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ON THE HYPER-WIENER INDEX OF THORNY-WHEEL GRAPHS

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ABSTRACT

Let G be the connected graph. The Wiener index W (G) is the sum of all distance between vertices of G, where as the hyper-Wiener index *V* is defined as $WW(G) = W(G) + \frac{1}{2}\sum_{\{u,v\}\subseteq V\{G\}} G$. In this paper we prove some general results on the hyper-Wiener index of thorny-wheel graphs.

Keywords: thorny-wheel graphs, Wiener index and hyper-Wiener index.
2000 Mathematics subject classification: 05C12.

INTRODUCTION

In mathematical terms a graph is represented as G = (V,E) where V is the set of vertices and E is the set of edges. Let G be an undirected connected graph without loops or multiple edges with n vertices, denoted by 1,2....n d (u,v) and it is the topological distance between the vertices u and v of V (G) is denoted by d(u; v) and it is defined as the number of edges in a minimal path connecting the vertices u and v.

The Wiener index W (G) of a connected graph G is defined as the sum of the distances between all unordered pair of vertices of G. It was put forward by Harold Wiener. The Wiener index is a graph invariant intensively studied both in mathematics and chemical literature, see for details [1, 2, 7, 9 and 10].

The hyper-wiener index was proposed by Randic [13] for a tree and extended by Klein et al. [3] to a connected graph. It is used to predict physicochemical properties of organic compounds. The Hyper-Wiener index defined as

$$WW(G)=\sum_{\{u,v\}\subseteq V(G)}\binom{d_{uv}+1}{2}=W(G)+\frac{1}{2}\sum_{\{u,v\}\subseteq V(G)}d^2(u,v)$$

The Hyper-Wiener index is studied both from a theoretical point of view and applications. We encourage the reader to consult [9-15] for further readings. The Hyper-Wiener index of Complete graph K_n, Path graph P_n, star-$K_{1;n\ 1}$ and cycle graph C_n is given by the expressions

$$WW(K_n)=\frac{n(n-1)}{2},WW(P_n)=\frac{n^4+2n^3-n^2-2n}{24},\quad WW(K_{1,(n-1)})=\frac{1}{2}(n-1)(3n-4)$$

And

$$WW(C_n)=\begin{cases}\frac{n^2(n+1)(n+2)}{48}, & if\ n\ is\ even\\ \frac{n(n^2-1)(n+3)}{48}, & if\ n\ is\ even\end{cases}$$

Let G be a connected n-vertex graph with vertex set V (G) = {v_1; v_2; ; v_n} and P = (p_1, $p_{2,...,}$ p_n) be an n-tuple of non-negative integers. The thorn graph G_P is the graph obtained by attached to the vertex V_i will be called the thorns of V_i. The concept of thorny graphs was introduced by Ivan Gutman.

RESULTS

Theorem 1: Let H be the wheel graph on k vertices. The graph G obtained by attaching s-number of pendent vertices to each vertex of H with common vertex then its hyper-Wiener index given by

$$WW(G)=\frac{1}{2}[k(10s^2k-25s^2+12sk-33s+3k-9)+25s^2-10s^2k-10sk+31s-2k+8]$$

Proof: To find hyper-Wiener index of the graph, we need to find following two parts

To find Wiener index: $W(G)=\frac{1}{2}\sum_{\{u,v\}\subseteq V(G)}$

$$W(G)=\frac{1}{2}\{\underbrace{1+2+2+\cdots+2}_{S+2\ times}+\underbrace{3+3+\cdots+3}_{k+2s-4\ times}+\underbrace{4+4+}_{s(k-4}$$

$$+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$+\underbrace{1+2+2+\cdots+}_{S+2\ times}\underbrace{2+3+3+}_{k+2s-4\ times}\cdots+\underbrace{3+4+4+}_{s(k-4}$$

$$+\underbrace{1+1+\cdots+1}_{3+s\ times}+\underbrace{2+2+\cdots+2}_{k+2s-4\ times}+\underbrace{3+3+}_{s(k-4}$$

$$+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\underbrace{+1+1+\cdots+}_{3+s\ times}\underbrace{1+2+2+\cdots+}_{k+2s-4\ times}\underbrace{2+3+3+}_{s(k-4}$$

$$\underbrace{+1+1+\cdots+}_{k-1\ times}\underbrace{1+2+2+}_{s(k-1}\}$$

$$W(G)=\frac{1}{2}\{[1+2(s+2)+3(k+2s-4)+4s(k-4)]+\cdots$$

$$+[1+2(s+2)+3(k+2s-4)+4s(k-4)]$$
$$+[3+s+2(k+2s-4)+3s(k-4)]+\cdots$$
$$+[3+s+2(k+2s-4)+3s(k-4)]$$
$$+[k-1+2s(k-1)]\}$$
$$W(G)=\frac{1}{2}\{\underbrace{[4sk-8s+3k-7]+\cdots+[4sk-8s+3}_{s(k-1}$$
$$\underbrace{+[3sk-7s+2k-5]+\cdots+[3sk-7s+2}_{(k-1)times}$$
$$+[2sk-2s+k-1]\}$$
$$W(G)=\frac{1}{2}\{s(k-1)[4sk-8s+3k-7]$$
$$+(k-1)[3sk-7s+2k-5]$$
$$+[2sk-2s+k-1]\}$$

$$W(G)=\frac{1}{2}[k(4s^2k-8s^2+6sk-14s+2k-5)+8s^2-4s^2k-4sk+12s-k+4]--(i)$$

To find $WW^*(G)$:

$$WW^*(G)=\frac{1}{2}\sum_{\{u,v\}\subseteq V(G)} d^2(u,v)$$
$$WW^*(G)=\frac{1}{2}\{\underbrace{1+1+\cdots+1}_{s+2\ \text{times}}+\underbrace{3+3+\cdots+3}_{k+2s-4\ \text{times}}+\underbrace{6+6+\cdots+6}_{s(k-4)\ \text{times}}$$
$$+\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$+\underbrace{1+1+\cdots+1}_{s+2\ \text{times}}+\underbrace{3+3+\cdots+3}_{k+2s-4\ \text{times}}+\underbrace{6+6+\cdots+6}_{s(k-4)\ \text{times}}$$
$$+\underbrace{1+1+\cdots+1}_{k+2s-4\ \text{times}}+\underbrace{3+3+\cdots+3}_{s(k-4)\ \text{times}}$$
$$+\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$+\underbrace{1+1+\cdots+1}_{k+2s-4\ \text{times}}+\underbrace{3+3+\cdots+3}_{s(k-4)\ \text{times}}$$
$$+\underbrace{1+1+\cdots+1}_{s(k-1)\ \text{times}}\}$$
$$WW^*(G)=\{[s+2+3(k+2s-4)+6s(k-4)]+\cdots$$
$$+[s+2+3(k+2s-4)+6s(k-4)]$$
$$+[k+2s-4+3s(k-4)]+\cdots$$
$$+[k+2s-4+3s(k-4)]+s(k-1)\}$$
$$WW^*(G)=\frac{1}{2}\{\underbrace{[6sk-17s+3k-10]+\cdots+[6sk-17s+3k-10]}_{s(k-1)\ \text{times}}$$
$$+\underbrace{[3sk-10s+k-4]+\cdots+[3sk-10s+k-4]}_{(k-1)\ \text{times}}$$
$$+s(k-1)\}$$

$$WW^*(G) = \frac{1}{2}\{s(k-1)[6sk - 17s + 3k - 10] + (k-1)[3sk - 10s + k - 4] + s(k-1)\}$$

$$WW^*(G) = \frac{1}{2}\{k(6s^2k - 17s^2 + 6sk - 19s + k - 4) + 17s^2 - 6s^2k - 6sk + 19s - k + 4\} - -(ii)$$

Combining (a) and (b) we get

$$WW(G) = \frac{1}{2}\{k(10s^2k - 25s^2 + 12sk - 33s + 3k - 9) + 25s^2 - 10s^2k - 10sk + 31s - 2k + 8\}$$

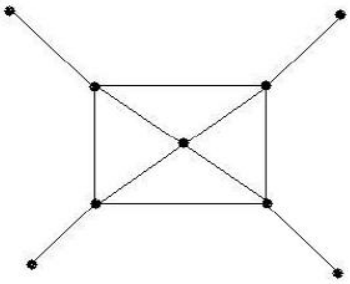

Figure 1: $k = 5$, $s = 1$, $W(G) = 72$ and $WW(G) = 122$.

Theorem 2: Let H be the wheel graph on k vertices (k is even). The graph G obtained by attaching s-number of pendent vertices to alternative vertex of graph H with common vertex then its hyper-Wiener index given by

$$WW(G) = \frac{1}{2}(6pk - 7sp + 10p^2 - 17p + k + 3p - 1) + \frac{k-1}{4}(6k - 14s + 6p + 3sk - 18)$$

Proof: To find hyper-Wiener index of the graph, we need to find following two parts:

To find Wiener index: $W(G) = \frac{1}{2}\sum_{\{u,v\}\subseteq V(G)}$

$$W(G) = \frac{1}{2}\{\underbrace{1 + 2 + 2 + \cdots}_{S+2\ times} + \underbrace{2 + 3 + 3 + \cdots}_{k-4\ times} + \underbrace{3 + 4 + 4 +}_{p-s\ times}$$

$$+ \cdots \ldots \ldots \ldots$$

$$\underbrace{+1 + 2 + 2 + \cdots}_{S+2\ times} + \underbrace{2 + 3 + 3 + \cdots}_{k-4\ times} + \underbrace{3 + 4 + 4 +}_{p-s}$$

$$\underbrace{+1 + 1 + \cdots}_{s+3\ times} + \underbrace{1 + 2 + 2 + \cdots}_{k-4\ times} + \underbrace{2 + 3 + 3 +}_{p-s}$$

$$+ \cdots \ldots \ldots \ldots$$

$$\underbrace{+1 + 1 + \cdots}_{s+3\ times} + \underbrace{1 + 2 + 2 + \cdots}_{k-4\ times} + \underbrace{2 + 3 + 3 +}_{p-s}$$

$$\underbrace{+1 + 1 + \cdots}_{k-1\ times} + \underbrace{1 + 2 + 2 +}_{p}$$

$$+1 + 1 \underbrace{+ 1 + 2 + 2 + \cdots}_{k+2s-4\ times} + \underbrace{2 + 3 + 3 +}_{s\left(\frac{k-1}{2} - 2\right)}$$

$$+ \cdots \ldots \ldots \ldots$$

$$+1 + 1 \underbrace{+ 1 + 2 + 2 + \cdots}_{} + \underbrace{2 + 3 + 3 +}_{}\}$$

$$k+2s-4\ times \quad s\left(\frac{k-1}{2}-2\right)$$

$$W(G)=\frac{1}{2}\{1+2(s+2)+3(k-4)+4(p-s)$$
$$+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$+1+2(s+2)+3(k-4)+4(p-s)$$
$$+3+s+2(k-4)+3(p-s)$$
$$+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$+3+s+2(k-4)+3(p-s)+(k-1+2p)$$
$$+3+2(k+2s-4)+3s\left(\frac{k-1}{2}-2\right)$$
$$+\cdots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$+3+2(k+2s-4)+3s\left(\frac{k-1}{2}-2\right)\}$$

$$W(G)=\frac{1}{2}\{\underbrace{3k-2s+4p-7+\cdots+3k-2s+}_{p}$$
$$\underbrace{+2k-2s+3p-5+\cdots+2k-2s+3p-5}_{\frac{k-1}{2}}+k+$$
$$\underbrace{+\frac{3}{2}sk+2k-\frac{7}{2}s-5+\cdots+\frac{3}{2}sk+2k-\frac{7}{2}s-5}_{\frac{k-1}{2}}$$

$$W(G)=\frac{1}{2}\left\{p[3k-2s+4p-7]+\frac{k-1}{2}[2k-2s+3p-5]+k+2p-1+\frac{k-1}{2}[\frac{3}{2}sk+2k-\frac{7}{2}s-5]\right\}$$

$$W(G)=\frac{1}{2}\{3pk-2ps+4p^2-7p+k+2p-1+\frac{k-1}{2}(4k-\frac{11}{2}s+3p-10+\frac{3}{2}sk)\}-----(i)$$

To find $WW^*(G)$:

$$WW^*(G)=\frac{1}{2}\sum_{\{u,v\}\subseteq V(G)}d^2(u,v)$$

$$WW^*(G)=\frac{1}{2}\{\underbrace{1+1+\cdots+1}_{s+2\text{ times}}+\underbrace{3+3+\cdots+3}_{k-4\text{ times}}+\underbrace{6+6+\cdots+6}_{(p-s)\text{ times}}$$
$$+\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$+\underbrace{1+1+\cdots+1}_{s+2\text{ times}}+\underbrace{3+3+\cdots+3}_{k-4\text{ times}}+\underbrace{6+6+\cdots+6}_{(p-s)\text{ times}}$$
$$+\underbrace{1+1+\cdots+1}_{k-4\text{ times}}+\underbrace{3+3+\cdots+3}_{(p-s)\text{ times}}$$
$$+\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$+\underbrace{1+1+\cdots+1}_{k-4\text{ times}}+\underbrace{3+3+\cdots+3}_{(p-s)\text{ times}}$$

$$
\begin{aligned}
&+ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&+ \underbrace{1+1+\cdots+1}_{k-4 \text{ times}} + \underbrace{3+3+\cdots+3}_{(p-s) \text{ times}} \\
&+ \underbrace{1+1+\cdots+1}_{p \text{ times}} \\
&+ \underbrace{1+1+\cdots+1}_{k+2s-4 \text{ times}} + \underbrace{3+3+\cdots+3}_{s[\frac{k-1}{2}-2] \text{ times}} \\
&+ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&+ \underbrace{1+1+\cdots+1}_{k+2s-4 \text{ times}} + \underbrace{3+3+\cdots+3}_{s[\frac{k-1}{2}-2] \text{ times}} \}
\end{aligned}
$$

$$
\begin{aligned}
WW^*(G) = \frac{1}{2}\{ & s+2+3(k-4)+6(p-3) \\
& + \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
& + s+2+3(k-4)+6(p-3) \\
& + k-4+3(p-s)+\cdots+k-4+3(p-s)+p \\
& + k+2s-4+3s(\frac{k-1}{2}-2) \\
& + \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
& + k+2s-4+3s(\frac{k-1}{2}-2)\}
\end{aligned}
$$

$$
\begin{aligned}
WW^*(G) = \frac{1}{2}\{ & \underbrace{3k+6p-5s-10+\cdots+3k+6p-5s-10}_{p \text{ times}} \\
& + \underbrace{k+3p-3s-4+\cdots+k+3p-3s-4}_{\frac{k-1}{2} \text{ times}} + p \\
& + \underbrace{\frac{3}{2}sk-\frac{11}{2}s+k-4+\cdots+\frac{3}{2}sk-\frac{11}{2}s+k-4}_{\frac{k-1}{2} \text{ times}}\}
\end{aligned}
$$

$$
\begin{aligned}
WW^*(G) = \frac{1}{2}\{ & p[3k+6p-5s-10]+\frac{k-1}{2}[k+3p-3s-4] \\
& + p+\frac{k-1}{2}[\frac{3}{2}sk-\frac{11}{2}s+k-4]\} -------(ii)
\end{aligned}
$$

Combining (i) and (ii) gives

$$
WW(G) = \frac{1}{2}(6pk-7sp+10p^2-17p+k+3p-1)+\frac{k-1}{4}(6k-14s+6p+3sk-18)
$$

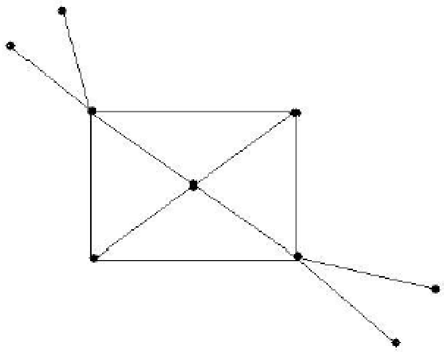

Figure 2: $k = 5$, $s = 2$, $p = 4$, $W(G) = 72$ and $WW(G) = 124$.

Theorem 3. Let H be the wheel graph on k vertices. The graph G obtained by attaching s-number of pendent vertices to any one vertex of graph H with common vertex then its hyper-Wiener index given by

$$WW(G) = \frac{1}{2}\left[s(3s + 6k - 17) + k(6s + 3k - 9) + 8 - 2k - 14s\right]$$

Proof. To nd Hyper-Wiener index of the graph, We need to find following two parts

To find Wiener index: $W(G) = \frac{1}{2} \sum_{\{u,v\} \subseteq V(G)} d(u,v)$

$$\begin{aligned}
W(G) = \frac{1}{2}\{&1 + \underbrace{2 + 2 + \cdots + 2}_{s+2 \text{ times}} + \underbrace{3 + 3 + \cdots + 3}_{k-4 \text{ times}} \\
&+ \cdots\cdots\cdots\cdots\cdots\cdots \\
&+ 1 + \underbrace{2 + 2 + \cdots + 2}_{s+2 \text{ times}} + \underbrace{3 + 3 + \cdots + 3}_{k-4 \text{ times}} \\
&+ \underbrace{1 + 1 + \cdots + 1}_{3+s \text{ times}} + \underbrace{2 + 2 + \cdots + 2}_{k-4 \text{ times}} + 1 + 1 + 1 + \underbrace{2 + 2 + \cdots + 2}_{k-4+s \text{ times}} \\
&+ 1 + 1 + 1 + \underbrace{2 + 2 + \cdots + 2}_{k-4+s \text{ times}} \\
&+ 1 + 1 + 1 + \underbrace{2 + 2 + \cdots + 2}_{k-4 \text{ times}} + \underbrace{3 + 3 + \cdots + 3}_{s \text{ times}} \\
&+ \cdots\cdots\cdots\cdots\cdots\cdots \\
&+ 1 + 1 + 1 + \underbrace{2 + 2 + \cdots + 2}_{k-4 \text{ times}} + \underbrace{3 + 3 + \cdots + 3}_{s \text{ times}} \\
&+ \underbrace{1 + 1 + \cdots + 1}_{k-1 \text{ times}} + \underbrace{2 + 2 + \cdots + 2}_{s \text{ times}}\}
\end{aligned}$$

$$\begin{aligned}
W(G) = \frac{1}{2}\{&\underbrace{2s + 3k - 7 + \cdots + 2s + 3k - 7}_{s \text{ times}} + s + 2k - 5 + 2(2s + 2k - 5) \\
&+ \underbrace{3s + 2k - 5 + \cdots + 3s + 2k - 5}_{k-4 \text{ times}} + k - 1 + 2s\}
\end{aligned}$$

$$W(G) = \frac{1}{2}\{s(2s + 3k - 7) + (k - 4)(3s + 2k - 5) + 7s + 7k - 16\}$$

$$W(G) = \frac{1}{2}\{s(2s + 3k - 7) + k(3s + 2k - 5) + 4 - k - 5s\} - - - - - (i)$$

To find $WW^*(G)$:

$$WW^*(G) = \frac{1}{2} \sum_{\{u,v\} \subseteq V(G)} d^2(u,v)$$

$$WW^*(G) = \frac{1}{2}\{\underbrace{1+1+\cdots+1}_{s+2 \text{ times}}+\underbrace{3+3+\cdots+3}_{k-4 \text{ times}}$$
$$+\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$+\underbrace{1+1+\cdots+1}_{s+2 \text{ times}}+\underbrace{3+3+\cdots+3}_{k-4 \text{ times}}$$
$$+\underbrace{1+1+\cdots+1}_{k-4 \text{ times}}+\underbrace{1+1+\cdots+1}_{k-4+s \text{ times}}+\underbrace{1+1+\cdots+1}_{k-4+s \text{ times}}$$
$$+\underbrace{1+1+\cdots+1}_{k-4 \text{ times}}+\underbrace{3+3+\cdots+3}_{s \text{ times}}$$
$$+\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$
$$+\underbrace{1+1+\cdots+1}_{k-4 \text{ times}}+\underbrace{3+3+\cdots+3}_{s \text{ times}}$$
$$+\underbrace{1+1+\cdots+1}_{s \text{ times}}\}$$

$$WW^*(G) = \frac{1}{2}\{\underbrace{s+3k-10+\cdots+s+3k-10}_{s \text{ times}}$$
$$+k-4+2(k-4+s)$$
$$+\underbrace{k+3s-4+\cdots+k+3s-4}_{k-4 \text{ times}}+s\}$$

$$WW^*(G) = \frac{1}{2}\{s(s+3k-10)+k(k+3s-4)+4-k-9s\} - - - - - - - - (ii)$$

Combining (i) and (ii) gives

$$WW(G) = \frac{1}{2}\left[s(3s+6k-17)+k(6s+3k-9)+8-2k-14s\right]$$

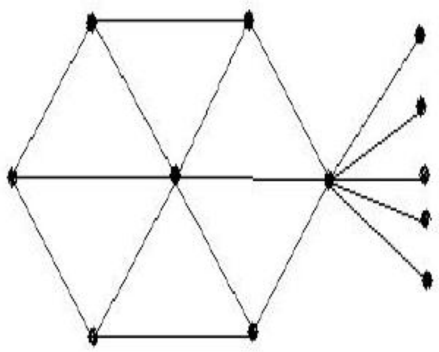

Figure 3: $k = 7$, $s = 5$, $W(G) = 130$ and $WW(G) = 209$.

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