range of real time applications for the fields of [1, 2, 3 & 4] Information systems, Computer science, Artificial Intelligence, Applied Mathematics and Decision making.

Salama A. A. [10] introduced Neutrosophic closed set and Neutrosophic continuous functions in Neutrosophic topological spaces. Further the basic sets like Neutrosophic regular-open sets, Neutrosophic semi-open sets, Neutrosophic pre-open sets, Neutrosophic α-open sets and Neutrosophic generalized closed sets are introduced in Neutrosophic topological space and their properties are studied by various authors [6], [9], [10], [12]. In this direction, we introduce and analyze a new class of Neutrosophic generalized closed set called Neutrosophic α-generalized semi closed sets and Neutrosophic α-generalized semi open sets in Neutrosophic topological spaces.

2. Preliminaries

We recall some basic definitions that are used in the sequel.

Definition 2.1:[5] Let X be a non-empty fixed set. A neutrosophic set (NS) A is an object having the form $A = \{x, \mu_A(x), \sigma_A(x), \nu_A(x): x \in X\}$ where $\mu_A(x), \sigma_A(x), \nu_A(x)$ represent the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element $x \in X$ to the set A.

A neutrosophic set $A = \{(x, \mu_A(x), \sigma_A(x), \nu_A(x)): x \in X\}$ can be identified as an ordered triple $<\mu_A(x), \sigma_A(x), \nu_A(x)>$ in $]^-0, 1^+[$ on X.

Definition 2.2[5]: A neutrosophic topology (NT) on a non-empty set X is a family ι of neutrosophic subsets in X satisfies the following axioms:

(NT1) $0_N , 1_N \in \iota$

(NT2) $G_1 \cap G_2 \in \iota$ for any $G_1, G_2 \in \iota$

(NT3) $\cup G_i \in \iota, \forall\ G_i \in \iota$

In this case the pair (X, ι) is a neutrosophic topological space (NTS) and any neutrosophic set in ι is known as a neutrosophic open set (NOS) in X. A neutrosophic set A is a neutrosophic closed set (NCS) if and only if its complement C(A) is a neutrosophic open set in X. Here the empty set (0_N) and the whole set (1_N) may be defined as follows:

$0_N = \{(x, 0, 0, 1): x \in X\}$

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$0_N = \{(x, 0, 0, 0): x \in X\}$

$1_N = \{(x, 1, 0, 0): x \in X\}$

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Definition 2.3: [5] Let X be a non-empty set and Neutrosophic sets A and B in the form

$A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle: x \in X\}$ and $B = \{\langle x, \mu_B(x), \sigma_B(x), \nu_B(x)\rangle: x \in X\}$. Then we may consider two possible definitions for subsets ($A \subseteq B$).

(i) $A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x), \sigma_A(x) \le \sigma_B(x)$ and $\mu_A(x) \ge \mu_B(x)\ \forall\, x \in X$

(ii) $A \subseteq B \Leftrightarrow \mu_A(x) \le \mu_B(x), \sigma_A(x) \ge \sigma_B(x)$ and $\mu_A(x) \ge \mu_B(x)\ \forall\, x \in X$

Definition 2.4: [5] Let X be a non-empty set and $A = \{<x, \mu_A(x), \sigma_A(x), v_A(x)>: x \in X\}$,
$B = \{<x, \mu_B(x), \sigma_B(x), v_B(x)>: x \in X\}$ are NSs. Then $A \cap B$ may be defined as:

(I1) $A \cap B = <x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x)$ and $v_A(x) \vee v_B(x)>$

(I2) $A \cap B = <x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x)$ and $v_A(x) \vee v_B(x)>$

$A \cup B$ may be defined as:

(U1) $A \cup B = <x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x)$ and $v_A(x) \wedge v_B(x)>$

(U2) $A \cup B = <x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x)$ and $v_A(x) \wedge v_B(x)>$

Definition 2.5: [5] For all A and B are two Neutrosophic sets then the following conditions are true: $C(A \cap B) = C(A) \cup C(B)$; $C(A \cup B) = C(A) \cap C(B)$.

Now, we define the Neutrosophic closure and Neutrosophic interior operations in Neutrosophic topological spaces:

Definition 2.6: [5] Let (X, τ) be NTS and $A = \{<x, \mu_A(x), \sigma_A(x), v_A(x)>: x \in X\}$ be a NS in X. Then the Neutrosophic closure and Neutrosophic interior of A are defined by

(i) $NCl(A) = \cap\{K : K$ is a NCS in X and $A \subseteq K\}$

(ii) $NInt(A) = \cup\{G : G$ is a NOS in X and $G \subseteq A\}$

It can be also shown that NCl(A) is NCS and NInt(A) is a NOS in X.

(i) A is NOS if and only if A = NInt(A),

(ii) A is NCS if and only if A = NCl(A)

3. Neutrosophic contra αgs-continuous functions

In this section we have introduced neutrosophic contra αgs-continuous functions and studied some of their basic properties.

Definition 3.1: A function $f : (X, \tau) \rightarrow (Y,\sigma)$ is called a neutrosophic contra generalized continuous function if the inverse image of every neutrosophic open set in (Y,σ) is a neutrosophic generalized closed set in (X,τ).

Definition 3.2: A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a neutrosophic contra αgs-continuous function if $f^{-1}(A)$ is a Nαgs-closed set in (X,τ) for every neutrosophic open set A in (Y,σ).

Remark 3.3: Every neutrosophic contra continuous function is a neutrosophic contra αgs-continuous function.

Proof: Let $f: X \rightarrow Y$ be neutrosophic contra continuous function and A be a neutrosophic open set in Y. From hypothesis, $f^{-1}(A)$ is a neutrosophic closed set in X. As every neutrosophic closed set is a neutrosophic αgs-closed, then $f^{-1}(A)$ neutrosophic αgs-closed set in X and hence f is a neutrosophic contra αgs-continuous function.

Theorem 3.4: Every neutrosophic contra α-continuous function is a neutrosophic contra αgs-continuous function but not conversely in general.

Proof: Let $f: X \rightarrow Y$ be neutrosophic contra α-continuous function. Let A be a neutrosophic open set in Y. From hypothesis, $f^{-1}(A)$ is a neutrosophic α-closed set in X. As, every neutrosophic α-closed set is a neutrosophic αgs-closed and so $f^{-1}(A)$ is a neutrosophic αgs-closed set in X and hence f is a neutrosophic contra αgs-continuous function.

Theorem 3.5: A functions f: X → Y is a neutrosophic contra αgs-continuous functions if and only if the inverse image of each neutrosophic closed set in Y is a neutrosophic αgs-open set in X.

Proof: Necessity: Let A be a neutrosophic closed set in Y and so A^c is a neutrosophic open in Y. Since f is a neutrosophic contra αgs-continuous, $f^{-1}(A^c)$ is a neutrosophic αgs-closed set in X. We have, $f^{-1}(A^c) = (f^{-1}(A))^c$ and so $f^{-1}(A)$ is a neutrosophic αgs-open in X.

Sufficiency: Let A be a neutrosophic open set in Y and so A^c is a neutrosophic closed set in Y. From hypothesis, $f^{-1}(A^c)$ is neutrosophic αgs-open in X. We have, $f^{-1}(A^c) = (f^{-1}(A))^c$ and so $f^{-1}(A)$ is a neutrosophic αgs-closed in X. Thus, f is a neutrosophic contra α αgs-continuous function.

Theorem 3.6: Let f: X → Y be a neutrosophic contra αgs-continuous and g: Y → Z is a neutrosophic contra continuous functions. Then g∘f: X → Z is a neutrosophic αgs-continuous.

Proof: Let A be a neutrosophic open set in Z. From hypothesis, $g^{-1}(A)$ is a neutrosophic closed set in Y. As f is a neutrosophic contra αgs-continuous, $f^{-1}(g^{-1}(A))$ is a neutrosophic αgs-open in X, that is $(g∘f)^{-1}(A)$ is a neutrosophic αgs-open in X. Thus, g∘f is a neutrosophic αgs-continuous.

Theorem 3.7: Let f: X → Y be a neutrosophic αgs-continuous and g: Y → Z is neutrosophic contra continuous, then g∘f: X → Z is a neutrosophic contra αgs-continuous.

Proof: Let A be a neutrosophic open set in Z and so $g^{-1}(A)$ is a neutrosophic closed in Y. As f is a neutrosophic αgs-continuous, $f^{-1}(g^{-1}(A))$ is a neutrosophic αgs-closed in X, that is $(g∘f)^{-1}(A)$ is a neutrosophic αgs-closed in X and so g∘f is a neutrosophic contra αgs-continuous function.

Theorem 3.8: Let f: X → Y be neutrosophic contra αgs-continuous and g: Y → Z is neutrosophic continuous functions. Then g∘f: X → Z is a neutrosophic contra αgs-continuous function.

Proof: Let A be a neutrosophic open set in Z. By hypothesis, $g^{-1}(A)$ is a neutrosophic open in Y. As f is a neutrosophic contra αgs-continuous, $f^{-1}(g^{-1}(A))$ is a neutrosophic αgs-closed in X. That is $(g∘f)^{-1}(A)$ is a neutrosophic αgs-closed in X and so g∘f is a neutrosophic contra αgs-continuous function.

4. Neutrosophic almost contra αgs-continuous functions

Definition 4.1: A functions f: X → Y is called a neutrosophic almost contra continuous functions if $f^{-1}(A)$ is a N-closed set in X for every NR-open set A in Y.

Definition 4.2: A functions f: X → Y is called a neutrosophic almost contra αgs-continuous functions if $f^{-1}(A)$ is a Nαgs-closed in X for every NR-open set A in Y.

Theorem 4.3: A functions f: X → Y is a neutrosophic almost contra αgs-continuous if and only if the inverse image of each neutrosophic regular closed set in Y is a neutrosophic open set in X.

Proof: Necessary`: Let A be a neutrosophic regular closed set in Y, that is A^c is a neutrosophic regular open in Y. As f is a neutrosophic almost contra αgs-continuous functions, $f^{-1}(A^c)$ is a neutrosophic αgs-closed in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, so $f^{-1}(A)$ is a Nαgs-open set in X.

Sufficiency: Let A be a neutrosophic regular open set in Y, that is A^c is a neutrosophic regular closed in Y. From hypothesis, $f^{-1}(A^c)$ is Nαgs-open set in X. We have, $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is a Nαgs-closed in X and so f is a neutrosophic almost contra αgs-continuous functions.

Theorem 4.4: Every neutrosophic contra continuous functions is a neutrosophic almost contra αgs-continuous functions but not conversely in general.

Proof: Let f: X → Y be a neutrosophic contra continuous functions and A be a neutrosophic regular open set in Y and so A is a neutrosophic open set in Y. Since f is a neutrosophic contra continuous functions, $f^{-1}(A)$ is a neutrosophic closed in X and as every neutrosophic closed set is a Nαgs-closed set in X. Thus, f is a neutrosophic almost contra αgs-continuous functions.

Theorem 4.5: Every neutrosophic contra αgs-continuous functions is a neutrosophic almost contra αgs-continuous functions.

Proof: Let f: X → Y be a neutrosophic contra αgs-continuous functions and A be a neutrosophic regular open set in Y. This implies neutrosophic open set in Y. Then by hypothesis, $f^{-1}(A)$ is a neutrosophic αgs-closed set in X. Since every neutrosophic α-closed set is a neutrosophic αgs-generalized closed set, $f^{-1}(A)$ is a neutrosophic αgs-closed set in X. Hence f is a neutrosophic almost contra αgs-continuous functions.

5. Neutrosophic contra αgs-irresolute functions

In this section, we have introduced neutrosophic contra αgs-irresolute functions and studied some of their basic properties.

Definition 5.1: A function f: X → Y is called a neutrosophic contra αgs-irresolute functions if $f^{-1}(A)$ is a neutrosophic αgs-closed in X for every Nαgs-open set A in Y.

Theorem 5.2: If f: X → Y is called a neutrosophic contra α*gs-irresolute, then f is a neutrosophic contra αgs-continuous function.

Proof: Let f be a neutrosophic contra αgs-irresolute functions and A be any neutrosophic open set in Y. As every neutrosophic open set is a N αgs-open and so A is a Nαgs-open in Y. From hypothesis, $f^{-1}(A)$ is a Nαgs-closed in X. Hence, f is a neutrosophic contra αgs-continuous.

Theorem 5.3: Let f: X → Y and g: Y → Z are neutrosophic contra αgs-irresolute functions. Then g∘f : X → Z is a neutrosophic αgs-irresolute functions.

Proof: Let A be a Nαgs-open in Z and so $g^{-1}(A)$ is a N αgs-closed set in Y. Since f is a neutrosophic contra αgs-irresolute, $f^{-1}(g^{-1}(A))$ is a Nαgs-open in X, that is $(g∘f)^{-1}(A)$ is a Nαgs-open set in X. Hence g∘f is a neutrosophic αgs-irresolute functions.

Theorem 5.4: If f: X → Y is a neutrosophic contra αgs-irresolute functions and g: Y → Z is a neutrosophic contra αgs-continuous functions, then g∘f: X → Z is a neutrosophic αgs-continuous functions.

Proof: Let A be Nαgs-open set in Z. From hypothesis, $g^{-1}(A)$ is a Nαgs- closed in Y. As f is a neutrosophic contra αgs-irresolute, $f^{-1}(g^{-1}(A))$ is a neutrosophic Nαgs-open in X, that is $(g \circ f)^{-1}(A)$ is a Nαgs-open set in X and so g∘f is a neutrosophic αgs-continuous functions.

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