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RESEARCH ARTICLE

A COMMON FIXED POINT THEOREM FOR TWO SELF MAPS IN CONE RECTANGULAR METRIC SPACE

MALLIKARJUN REDDY.P[1]*, RANGAMMA.M[2]
[1,2]Department of Mathematics, Osmania University
Hyderabad, Telangana, India.

MALLIKARJUN REDDY.P

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ABSTRACT

The purpose of this paper is to establish a common fixed point theorem for two self mappings in cone rectangular metric space. Our result extends Kannan's fixed point theorem in cone rectangular metric space.

Key words: cone rectangular metric space, common fixed point theorem, coincidence point.

INTRODUCTION

In 1906, M. Frenchet [4] introduced the concept of metric spaces. Recently, Huang and Zhang [5] introduce the notion of cone metric spaces. They have replaced real number system by an ordered Banach space and established some fixed point theorems for contractive type mappings in a normal cone metric space. The study of fixed point theorems in such spaces is followed by some other mathematicians; see [1], [6], [12], [15], [16], [18].

Following idea of Azam, Arshad and Beg [3] extended the notion of cone metric spaces by replacing the triangular inequality by a rectangular inequality and they proved Banach contraction Principle in a complete normal cone rectangular metric space. Several authors proved some fixed point theorems in such spaces see; [2], [9], [10], [11], [13], [14]. In 2009, Jleli, Samet [7] extended the Kannan's fixed point theorem in a complete normal cone rectangular metric space. In this paper we prove a common fixed point theorem for two self mappings in cone rectangular metric space.

2. PRELIMINARIES

First, we recall some standard definitions and other results that will be needed in the sequel.

2.1. Definition [5]: A subset P of a real Banach space E is called a *'cone'* if it has following properties:

(1) P is nonempty, closed and $P \neq \{\theta\}$;

(2) $a, b \in R,\ a,\ b \geq 0,\ x,\ y \in P$ implies $ax + by \in P$;

(3) $\mathrm{P} \cap (-\mathrm{P}) = \{\theta\}$.

For a given cone $\mathrm{P} \subseteq \mathrm{E}$, we can define a partial ordering $\leq$ on E with respect to P by $\mathrm{x} \leq \mathrm{y}$ if and only if $\mathrm{y} - \mathrm{x} \in \mathrm{P}.$ We shall write $\mathrm{x} < \mathrm{y}$ if $\mathrm{x} \leq \mathrm{y}$ and $\mathrm{x} \neq \mathrm{y},$ while x << y will stands for $\mathrm{y} - \mathrm{x} \in \operatorname{int}(\mathrm{P}),$ where $\operatorname{int}(\mathrm{P})$ denotes the interior of P.

2.2. Definition [5]: The cone P is called *'normal'* if there is a number $\mathrm{k} > 1$ such that for all x, y $\mathrm{x}, \mathrm{y} \in \mathrm{E},\ 0 \leq x \leq y$ implies $\|x\| \leq k\|y\|.$

The least positive number *k* satisfying the above condition is called the *normal constant* of P.

2.3. Definition [5]: Let X is a nonempty set, E is a real Banach space and P is a cone in E with $\operatorname{int} \mathrm{P} \neq \Phi$ and $\leq$ is a partial ordering with respect to P. Suppose that the mapping $d : \mathrm{X} \times \mathrm{X} \rightarrow \mathrm{E}$ satisfies:

(1) $\theta \leq \mathrm{d}(\mathrm{x},\ \mathrm{y}),$ for all $\mathrm{x}, \mathrm{y} \in \mathrm{X}$ and $\mathrm{d}(\mathrm{x},\ \mathrm{y}) = \theta$ if and only if $\mathrm{x} = \mathrm{y};$

(2) $\mathrm{d}(\mathrm{x}, \mathrm{y}) = \mathrm{d}(\mathrm{y},\ \mathrm{x}),$ for all $\mathrm{x}, \mathrm{y} \in \mathrm{X};$

(3) $\mathrm{d}(\mathrm{x}, \mathrm{y}) \leq \mathrm{d}(\mathrm{x}, \mathrm{z}) + \mathrm{d}(\mathrm{z},\ \mathrm{y}),$ for all $\mathrm{x}, \mathrm{y}, \mathrm{z} \in \mathrm{X}.$

Then *d* is called a *'cone metric'* on X and (X, d) is called a *'cone metric space'*.

2.4. Definition [3]: Let X is a nonempty set, E is a real Banach space and P is a cone in E with $\operatorname{int} \mathrm{P} \neq \Phi$ and $\leq$ is a partial ordering with respect to P. Suppose that the mapping $d : \mathrm{X} \times \mathrm{X} \rightarrow \mathrm{E}$ satisfies:

(1) $\theta \leq \mathrm{d}(\mathrm{x},\ \mathrm{y}),$ for all $\mathrm{x},\ \mathrm{y} \in \mathrm{X}$ and $\mathrm{d}(\mathrm{x},\ \mathrm{y}) = \theta$ if and only if x = y;

(2) $\mathrm{d}(\mathrm{x}, \mathrm{y}) = \mathrm{d}(\mathrm{y},\ \mathrm{x}),$ for all $\mathrm{x}, \mathrm{y} \in \mathrm{X};$

(3) $\mathrm{d}(\mathrm{x},\ \mathrm{y}) \leq \mathrm{d}(\mathrm{x}, \mathrm{w}) + \mathrm{d}(\mathrm{w},\ \mathrm{z}) + \mathrm{d}(\mathrm{z},\ \mathrm{y}),$ for all $\mathrm{x},\ \mathrm{y} \in \mathrm{X}$ and for all distinct points $\mathrm{w},\ \mathrm{z} \in \mathrm{X} - \{\mathrm{x},\ \mathrm{y}\}$ [rectangular property].

Then *d* is called a *'cone rectangular metric'* on X and (X, d) is called a *'cone rectangular metric space'*.

2.5. Definition [3]: Let (X, d) be a cone rectangular metric space. Let $\{\mathrm{x}_n\}$ be a sequence in (X, d) and $\mathrm{x} \in \mathrm{X}.$ If for every $\mathrm{c} \in \mathrm{E},$ with $\theta << \mathrm{c}$ there is $\mathrm{n}_0 \in \mathrm{N}$ such that for all $\mathrm{n} > \mathrm{n}_0,\ \mathrm{d}(\mathrm{x}_n,\ \mathrm{x}) << \mathrm{c},$ then $\{\mathrm{x}_n\}$ is said to be convergent, $\{\mathrm{x}_n\}$ converges to x and x is the limit of $\{\mathrm{x}_n\}$. We denote this by $\lim_{n \to \infty} \mathrm{x}_n = \mathrm{x}$ or $\mathrm{x}_n \rightarrow \mathrm{x},$ as $\mathrm{n} \rightarrow \infty.$

2.6. Definition [3]: Let (X, d) be a cone rectangular metric space. Let $\{\mathrm{x}_n\}$ be a sequence in (X, d). If for every $\mathrm{c} \in \mathrm{E},$ with $\theta << \mathrm{c}$ there is $\mathrm{n}_0 \in \mathrm{N}$ such that for all $\mathrm{m}, \mathrm{n} > \mathrm{n}_0,\ \mathrm{d}(\mathrm{x}_\mathrm{n},\ \mathrm{x}_\mathrm{m}) << \mathrm{c},$ then $\{\mathrm{x}_n\}$ is called a Cauchy sequence in (X, d).

2.7. Definition [3]: Let (X, d) be a cone rectangular metric space. If every Cauchy sequence is convergent in (X, d), then (X, d) is called a complete cone rectangular metric space.

2.8. Lemma [3]: Let (X, d) be a cone rectangular metric space and P be a normal cone with normal constant *k*, let $\{\mathrm{x}_n\}$ be a sequence in X. Then $\{\mathrm{x}_n\}$ converges to x if and only if $\|d(\mathrm{x}_\mathrm{n}, \mathrm{x})\| \rightarrow 0,$ as $n \rightarrow \infty$.

2.9. Lemma [3]: Let (X, d) be a cone rectangular metric space, P be a normal cone with normal constant *k*. Let $\{\mathrm{x}_n\}$ be a sequence in X. Then $\{\mathrm{x}_n\}$ is a Cauchy sequence if and only if $\|\mathrm{d}(\mathrm{x}_\mathrm{n},\ \mathrm{x}_\mathrm{m})\| \rightarrow 0,$ as $\mathrm{n}, \mathrm{m} \rightarrow \infty.$

2.10. Definition [8]: Let T and S are self maps of a nonempty set X. If $w = Tx = Sx,$ for some $x \in X$, then *x* is called a *coincidence point* of T and S and *w* is called a *'point of coincidence'* of T and S.

2.11. Definition [8]: Two self mappings T and S are said to be *'weakly compatible'* if they commute at their coincidence points, that is, $Tx = Sx$ implies that $TSx = STx.$

3. MAIN RESULT

In this section we establish a common fixed point theorem for two self mappings in cone rectangular metric space. The following theorem extends and improves Theorem 2.1 in [7].

3.1. Theorem: Let (X, d) be a cone rectangular metric space and P be a normal cone with normal constant *k*. Let S, T: X →X be two self mappings of X satisfying the following condition:

$$\mathrm{d}(\mathrm{Sx},\mathrm{Sy}) \le \lambda\left[\mathrm{d}(\mathrm{Sx},\ \mathrm{Tx}) + \mathrm{d}(\mathrm{Sy},\mathrm{Ty})\right], \tag{1}$$

for all x, y ∈X, where $\lambda \in [0,1/2).$ If $\mathrm{S}(\mathrm{X}) \subseteq \mathrm{T}(\mathrm{X})$ and S(X) or T(X) is a complete subspace of X, then the mappings S and T have a unique coincidence point in X. Moreover, if S and T are weakly compatible pairs then S and T have a unique common fixed point in X.

Proof: Let x_0 be any arbitrary point of X. Since, $\mathrm{S}(\mathrm{X}) \subseteq \mathrm{T}(\mathrm{X}),$ we can choose a point x_1 in X such that $\mathrm{Sx}_0 = \mathrm{Tx}_1.$ Continuing in this way, for x_n in X we find x_{n+1} in X such that $\mathrm{Sx}_n = \mathrm{Tx}_{n+1},\ n = 0, 1, 2, \ldots$

Now, we define a sequence $\{y_n\}$ in X such that, $y_n = Sx_n = Tx_{n+1},$ for n = 0, 1, 2, ...

If $y_m = y_{m+1},$ for some $m \in N,$ then $y_m = Tx_{m+1} = Sx_{m+1}.$

That is, S and T have a coincidence point x_{m+1} in X.

Assume $y_n \ne y_{n+1},$ for all $n \in N.$

Then from (1) it follows that,

$$\begin{aligned} d(y_n, y_{n+1}) &= d(Sx_n, Sx_{n+1}) \\ &\le \lambda\left[\mathrm{d}(\mathrm{Sx}_n,\ \mathrm{Tx}_n) + \mathrm{d}(\mathrm{Sx}_{n+1},\mathrm{Tx}_{n+1})\right] \\ &= \lambda\left[\mathrm{d}(y_n,\ \mathrm{y}_{n-1}) + \mathrm{d}(y_{n+1},\ y_n)\right] \end{aligned}$$

which implies that,

$$d(y_n, y_{n+1}) \le \frac{\lambda}{1-\lambda} d(y_{n-1}, y_n), \text{ for all } \mathrm{n} = 0, 1, 2, \ldots$$

Thus,
$$\begin{aligned} d(y_n, y_{n+1}) &\le \mu d(y_{n-1}, y_n) \\ &\le \mu^2 d(y_{n-2}, y_{n-1}) \\ &\ \ \vdots \\ &\le \mu^n d(y_0, y_1), \end{aligned} \tag{2}$$

for all n ≥ 0, where $\mu = \dfrac{\lambda}{1-\lambda} < 1.$

From (1), (2) and using the facts $\lambda \le \mu$ and $0 \le \lambda < ½ < 1$, we get,

$$\begin{aligned} d(y_n, y_{n+2}) &= d(Sx_n, Sx_{n+2}) \\ &\le \lambda\left[\mathrm{d}(\mathrm{Sx}_n,\ \mathrm{Tx}_n) + \mathrm{d}(\mathrm{Sx}_{n+2},\mathrm{Tx}_{n+2})\right] \\ &= \lambda\left[\mathrm{d}(y_n,\ \mathrm{y}_{n-1}) + \mathrm{d}(y_{n+2},\ y_{n+1})\right] \\ &\le \lambda\left[\mu^{n-1}\mathrm{d}(y_0,\ \mathrm{y}_1) + \mu^{n+1}\mathrm{d}(y_0,\ y_1)\right] \\ &\le \mu^n\mathrm{d}(y_0,\ \mathrm{y}_1) + \mu^{n+1}\mathrm{d}(y_0,\ y_1) \\ &= (1+\mu)\mu^n d(y_0, y_1), \text{ for all } n \ge 0 \end{aligned}$$

(3) For the sequence $\{y_n\}$ we consider $\mathrm{d}(\mathrm{y}_n, \mathrm{y}_{n+p})$ in two cases.

If p is odd say *2m+1*, for *m≥1*, then by using rectangular inequality and (2) we get,

$$\begin{aligned} d(y_n, y_{n+2m+1}) &\leq d(y_{n+2m+1}, y_{n+2m}) + d(y_{n+2m}, y_{n+2m-1}) + d(y_{n+2m-1}, y_n) \\ &\leq d(y_{n+2m}, y_{n+2m+1}) + d(y_{n+2m-1}, y_{n+2m}) + d(y_{n+2m-1}, y_{n+2m-2}) \\ &\quad + d(y_{n+2m-2}, y_{n+2m-3}) + \dots + d(y_{n+2}, y_{n+1}) + d(y_{n+1}, y_n) \\ &= d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) + \dots + d(y_{n+2m-1}, y_{n+2m}) + d(y_{n+2m}, y_{n+2m+1}) \\ &\leq \mu^n d(y_0, y_1) + \mu^{n+1} d(y_0, y_1) + \mu^{n+2} d(y_0, y_1) + \dots \\ &\quad + \mu^{n+2m-1} d(y_0, y_1) + \mu^{n+2m} d(y_0, y_1) \\ &\leq \left[1 + \mu + \mu^2 + \mu^3 + \dots\right] \mu^n d(y_0, y_1) \end{aligned}$$

$$\text{Hence, } d(y_n, y_{n+2m+1}) \leq \frac{\mu^n}{1-\mu} d(y_0, y_1), \text{ for all } n \geq 1, m \geq 1. \tag{4}$$

If p is even say *2m*, for *m ≥1*, then by using rectangular inequality, (2), (3) and the fact that μ <1 we get,

$$\begin{aligned} d(y_n, y_{n+2m}) &\leq d(y_{n+2m}, y_{n+2m-1}) + d(y_{n+2m-1}, y_{n+2m-2}) + d(y_{n+2m-2}, y_n) \\ &\leq d(y_{n+2m-1}, y_{n+2m}) + d(y_{n+2m-2}, y_{n+2m-1}) + \dots + \\ &\quad d(y_{n+4}, y_{n+3}) + d(y_{n+3}, y_{n+2}) + d(y_{n+2}, y_n) \\ &= d(y_n, y_{n+2}) + d(y_{n+2}, y_{n+3}) + \dots + d(y_{n+2m-2}, y_{n+2m-1}) + d(y_{n+2m-1}, y_{n+2m}) \\ &\leq (1+\mu)\mu^n d(y_0, y_1) + \Big[\mu^{n+2} d(y_0, y_1) + \mu^{n+3} d(y_0, y_1) \\ &\qquad + \dots + \mu^{n+2m-2} d(y_0, y_1) + \mu^{n+2m-1} d(y_0, y_1)\Big] \\ &\leq \left[1 + \mu + \mu^2 + \mu^3 + \dots\right] \mu^n d(y_0, y_1) \end{aligned}$$

$$\text{Hence, } d(y_n, y_{n+2m}) \leq \frac{\mu^n}{1-\mu} d(y_0, y_1), \tag{5}$$

for all $n \geq 1, m \geq 1.$

Thus combining all the cases we have,

$$d(y_n, y_{n+p}) \leq \frac{\mu^n}{1-\mu} d(y_0, y_1), \tag{6}$$

for all $n \in N, p \in N.$

Since, P is a normal cone with normal constant k and $\mu < 1$, we have

$$\left\| d(y_n, y_{n+p}) \right\| \leq \frac{k\mu^n}{1-\mu} \left\| d(y_0, y_1) \right\| \to \theta, \text{ as n} \to \infty,$$

$$i.e., \left\| d(y_n, y_{n+p}) \right\| \to \theta, \text{ as n} \to \infty, \quad \forall\, p \in N. \tag{7}$$

Hence, $\{y_n\}$ is a Cauchy sequence in X.

Case I: Suppose S(X) is a complete subspace of X, there exists $z \in S(X) \subseteq T(X)$ such that

$$\lim_{n\to\infty} y_n = \lim_{n\to\infty} Sx_n = z \tag{8}$$

Also, we can find $x \in X$ such that $z = Tx$ (9)

We shall show that $z = Sx$

Using rectangular inequality, (1) and (9) we get,

$$d(z, Sx) \leq d(z, y_n) + d(y_n, y_{n+1}) + d(y_{n+1}, Sx)$$

$$= d(z, y_n) + d(y_n, y_{n+1}) + d(Sx_{n+1}, Sx)$$
$$\leq d(z, y_n) + d(y_n, y_{n+1}) + \lambda\left[d(Sx_{n+1}, Tx_{n+1}) + d(Sx, Tx)\right]$$
$$\leq d(z, y_n) + d(y_n, y_{n+1}) + \lambda d(y_{n+1}, y_n) + \lambda d(Sx, Tx)$$

which implies that,

$$d(z, Sx) \leq \frac{1}{1-\lambda}\left[d(z, y_n) + d(y_n, y_{n+1}) + \lambda d(y_n, y_{n+1})\right], \text{ for all } n \geq 1.$$

Since, P is a normal cone with normal constant *k*, using (7) and (8) we get,

$$\|d(z, Sx)\| \leq \frac{k}{1-\lambda}\left[\|d(y_n, z)\| + \|d(y_n, y_{n+1})\| + \lambda\|d(y_n, y_{n+1})\|\right] \to \theta, \text{as n} \to \infty,$$

$$i.e., \ \|d(z, Sx)\| = \theta$$

$$i.e., \ z = Sx.$$

Therefore, $Sx = Tx = z$

Hence, S and T have a coincidence point *x* in X.

Case II: Suppose T(X) is a complete subspace of X, there exists $z' \in T(X)$ such that

$$\lim_{n\to\infty} y_n = \lim_{n\to\infty} Tx_{n+1} = z' \quad (10)$$

Also, we can find $x' \in X$ such that $z' = Tx'$ (11)

We shall show that $Sx' = z'$

Using rectangular inequality, (1) and (11) we get,

$$d(Sx', z') \leq d(Sx', y_{n-1}) + d(y_{n-1}, y_n) + d(y_n, z')$$
$$= d(Sx', Sx_{n-1}) + d(y_{n-1}, y_n) + d(y_n, z')$$
$$\leq \lambda\left[d(Sx', Tx') + d(Sx_{n-1}, Tx_{n-1})\right] + d(y_{n-1}, y_n) + d(y_n, z')$$
$$\leq \lambda d(Sx', z') + \lambda d(y_{n-1}, y_n) + d(y_{n-1}, y_n) + d(y_n, z')$$

which implies that,

$$d(Sx', z') \leq \frac{1}{1-\lambda}\left[\lambda d(y_{n-1}, y_n) + d(y_{n-1}, y_n) + d(y_n, z')\right], \text{ for all } n \geq 1.$$

Since, P is a normal cone with normal constant *k*, using (7) and (10) we get,

$$\|d(Sx', z')\| \leq \frac{k}{1-\lambda}\left[\lambda\|d(y_{n-1}, y_n)\| + \|d(y_{n-1}, y_n)\| + \|d(y_n, z')\|\right] \to \theta, \text{as n} \to \infty,$$

$$i.e., \ \|d(Sx', z')\| = \theta$$

$$i.e., \ Sx' = z'.$$

Therefore, $Sx' = Tx' = z'$

Hence, S and T have a coincidence point x' in X.

Thus in all cases we have S and T have a coincidence point in X.

We shall prove point of coincidence of S and T is unique.

If *v* is a point of coincidence of S and T, then $Sy = Ty = v$ (12)

Suppose v^* is another point of coincidence of S and T.

Then we have, $v^* = Sy^* = Ty^*$, for some $y^* \in X$ (13)

Using (1), (12) and (13) we get

$$d(v, v^*) = d(Sy, Sy^*)$$

$$\leq \lambda\left[d(Sy,Ty)+d(Sy^*,Ty^*)\right]=\theta$$

which implies that, $d(v,v^*)=\theta.$

i.e., $v=v^*.$

Thus, S and T have a unique point of coincidence in X.

Suppose S and T are weakly compatible, then from (12) we get,

$$Sv=STy=TSy=Tv$$

Therefore, $Sv=Tv=w$ (say)

This shows that, *w* is another point of coincidence of S and T.

Therefore, by the uniqueness of point of coincidence, we must have *w = v.*

Hence, there exist unique point $v\in X$ such that $Sv=Tv=v.$

Thus *v* is a unique common fixed point of self mappings S and T.

To illustrate Theorem 3.1 we give the following example.

3.2. Example: Let X = {a, b, c, d}, where a, b, c, d ∈ ℝ, E = $\mathbb{R}^2$ and P = {(x, y): x, y ≥ 0} is a normal cone in E. Define $d: X\times X\to E$ as follows:

$$\begin{cases} d(x,\ x)=\theta, \quad for\ all\ x\in X, \\ d(a,\ b)=d(b,\ a)=(3,\ 9), \\ d(a,\ c)=d(c,\ a)=d(b,\ c)=d(c,\ b)=(1,\ 3), \\ d(a,\ d)=d(d,\ a)=d(b,\ d)=d(d,\ b)=d(c,\ d)=d(d,\ \mathrm{c})=(4,\ 12). \end{cases}$$

Then (X, d) is a complete cone rectangular metric space but (X, d) is not a cone metric space because it lacks the triangular property:

(3, 9) = d(a, b) > d(a, c) + d(c, b) = (1, 3) + (1, 3) = (2, 6),

as (3, 9) - (2, 6) = (1, 3) ∈ P.

Now we define mappings S and T: X → X as follows:

$$S(x)=\begin{cases} \mathrm{c} & if\ x\neq d \\ \mathrm{a} & if\ x=d \end{cases}$$

and $$T(x)=\begin{cases} b & if\ x=a \\ a & if\ x=b \\ c & if\ x=c \\ d & if\ x=d \end{cases}$$

It is clear that, $S(X)\subseteq T(X)$, S and T are weakly compatible mappings. However, S and T satisfy contractive condition (1) of Theorem 3.1 with λ = 1/4. Hence, all the conditions of Theorem 3.1 are satisfied and *c* is a unique common fixed point of S and T.

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