Vol.3.Issue.1.2015

http://www.bomsr.com

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

ISSN:2348-0580

RESEARCH ARTICLE

OPTIMAL INVENTORY FOR CLEARANCE- TYPE OF SALES SITUATION (PREVAILING IN DEVELOPING COUNTRIES LIKE INDIA)

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Article Info:
Article received :18/01/2015
Revised on:26/01/2015
Accepted on:01/02/2015

ABSTRACT

This paper deals with an inventory model related to a clearance type of sales situation. Optimal inventory model for obtaining optimal order rate and optimal profit based on the assumption that the demand follows an exponential pattern have been studied. Many qualitative aspects are extracted from the theoretical results that are presented in this paper.

Key words: Optimal order rate, optimal profit.

INTRODUCTION

In many recent investigations optimal inventory models are scarcely developed, based on the assumption that demand rate, inventory level and price patterns are independent of each other. For example, a manufacturing company dealing with the marketing of an item may go in for some attractive advertisements investing suitable or necessary money/funds per unit item. Such costs therefore should naturally be proportional to the stock level of the item and it may reasonably be included in the holding cost. In return to this cost in advertisement, the demand rate is boosted up in proportion to the stock level of the item. Further, in case of certain type of consumer items, some customers may be motivated to acquire stocks in abundance when prices are low. Such a behavior of customers is approximated reasonably well through an exponential type of demand pattern.

Rakesh Guptha and Prem Vrat (1986) developed an EOQ model by considering cost minimization technique based on the assumption that inventories are dependent on the consumption rate. Mandal and Phaujdar (1989) investigated into inventory models with stock dependent consumption rate employing the technique of profit maximization. Venugopal and Chandra Kesavulu Naidu (1991) developed optimal inventory systems employing concurrently the techniques of profit maximization and cost minimization. However, in the works cited above situations as are indicated in the earlier data are not dealt with. We observe that situations are particularly relevant to developing countries, like India. We thus are motivated into investigating a problem concerning a clearance- sales type of situation. In such cases, exponential types of demand patterns are well suited.

In the following we consider such demand patterns. Naturally then, we are led to consider appropriately low values for the ordering cost, the holding cost and the decay rate. First, optimal inventory model is developed and later the results are supported with empirical work choosing typical parametric values. The qualitative aspects of the research are then brought out in detail in the discussion.

2. MODEL, NOTATIONS AND NOMENCLATURE

In this section we develop optimal inventory model for obtaining optimal order rate and optimal profit based on the assumption that the demand follows an exponential pattern. We assume quick/clearance sale of an item during short lengths/small periods of time. Here perishability is assumed to be negligible (in keeping with real life physical situation) for if at all perishability occurs this may be due to theft or damage of an item in transit. Set-up cost and holding cost (room rent, transportation charges, electricity charges and so on) are also assumed to be small. However, it may be observed that these assumptions do have role in empirical analysis but its role is little in analytic results.

NOTATIONS

d (p)	exponential demand when the price is ' p '
I(t)	inventory level at time 't'
λ	constant decay rate; in keeping with reality we assume 'λ' to be very small quantity, that is, λ<<1
Z(t)	stock loss due to decay in the closed time interval [0,t]
α	constant
β	constant and positive
p	selling price of an item
C	cost price of an item
K	set-up cost
h	holding cost / unit
T	length of each cycle
C (T,p)	cost / unit time
$\hat{C}$(T,p)	cost/cycle
Q_T	order quantity in a cycle of length T
P^*	optimal price
T^*	optimal cycle length
Q_0	optimal order quantity
π^*	optimal profit
π^*/ Q_0	optimal profit per unit of optimal order quantity
Q_0 / T^*	optimal order quantity per unit of optimal cycle length

We consider a continuous review, deterministic exponential demand with a very small quantity of decay rate. In case of continuous review, it is logical to assume that the depletion due to such decay and the depletion due to meeting the demand occurs simultaneously. Accordingly, the differential equation describing the time behavior of the system is given by:

$$\frac{d\,I(t)}{dt} = -\lambda\, I(t) - d(p) \quad \ldots\ldots\ldots\ldots(1)$$

This is a first order linear differential equation, the solution to which is given by :

$$I(t) = I(O)\, e^{-\lambda t} - \frac{d(p)}{\lambda}\left[1 - e^{-\lambda t}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Now , Z (t), the stock loss due to decay in the time interval [0, t] is the difference between the inventory position at time 't' which would prevail if there were no decay and the inventory position when there is decay. Thus,

$$Z(t) = I(t)\,[e^{\lambda t} - 1] - d(p)\,t + \frac{d(p)}{\lambda}[e^{\lambda t} - 1] \quad \text{.........(3)}$$

The cost structure to be defined includes holding cost, it is appropriate and optimal to set I(T) = 0, where T is the cycle length. Now the expression for quantity ordered in a cycle of T is given by:

$$Q_T = Z(T) + d(P)\,T$$

$$= -d(p)\,T + \frac{d(p)}{\lambda}[e^{\lambda T} - 1] + d(p)\,T$$

$$= \frac{d(p)}{\lambda}[e^{\lambda T} - 1] \quad \text{.......................... (4)}$$

Since $I(0) = Q_T$, we have

$$I(t) = \frac{d(p)}{\lambda}[e^{\lambda(T-t)} - 1]$$

Now $\hat{C}(T, p)$ is given by

$$\hat{C}(T, p) = K + CQ_T + h\int_0^T I(t)\,dt$$

$$= K + \frac{C\,d(p)}{\lambda}[e^{\lambda T} - 1] + \frac{h\,d(p)}{\lambda^2}[e^{\lambda T} - \lambda T - 1]$$

Cost per unit time is given by

$$C(T, p) = \frac{\hat{C}(T, p)}{T}$$

$$= K/T + [C\lambda + h]\,d(p)\,[e^{\lambda T} - 1]/\lambda^2 T - h\,d(p)/\lambda \quad \text{............(5)}$$

Minimizing C(T, p) with respect to T, by holding p fixed we obtain the optimal cycle length T *. Therefore the following conditions must be satisfied.

(i.) $$\left.\frac{\partial C(T,p)}{\partial T}\right|_{T=T^*} = 0, \text{ and } \quad \text{.............................(6)}$$

(ii.) $$\left.\frac{\partial^2 C(T,p)}{\partial T^2}\right|_{T=T^*} > 0 \quad \text{................................... (7)}$$

Minimizing (5) with respect to T, we get

$$\frac{\partial C(T,p)}{\partial T} = -\frac{K}{T^2} + \frac{[(C\lambda+h)\,d(p)(T\lambda e^{\lambda T} - e^{\lambda T} + 1)]}{\lambda^2 T^2}$$

$$= 0$$

After some calculations we obtain:

$$e^{\lambda T}(\lambda T - 1) = K\lambda^2/[d(p)(C\lambda + h)] - 1 \quad \text{.................(8)}$$

An approximate solution to (8) is obtained by using truncated Taylor series expansion for the exponential function of the following type:

$$e^{\lambda T} = 1 + \lambda T + \lambda^2 T^2/2$$

Which is valid for small values of λT $(since\ \lambda << 1, see\ notations)$. Thus it follows from (5), that the cost per unit time now turns out to be:

$$C(T, p) = C\,d(p)\,[1 + \lambda T/2] + K/T + h\,T\,d(p)/2$$

$$= C\,d(p) + (C\lambda + h)\,d(p)\,T/2 + K/T \quad \text{...................... (9)}$$

From (9) we obtain:

$$\left.\frac{\partial C(T,p)}{\partial T}\right|_{T=T^*} = (C\lambda + h)\,d(p)/2 - K/T^2$$

$$= 0 \quad \text{... (10)}$$

From (10), we obtain optimal cycle length T*

$T^* = [\, 2K/ (C\lambda + h)\, d\,(p)\,]^{½}$ .... (11)

Using (4) & (11), we obtain optimal order quantity (quantity per unit time).

$Q_0 = Q/T^* = d\,(p)\,(1+ \lambda\, T^*/2)$ .... (12)

In order to consider the optimal policy for pricing, we maximize the profit function $\pi\ (T,p)$ with respect to 'p'.

$\pi\ (T,p)\ is$ now given by :

$\pi\ (T,p) = p\, d\,(p) - C(T, p)$

$= p\, d\,(p) - [\, C\, d\,(p) + K/T + (C\lambda + h)\, d\,(p)\, T/2]$ .... (13)

Maximizing $\pi\ (T,p)$ with respect to p, we obtain the optimal price p*.

This process in turn leads to the following conditions:

(i) $\left.\dfrac{\partial\, \pi\ (T,p)}{\partial\, p}\right|_{p = p^*} = 0$, and .... (14)

(ii) $\left.\dfrac{\partial^{\,2} \pi\ (T,p)}{\partial\, p^2}\right|_{p = p^*} < 0$ .... (15)

Thus we obtain optimal price, p* as

$P^* = C\,(1+ \lambda\, T^*/2) + \lambda\, T^*/2 - d\,(p\,)/\, d'\,(\,p)$ .... (16)

In the following section, we consider the quick / on-the-spot / clearance sales type of physical situation and derive optimal inventory systems. We observe, as indicated earlier, that exponential type of demand is appropriate in these situations. Hence, specializing to the form: d (p) = $\alpha\ e^{-\beta p}$, we proceed to obtain analytic results in the following section.

3. ANALYTIC RESULTS FOR EXPONENTIAL DEMAND

Let the exponential demand pattern be set as:

d (p)= $\alpha\ e^{-\ \beta\ p}$, $\beta > 0$, α is constant,

using (9) we have,

$C\,(T, p\,) = C\,\alpha\ e^{-\ \beta\ p} + (C\lambda + h\,)\ \alpha\ e^{-\ \beta\ p}\ T/2 + K/T$ .... (17)

Minimizing (17) with respect to T, we obtain optimal cycle length T*:

$\left.\dfrac{\partial\, C\ (T,p)}{\partial\, T}\right|_{T = T^*} = (C\lambda + h\,)\ \alpha\ e^{-\ \beta\ p} / 2 - K/T^2\ _{T=T^*} = 0\ \Big|$............... (18)

So that, $T^* = [2K/ (C\lambda + h\,)\ \alpha\ e^{-\ \beta\ p}\,]^{1/2}$ .... (19)

Further it is clear from (18) that:

$\left.\dfrac{\partial^{\,2} C\ (T,p)}{\partial\, T^2}\right|_{T=T^*} > 0$

The optimal price, p*, is obtained by maximizing $\pi\ (T,p)$ which is given by:

$\pi\ (T,p) = p\,\alpha\ e^{-\ \beta\ p} - C\,\alpha\ e^{-\ \beta\ p} - (C\lambda + h\,)\ \alpha\ e^{-\ \beta\ p}\ T/2 - K/T$ .... (20)

Differentiating $\pi\ (T,p)$ with respect to p , we have

$\dfrac{\partial\, \pi\ (T,p)}{\partial\, p} = \alpha\ [\,p\, e^{-\ \beta\ p}(-\beta) + \ e^{-\ \beta p}\,] - C\,\alpha\ e^{-\ \beta\ p}\ (-\beta) - (C\lambda + h\,)\ \alpha\ e^{-\ \beta p}\ T\,(-\beta)/2$

$= \alpha\ \beta e^{-\ \beta\ p}\ [\, - p + 1/\ \beta + C + (C\lambda + h\,)\ T/2]$

Now p* is obtained thus:

$\left.\dfrac{\partial\, \pi\ (T,p)}{\partial\, p}\right|_{p = p^*} = \alpha\ \beta e^{-\ \beta\ p}\ [-p + 1/\ \beta + C + (C\lambda + h\,)\ T/2] = 0$ .... (21)

So that, $p^* = 1/\ \beta + C + (C\lambda + h\,)\ T/2$ .... (22)

Also,

$$\frac{\partial^2 \pi(T,p)}{\partial p^2} = \alpha \beta e^{-\beta p}[p\beta - 2 - C\beta - (C\lambda + h)\beta T/2] \text{ .... (23)}$$

Using (22) in (23) we notice that

$$\left.\frac{\partial^2 \pi(T,p)}{\partial p^2}\right|_{p=p^*} < 0$$

Hence, using (12), optimal order quantity Q_0 is obtained as:

$$Q_0 = \alpha e^{-\beta p}(1+\lambda T^*/2) \text{ .... (24)}$$

Optimal profit π^* is given by:

Using (13) and (17)

$$\pi^* = p^* \alpha e^{-\beta p^*} - C\alpha e^{-\beta p^*} - [2K(C\lambda + h)\alpha e^{-\beta p^*}]^{1/2} \text{ ..... (25)}$$

In the following section we present empirical work basing on the theoretical results reported in the above.

4. EMPIRICAL WORK

In keeping with the remarks on modeling as discussed in the beginning, we choose the typical values for α, β, h, C, K and λ as given in the following table.

The empirical results are tabulated in the following:

TABLE 1: P*, T*, π^* and other optimal values

α	β	h	C	K	λ	P*	T*	Q_0	π^*	Q_0/T*	π^*/Q_0
50	0.50	0.05	1	10	0.005	3.1635	5.9475	10.4339	18.8794	1.7543	1.8094
55	0.55	0.06	1	12	0.006	3.0125	5.8874	10.6760	17.0355	1.8134	1.5957
60	0.60	0.07	1	14	0.007	2.8923	5.8628	10.7968	15.2443	1.8416	1.4119
65	0.65	0.08	1	16	0.008	2.7968	5.8699	10.8016	13.5112	1.8402	1.2508
70	0.70	0.09	1	18	0.009	2.7201	5.9074	10.7047	11.8399	1.8121	1.1060
75	0.75	0.10	1	20	0.010	2.6619	5.9748	10.4908	10.2341	1.7558	0.9755
80	0.80	0.12	1	22	0.012	2.6368	5.8609	10.0460	8.3768	1.7141	0.8338
85	0.85	0.14	1	24	0.014	2.6268	5.8479	9.4873	6.6192	1.6223	0.6977
90	0.90	0.16	1	26	0.016	2.6324	5.9237	8.8194	4.9668	1.4888	0.5632
95	0.95	0.18	1	28	0.018	2.6557	6.0918	8.0390	3.4260	1.3196	0.4262
100	1.00	0.20	1	30	0.020	2.7012	6.3742	7.1401	2.0063	1.1201	0.2809

5. DISCUSSION

A general but basic motivation to carry out empirical work based on analytic results is to bring out qualitative (verbal) analysis from the otherwise dumb expression. With this intention, the theoretical work presented in section 3 is used to obtain some empirical results, for certain suitable/typical choices for values of the concerned parameters.

From the tabulated values in Table-1 presenting the optimal values of P*, T*, Q_0, π^* and other optimal per unit values (that is optimal quantity per optimal time unit and optimal profit per optimal quantity unit). We may draw, for instance the following type of conclusions, constituting a qualitative analysis:

(i.) When the set up costs are low (say, K=10 or 12) as well as for small values for exponential demand parameters (α, β each one say 0.50/0.55) the entrepreneur can well set a profitable price as high as say about 3 units as (optimal p*) to obtain optimal high profits (π *) of more than17 units.

(ii.) However, for minor changes in the parametric values and jointly considering the optimal values of p* and T* (taking initial value at p=3 units) the optimal price p* and optimal cycle length T* shows some minor fluctuations, but T* appears to be unchanged (at T*=6 units), while the optimal ordering quantity Q_0 shows some noticeable fluctuations (from 7 units to 11 units).

(iii.) Now the per unit optimal profit per unit of optimal quantity attains the maximum (that is, π^*/Q_0 is highest) where π^* is nearly 19 units and it declines with the increases in the values of the other parameters.

Much more qualitative analysis of the type reported in the above can be extracted with more/elaborated empirical work. A qualitative analysis, such as the above, thus profitably leads the entrepreneurs to take optimal decisions as well as to accomplish targeted profits by cleverly/judiciously manipulating or setting the values of the rest of the concerned parameters. Our basic aim thus comprises in focusing the value of empirical work (based, however, on some theoretical results) in arriving at profitable/optimal/useful entrepreneur's decisions with the help of suitable qualitative analysis.

ACKNOWLEDGEMENT

This work was financially supported by the University Grants Commission under Minor Research Project Scheme **No.F. MRP- 4849/14(SERO/UGC)** for distinguished teachers working in Degree Colleges. In this regard one of the authors (CSR) would like to sincerely thank the Joint Secretary (UGC- SERO, Hyderabad, A.P., India.) for his kind co-operation and support for this present work.

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