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## NUMERICAL SIMULATION OF HEAT TRANSFER IN THE WALLS OF AN AIR HOLLOW MANY-LAYER COMBUSTION FURNACE USING THE FINITE DIFFERENCE METHOD

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### ABSTRACT

The purpose of conducting a numerical simulation is to analyse the heat transfer occurring on a multilayer wall of a furnace that contains a cavity. This simulation aims to determine the temperature distribution and airflow pattern on the furnace wall, considering both isothermal and adiabatic boundary conditions. The interior of the wall exhibits a higher temperature compared to the exterior of the wall. This task involves solving a problem that is characterised by two dimensions and is subject to changes over time. The ADI (Alternating Directional Implicit) method is employed to discretize the heat transfer equation for conduction and natural convection within the boiler wall. The Thomas Algorithm is employed to calculate the temperature distribution and airflow pattern on the wall. The current methodology is validated through a comparison of its numerical outcomes with existing data in the literature. A high level of consensus had been reached.

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### 1. INTRODUCTION

Efficient furnace design is important to reduce fuel consumption and heat losses, as well as to reduce the negative impact on the environment due to the use of hydrocarbon fuels. Air-hollow multilayer walls that act like heat insulators can be applied to small industrial combustion furnaces to reduce heat losses.

One method of solving heat transfer can be done using numerical methods. The advantage of numerical methods is that they can provide fast and accurate results. The choice of heat transfer solution method must be considered and adapted to the boundary conditions and geometry. In this research, the combustion furnace wall experiences conduction heat transfer in the solid layer and natural convection in the air layer with isothermal and adiabatic boundary conditions. The finite difference method is the method used in this research.

### 2. PURPOSE

The objectives of this research are:

1. Shows the temperature distribution on the walls of a multi-layer combustion furnace which experiences unsteady state conduction and convection heat transfer.
2. Knowing the effect of the thermal conductivity value of the materials making up the walls of the combustion furnace on the temperature distribution.
3. Know the effect of air layer thickness on the insulation capacity.

### 3. LITERATURE REVIEW

Optimal thickness of the air layer in windows with double glazing. The thickness of the air layer should be such that it does not allow air movement. The free movement of air increases the convection heat transfer coefficient, causing the natural convection heat transfer in the air layer to become greater, thereby reducing the insulation capacity.

Research on natural convection problems in two-dimensional boxes with variations in slope was carried out by Aris (2006) by solving it using the finite difference method to obtain velocity vectors, temperature distribution and pressure distribution with air fluid.

Balderas et al. (2007) conducted heat transfer research in multilayer walls where he observed a critical thickness that identified the beginning of the natural convection process in the air layer.

Armando et al. (2011) developed research on simulating heat transfer on furnace walls using computational fluid dynamics software (Fluent 6.2.16). The heat transfer studied is conduction heat transfer and two-dimensional steady state natural convection heat transfer with isothermal and adiabatic boundary conditions on the walls of a four-layer combustion furnace material.

### 4. METHODS

#### 4.1 Research Procedures

This research uses Fortran and MATLAB supporting software in modelling. Determining the geometry, boundary conditions and iteration process of the furnace walls using Fortran and then visualizing the temperature distribution and air flow patterns using MATLAB.

The research was carried out by literature study with the research steps outlined as follows:

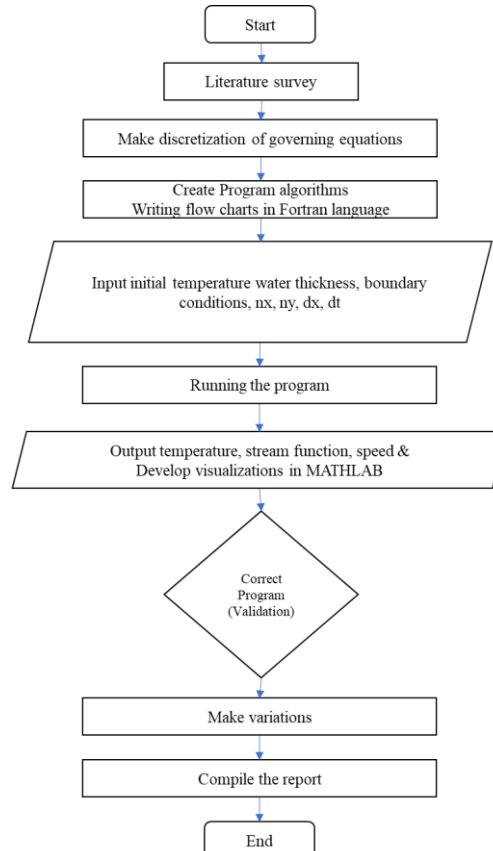


Figure 1. Research flow diagram

#### 4.2 Finite Difference Governing Equations

The governing equations in this research are solved using the ADI (Alternating Direction Implicit) method. The governing equation for natural convection is as follows:

Continuity Equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Navier Stokes equation:

□ Momentum Equation in x direction:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial p}{\partial x} + \frac{Pr}{Ra^{0.5}} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + Pr \theta \cos \theta$$

□ Momentum Equation in y direction:

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = \frac{\partial p}{\partial y} + \frac{Pr}{Ra^{0.5}} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + Pr \theta \sin \theta$$

Energy Equation:

$$\frac{\partial \theta}{\partial t} + u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{Ra^{0.5}} \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right)$$

The conduction governing equation uses the energy equation with the velocity equal to zero, as follows:

$$\frac{\partial \theta}{\partial t} = \frac{1}{Ra^{0.5}} \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right)$$

Stream function is calculated using the Line Gauss-Siedel method. The equation for finding the stream function in the air layer is as follows:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = - \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)$$

The Rayleigh number is defined as a dimensionless unit of the product of the Grashof number and the Prandtl number (Pr) which is formulated as follows:

$$Ra = Gr \cdot Pr = \frac{g \beta (T_i - T_o) L^3 \cdot Pr}{\nu^2}$$

The heat flux through the multilayer combustion furnace wall is calculated using the following equation:

$$q'' = \frac{T_i - T_o}{\frac{l_1}{k_1} + \frac{l_2}{k_2} + \frac{l_3}{k_3} + \frac{l_4}{k_{eff}}}$$

The convection heat transfer coefficient can be calculated using the Nusselt number which is based on the Churchill and Chu equation as follows:

$$\bar{N}_{uL} = 1.333 \left\{ \frac{Gr_L}{4} \right\}^{0.35} g(Pr)$$

$$g(Pr) = 0.75 \frac{Pr^{0.5}}{(0.609 + 1.221 Pr^{0.5} + 1.238 Pr)^{0.25}}$$

The average convection heat transfer coefficient is formulated:

$$\bar{h}_L = \bar{N}_{uL} \frac{k_{eff}}{H}$$

Where,

$$k_{eff} = \bar{N}_{uL} k_{air}$$

#### 4.3 Geometry of Combustion Furnace Walls

The furnace walls studied by Armando et al. (2011) has properties as in table 1.

Table 1. Wall properties Armando et al. (2011)

Layer	Material	Thickness (m)	Thermal Conductivity (W/m.K)
1	Fire bricks	0.1	1.1
2	Ceramic fiber	0.1	0.22
3	Air	0.1	0.05298
4	Common brick	0.2	0.72

The walls of the multilayer combustion furnace studied by Armando et al. (2011) consists of four layers of material, namely: fire brick, ceramic fiber, air and common brick with a height of 1.9 m.

Armando et al. (2011) conducted a two-dimensional steady state heat transfer simulation study on the walls of a four-layer combustion furnace with isothermal boundary conditions. On the top and bottom walls of this multilayer combustion furnace are assumed to be adiabatic.

The furnace walls studied in this study have properties as in table 2.

Table 2. Wall properties studied

Layer	Material	Thickness (m)	Thermal Conductivity (W/m.K)
1	Fire brick	0.1	1.1
2	Ceramic fiber	0.1	0.22
3	Air	0.05 to 0.1	0.05298
		0.15 to 0.2	
4	Common brick	0.2	0.72

In this study, the walls of the four-layer furnace studied had the same material as the walls of the furnace studied by Armando et al. (2011) but with a height of 1.1 m. The thickness of each layer is also the same except for the air layer. The thickness of the air layer was varied from 0.05 m to 0.2 m. Iterate temperature values and air flow patterns using the ADI method (Alternating Direction Implicit).

The assumptions in this research are:

- i. The problem under study is a non-steady condition
- ii. Natural conduction and convection heat transfer in walls is two-dimensional heat transfer
- iii. The boundary conditions at the top and bottom of the wall are adiabatic
- iv. The inner and outer boundary conditions of the wall are isothermal ( $T_i = 1173$  K,  $T_o = 300$  K)
- v. The thermal conductivity of the wall material is constant
- vi. The interface between the wall materials is in perfect contact

## 5. RESULTS AND DISCUSSION

### 5.1 Validation of the ADI (Alternating Direction Implicit) Method

To test the validity of the ADI (Alternating Direction Implicit) method used in this research, it needs to be compared with other methods. Fluid dynamics computational software (Fluent 6.2.16) will be used to validate this research.

#### 5.1.1 Validation of Computational Fluid Dynamics Software (Fluent 6.2.16)

Armando et al. (2011) developed research on simulating heat transfer on furnace walls using computational fluid dynamics software (Fluent 6.2.16). The heat transfer studied is two-dimensional steady state natural conduction and convection heat transfer with isothermal boundary conditions on the inner and outer walls, while the top and bottom have adiabatic boundary conditions. The temperature of the inner side of the furnace wall is 1173 K and the temperature of the outer side of the furnace wall is 300 K.

The two-dimensional unsteady state simulation of natural conduction and convection heat transfer on the walls of a multi-layer combustion furnace in this research uses the finite difference method. The ADI (Alternating Direction Implicit) method is used to iteratively calculate the temperature distribution and air flow patterns on the walls of a multi-layer combustion furnace.

A comparison of the visualization results of the ADI (Alternating Direction Implicit) method with computational fluid dynamics software (Fluent 6.2.16) can be seen in the image below.

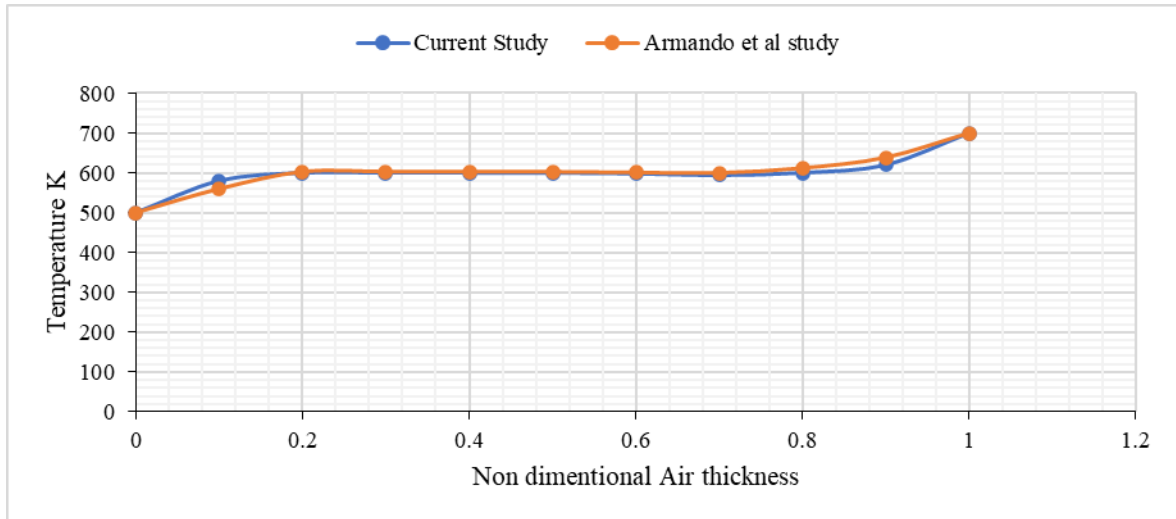


Figure 2. Visualization of temperature distribution on the furnace wall (a) ADI method (b) Fluent 6.2.16 software (Armando et al.)

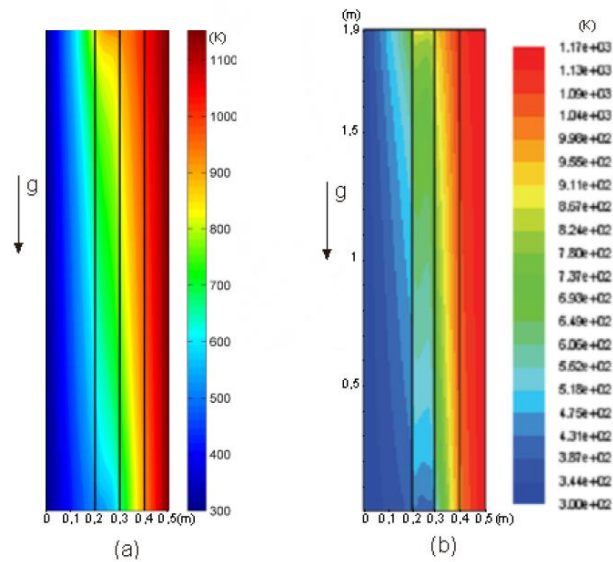


Figure 3. Comparison graph of temperature distribution in the air layer of the current research with the research of Armando et al.

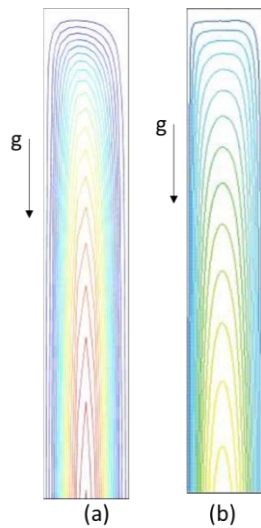


Figure 4. Visualization of air flow patterns on the furnace walls (a) ADI method (b) Fluent 6.2.16 software (Armando et al.)

Figure 2 shows that the temperature distribution of the furnace walls is influenced by the thermal conductivity value of the constituent materials. The lower the thermal conductivity value of the material, the more difficult the temperature propagation is. This is indicated by the increasingly dense color contours. The temperature distribution visualization results above between the ADI (Alternating Direction Implicit) method and computational fluid dynamics software (Fluent 6.2.16) show relatively similar temperature distribution results.

Figure 3 is a comparison graph of the temperature distribution in the air layer of the multi-layer combustion furnace wall from the results of the current research with the research of Armando et al. The graph shows relatively similar patterns and values with the largest error: 7.2% and average error: 3.6%. This can be used for validation because research results are considered valid if the error value is below 10%.

Figure 4 shows the air flow pattern that occurs in the air layers on the walls of a multi-layer combustion furnace. This air movement occurs due to the buoyant force that arises due to differences in density due to differences in pressure in the flow. Visualization results of air flow patterns on the multi-layer combustion furnace walls above between the ADI (Alternating Direction Implicit) method and computational fluid dynamics software (Fluent 6.2.16) shows relatively the same air flow pattern results.

**5.2 Simulation of 2D Unsteady State Heat Transfer on Four-Layer Combustion Furnace Walls with Varying Air Layer Thickness**

The simulation of the case of conduction and convection heat transfer on the walls of a two-dimensional four-layer combustion furnace was carried out using the ADI (Alternating Direction Implicit) method with boundary conditions on the inside of the wall of 1173 K and on the outside of the wall 300 K. The thickness of the air layer varied with the height of the furnace wall constant combustion is 1.1 m and  $\Delta x = \Delta y$  is 0.005 m. Variations in air layer thickness and number of grids are shown in table 3.

Table 3. Variations in air layer thickness and number of grids

Air Layer Thickness (m)	Number of X Direction Grids	Number of Y Direction Grids
0.05	81	231
0.1	91	231
0.15	101	231
0.2	111	231

Visualization of the temperature distribution on the walls of a four-layer combustion furnace with variations in the thickness of the air layer experiencing heat transfer is as follows:

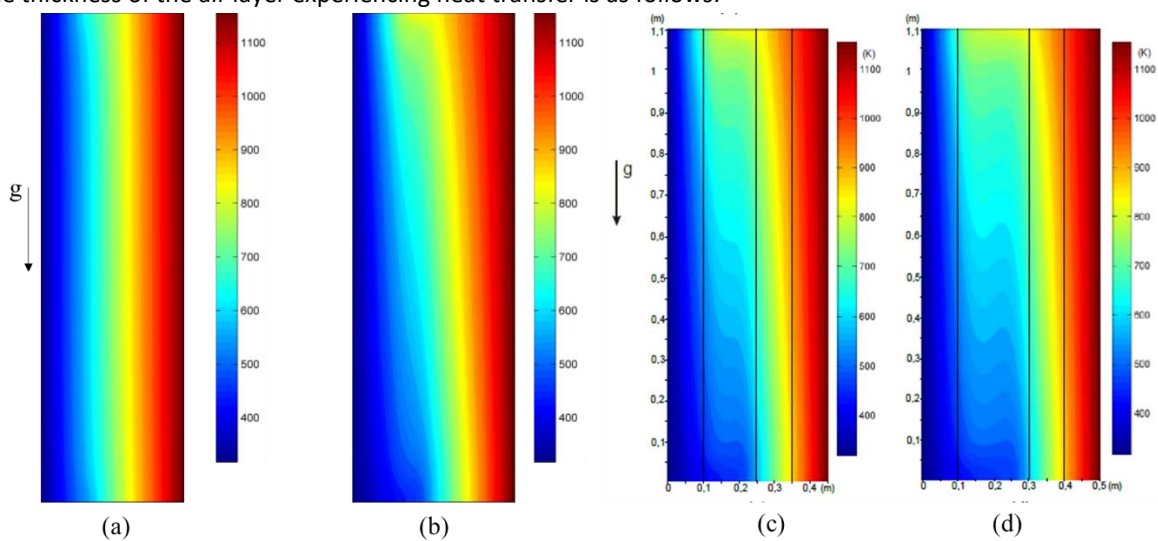


Figure 5. Visualization of temperature distribution on the walls of a combustion furnace with an air layer thickness (a) 0.05 m (b) 0.1 m (c) 0.15 m (d) 0.2m.

Figure 5 shows that the visualization of the temperature distribution of the furnace walls varies with the thickness of the air layer. In the picture it can be seen that the greater the thickness of the air, the more visible the natural convection heat transfer will be in the air layer.

The relationship between the thickness of the air layer and the temperature distribution on the walls of the multilayer combustion furnace at  $Y=0.55$  with variations in thickness can be seen in the graph below.

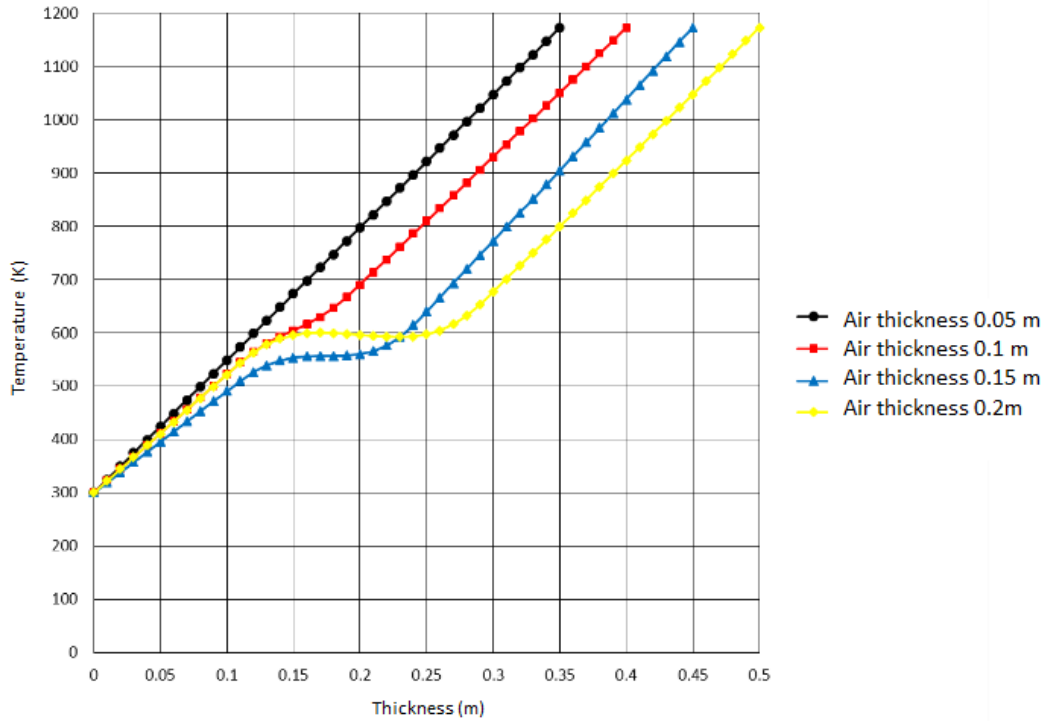


Figure 6. Graph of the relationship between air layer thickness and temperature distribution on the furnace wall at  $Y=0.55$  m

Figure 6 shows the relationship between the thickness of the air layer and the temperature distribution on the multi-layer combustion furnace wall at  $Y=0.55$ . The first variation with an air layer thickness of 0.05 m has a linear graph in the air layer. This shows that at an air layer thickness of 0.05 m, the heat transfer that occurs in the air layer is dominated by conduction heat transfer. Meanwhile, the second, third and fourth variations have graphs that are not linear (curved) in the air layer. This shows that at air layer thicknesses of 0.1 m, 0.15 m and 0.2 m, the heat transfer that occurs in the air layer is natural convection heat transfer and conduction heat transfer only occurs in the air attached to the wall.

The air flow pattern that occurs in the air layer on the four-layer combustion furnace wall with variations in the thickness of the air layer is as follows:

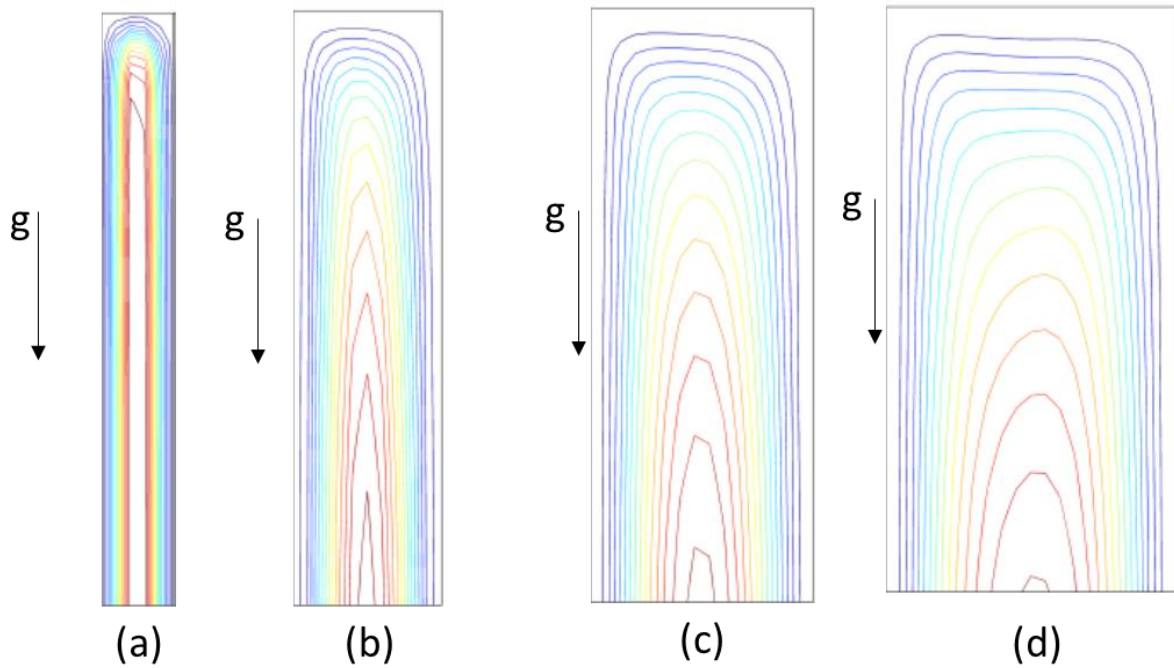


Figure 7. Visualization of flow patterns on the walls of a combustion furnace with an air layer thickness of (a) 0.05 m (b) 0.1 m (c) 0.15 m (d) 0.2 m.

Figure 7 shows the air flow pattern that occurs in the air layer on the furnace wall with variations in the thickness of the air layer. In the picture you can see that the air close to the right wall moves upwards while the air close to the left wall moves downwards. This happens because the right wall is hotter than the left wall, so the air close to the right wall will have a higher temperature and lower density, making the air move upwards, while the air close to the left wall will have a lower temperature and Greater density means air will move downwards.

The relationship between variations in air layer thickness with the average Nusselt number, average convection heat transfer coefficient and heat flux is as follows:

Table 4. Air layer thickness, average Nusselt number, average convection heat transfer coefficient and heat flux

Air Thickness (m)	The average Nusselt number	Average Heat Transfer Coefficient (W/m <sup>2</sup> K)	Heat Flux (W/m <sup>2</sup> )
0.05	10.959	0.527	1135.07
0.1	18.431	0.887	1111.52
0.15	24.981	1.203	1096.27
0.2	30.996	1.492	1084.76



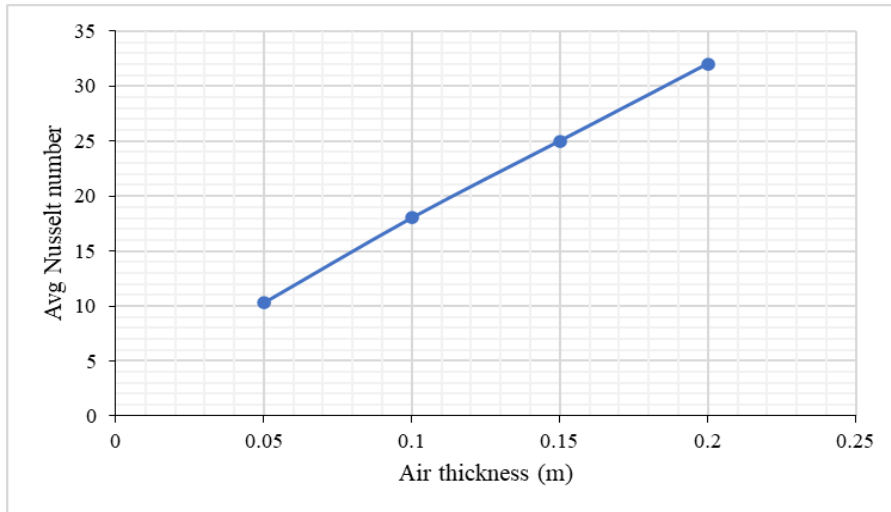


Figure 8. Relationship between air layer thickness and average Nusselt number

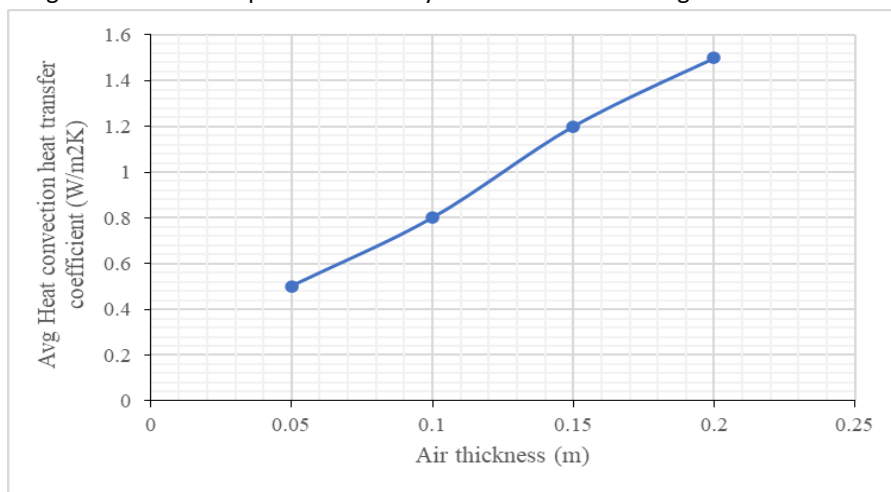


Figure 9. Relationship between air layer thickness and average heat transfer coefficient

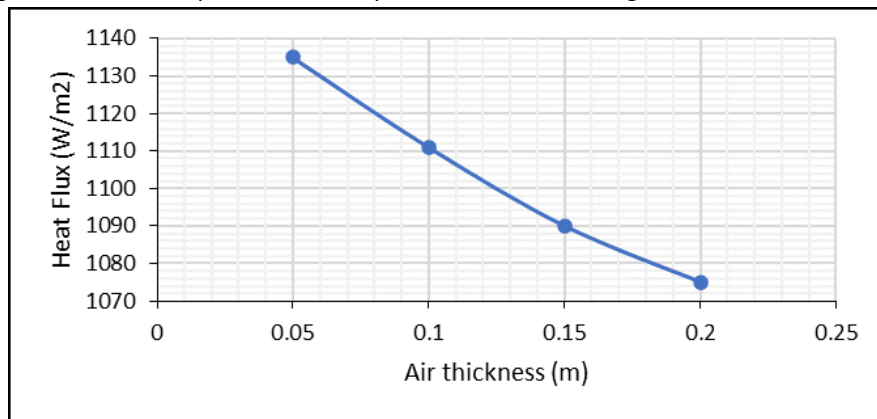


Figure 10. Relationship between air layer thickness and heat flux

Figure 8 shows the relationship between the thickness of the air layer and the average Nusselt number value. The data obtained shows that the greater the air thickness, the average Nusselt number value also increases. The Nusselt number is the ratio between the convection heat transfer coefficient ( $h$ ) and the thermal conductivity ( $k$ ) on a surface. So, the greater the Nusselt number, the greater the natural convection heat transfer.

Figure 9 shows the relationship between the thickness of the air layer and the average convection heat transfer coefficient. The data obtained shows that the greater the thickness of the air, the average convection

heat transfer coefficient ( $h$ ) also increases so that the convection heat transfer that occurs in the air layer is also greater. This causes a reduction in the insulating capacity of the air layer (Armando et al. 2009).

Figure 10 shows the relationship between air layer thickness and heat flux. The data obtained shows that the greater the air thickness, the smaller the heat flux ( $q''$ ). This happens because the total thickness of the wall also increases as the air thickness increases so that the heat flux through the wall decreases as the wall thickness increases.

## 6. CONCLUSION

Several things that can be concluded from the research and discussion of the results that have been carried out include:

- a. Different methods have succeeded in displaying the temperature distribution and air flow patterns on the walls of multi-layer combustion furnaces that experience heat transfer.
- b. The thermal conductivity value of a material making up the walls of a combustion furnace influences its temperature distribution. The smaller the thermal conductivity value of a material, the more difficult it is to conduct heat because the material has good heat insulating properties. In this study, the material making up the walls of the combustion furnace which had the smallest thermal conductivity was air with a thermal conductivity value of 0.05298 W/m.K.
- c. The thickness of the air layer affects the heat transfer that occurs in the air layer. The greater the thickness of the air layer, the greater the natural convection heat transfer that occurs. This is indicated by the increase in the average convection heat transfer coefficient value as the thickness of the air layer increases. The thickness of the air layer that has the best insulating capacity is an air layer thickness of 0.05 m with the smallest average heat transfer coefficient value of 0.527 W/m<sup>2</sup>K.

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