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RESEARCH ARTICLE

CONNECTEDNESS IN U- SPACES

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ABSTRACT

This is the sixth in a series of papers on U- spaces. Here connectedness has been introduced for U- spaces and many topological theorems related to connectedness have been generalized to U- spaces, as an extension of study of supratopological spaces.

Key Words: component, totally-disconnectedness, local connectedness, path connectedness, locally path connectedness, connectedness

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INTRODUCTION

In a previous paper [1] we have introduced U- spaces and studied some of their properties there and in [2],[6] and [7]. In this paper we use the terminology of [1]. Some study of these spaces was done previously in ([3],[4],[8],[10]) in less general form, and the spaces were called supratopological spaces. In this paper we have generalized the concepts of connectedness in U-space. The concepts of a component, totally-disconnectedness, local connectedness, path connectedness, locally path connectedness, connectedness in the topological spaces ([5], [9]) have been generalized to the case of
U- spaces.

We have constructed many examples and proved a number of theorems involving these concepts.

2. CONNECTEDNESS

Definition 2.1 Let X be the usual U-space R. A U-space X is said to be **connected** if X can not be written as a disjoint union of two nonempty U-open sets. i.e. if there do not exist nonempty U-open sets G and H such that $G \cap H = \Phi \text{ and } G \cup H = X$.

If X is not connected U-space then, it is called **disconnected U-space**.
Let A be a nonempty subset of X. Then A is said to be **connected** if A is connected as a U-subspace of X. Thus, A is connected if there do not exist U-open sets G and H in X such that $A\cap G\neq\Phi, A\cap H\neq\Phi, (A\cap G)\cap(A\cap H)=\Phi$ *and* $(A\cap G)\cup(A\cap H)=A. Or, (A\subseteq G\cup H)$
The empty set Φ and singleton sets {p} are obviously connected U-spaces.
Example 2.1 We consider N as a U-subspace of the usual U-space R. Let $n_0\in$ N.
Let G = {r $\in$ N : $-\infty < r < n_0 + 1$} and H = {r $\in$ N: $n_0 < r < \infty$}. Then G and H are U- open subsets of N and G $\cup$ H = N. So, N is a disconnected U-space.
Similarly we can prove that z is a disconnected U-space.
We prove here that Q is disconnected.
Let A = Q. Since $\sqrt{2}$ is irrational, G = $(-\infty,\sqrt{2})$ *and* $H=(\sqrt{2},\infty)$ are U-open in the usual U-space R. Now, $\Phi\neq G\cap A=\{q\in$ Q: $q<\sqrt{2}$ }, $\Phi\neq H\cap A=\{q\in$Q: $q>\sqrt{2}$ }.
So, $(G\cap A)\cap(H\cap A)=\Phi$ and $(G\cap A)\cup(H\cap A)=$ Q. Therefore Q is a disconnected U-subspace of R.
Example 2.2 R, $(-\infty, a)$, (b, ∞) and (a, b), (a, b], [a, b) and every interval in R are connected subsets of usual U-space R. In fact, these are the only connected U-subspace of R.
The following theorems generalize the corresponding theorems about topological spaces [5](p. 70 - 78). **Here we only give the statements of the theorem**. The proofs are almost exactly similar to those for topological spaces. The proof of Theorem 2.10 (Theorem 1.9, [5]) has been given to show that the arguments really hold. Also we have proved the proofs of the theorems about the continuous images, since these are different here from those in topology.
Theorem 2.1 If (X, U) is a U-space and A and B are connected U-subspace of X such that A $\cap$ B $\neq\Phi$, then A $\cup B$ is connected.
Theorem 2.2 Let (X, U) be a U-space and $\{A_i\}_{i\in I}$ a collection of connected U-subspace of X . If $\bigcap_{i\in I} A_i\neq\Phi$, then $\bigcup_{i\in I} A_i$ is connected.
Theorem 2.3 The U-space R and each interval of R is connected and these are the only connected U-subspace of R.
Theorem 2.4 A U-continuous image of a connected U-space is connected.
Proof: Let X be a connected U-space and Y a U-space and f: X $\rightarrow$ Y is a U-continuous mapping. We shall show that f(X) is connected. If f(X) is not connected, let f(X) = (f(X) $\cap$ G) $\cup$ (f(X) $\cap$ H) be separation of f(X). G and H be nonempty U-open sets of Y and f is a U-continuous function.
Therefore $f^{-1}(G)$ and $f^{-1}(H)$ are U-open sets of X and X = (X $\cap$ $f^{-1}(G)$) $\cup$ (X $\cap$ $f^{-1}(H)$)
= $f^{-1}(G)\cup f^{-1}(H)$, $f^{-1}(G)\neq\Phi$, $f^{-1}(H)\neq\Phi$ and $f^{-1}(G)\cap f^{-1}(H)=\Phi$.
Hence X is disconnected, contradicting the assumption.
Therefore f(X) is connected.
Theorem 2.5 A Ū-continuous image of a connected U-space is connected.
Proof: Let X be a U-space and Y a connected space and f: X $\rightarrow$ Y is a Ū-continuous mapping. We shall show that f(X) is connected. If f(X) is not connected, let f(X) = (f(X) $\cap$ G) $\cup$ (f(X) $\cap$ H) be separation of f(X). G and H be nonempty open sets of Y and f is a Ū-continuous function.
Therefore $f^{-1}(G)$ and $f^{-1}(H)$ are U- open sets of X and X = (X $\cap$ $f^{-1}(G)$) $\cup$ (X $\cap$ $f^{-1}(H)$)
= $f^{-1}(G)\cup f^{-1}(H)$, $f^{-1}(G)\neq\Phi$, $f^{-1}(H)\neq\Phi$ and $f^{-1}(G)\cap f^{-1}(H)=\Phi$.
Hence X is disconnected, contradicting the assumption. Therefore f(X) is connected.
Theorem 2.6 A U*-continuous image of a connected U-space is connected.
Proof: Let X be a connected space and Y a U-space and f: X $\rightarrow$ Y is a U*-continuous mapping. We shall show that f(X) is connected U-space. If f(X) is not connected, let f(X) = (f(X) $\cap$ G) $\cup$ (f(X) $\cap$ H) be separation of f(X). G and H be nonempty U-open sets of Y and f is a U*- continuous function.

Therefore $f^{-1}(G)$ and $f^{-1}(H)$ are open sets of X and $X = (X \cap f^{-1}(G)) \cup (X \cap f^{-1}(H))$
$= f^{-1}(G) \cup f^{-1}(H)$, $f^{-1}(G) \neq \Phi$, $f^{-1}(H) \neq \Phi$ and $f^{-1}(G) \cap f^{-1}(H) = \Phi$.
Hence X is disconnected, contradicting the assumption.
Therefore f(X) is connected.

Theorem 2.7 Let X be a connected U-space. Then there does not exist U-closed-open subsets of X except X and Φ.

Theorem 2.8 Let X be a U-space and A is a connected U-subspace of X .If B is a U-subspace of X such that $A \subseteq B \subseteq \overline{A}$, then B is connected;
in particular $\overline{A}$ is connected.

Theorem 2.9 A U-space X is disconnected if and only if there exists a $\overline{U}$-continuous mapping X onto the discrete two point space {0,1}.

Proof: Let X be a U-space and E is the discrete two point space {0, 1}. Suppose that X is disconnected. Then X has two disjoint U-open sets G and H such that $X = G \cup H$.
Let us define a map $f: X \longrightarrow E$ such that $f(x) = 0, x \in G$
$= 1, x \in H$
Also G and H are U-open sets. This implies that f is $\overline{U}$-continuous.
Conversely, suppose that there exists a $\overline{U}$-continuous map $f: X \longrightarrow E$. Then $f^{-1}(\{0\})$ and $f^{-1}(\{1\})$ are disjoint U-open sets of X and $X = f^{-1}(\{0\}) \cup f^{-1}(\{1\})$. So X is disconnected.

Theorem 2.10 A finite Cartesian product of connected U-spaces is connected.

Proof: Let $X_1, X_2, X_3, \text{-------}, X_n$ be connected U-spaces and
$X = X_1 \times X_2 \times X_3 \times \text{-------} \times X_n$.
We shall use induction.
Let n = 2, then $X = X_1 \times X_2$. Let $(a, b) \in X_1 \times X_2$. Since $X_1 \times \{b\}$ and X_1 and for each $x_1 \in X_1$, $\{x_1\} \times X_2$ and X_2 are homeomorphic. So $X_1 \times \{b\}$ and $\{x_1\} \times X_2$ are U-connected.
Again since $(x_1, b) \in (X_1 \times \{b\}) \cap (\{x_1\} \times X_2)$.
This implies that $U_{x_1} = (X_1 \times \{b\}) \cup (\{x_1\} \times X_2)$ is connected.
Let $U = \bigcup_{x_1 \in X_1} U_{x_1}$. This union is connected because it is the union of a collection of connected U-spaces that have the point (a, b) in common. Since this union $U = X_1 \times X_2$, the space $X_1 \times X_2$ is connected.
Now let $X_1 \times X_2 \times X_3 \times \text{-------} \times X_{n-1}$ be connected for n > 2. Since $X_1 \times X_2 \times X_3 \times \text{-------} \times X_{n-1} \times X_n$ and $(X_1 \times X_2 \times X_3 \times \text{-------} \times X_{n-1}) \times X_n$ are homeomorphic, $X_1 \times X_2 \times X_3 \times \text{-------} \times X_n$ is connected as in the case of $X_1 \times X_2$.

Theorem 2.11 Let $\{X_i\}_{i \in I}$ be collection of nonempty connected U-space and $X = \prod X_i$, then X is connected.

Theorem 2.12 (The generalized form of the Intermediate value theorem).

Let $f: X \longrightarrow Y$ be a $\overline{U}$-continuous map, where X is a connected U-space and Y is an ordered set with the order U-structure. If a and b are two points of X and if r is a point of Y lying between f(a) and f (b), then there exists a point c of X such that f(c) = r.

Proof: Let $A = f(X) \cap \{y \in Y: y < r\}$ and $B = f(X) \cap \{y \in Y: r < y\}$. So,
$A \cap B = \Phi$ and $A \neq \Phi$, $B \neq \Phi$ because $f(a) \in A$ and $f(b) \in B$. Since A and B are U-open, if there were no point c of X such that f(c) = r, then $f(X) = A \cup B$ and f(X) is disconnected, contradicting the fact that the image of a connected U-space under a $\overline{U}$-continuous map is connected.

Definition 2.2 Let X be a U-space. A subset M of X is said to be **U-component or connected component** if (i) M is connected, (ii) if A is a connected subset of X such that $M \subseteq A \subseteq X$, then A = M or A = X, i.e. M is a maximal subset of a U-space X.

Example - 2.3 Let $X = [3, 5) \cup (6, 9)$ be a subspace of the usual U-space R. Here X is a disconnected U-space and [3, 5) and (6, 9) are two components of X.

Example- 2.4 N, Z, Q are disconnected subspaces of usual U-space R. Singleton subsets are the components of the above U-subspaces.

Example 2.5 Let X = {a,b,c,d,e}, **U** ={X, Φ ,{e},{a, b}, {c, d}, {a, b, c}, {a, b, e}, {c, d, e},{a, b, c, d}, {a, b, c, e}}. Then X is a disconnected U-space and {a, b}, {c, d}, {e}, are the components of X.

Theorem 2.13 Let X be a U-space.

(i) Every connected U-closed-open subset of X is a component of X.

(ii) Every component of X is U-closed.

(iii) Every element of X is contained in a unique component of X.

(iv) Every connected subset of U-space X is contained in a unique component of U-space X.

Definition 2.3 Let X be a U-space. A U-space X is called **totally disconnected** U-space if for every pair of distinct points x and y (x $\neq$ y), there exists a non-empty disjoint U-open set A, B such that X = A $\cup$ B with x $\in$ A and y $\in$ B.

Example 2.6 The U-subspaces N, Z, Q and $Q^{/}$ (the set of irrational numbers) of the usual U-space R are totally disconnected. We prove the statement here. Prove this statement below:

(i) Let m, n $\in$ N with m < n.

Then, {1, 2, 3,........., m} $\cup$ {m + 1,m + 2, m + 3,.........} is a disconnection of N . Here, {1, 2, 3, ...m} = N $\cap$ (-∞, m + $\frac{1}{2}$) and {m + 1,m + 2, m + 3,.........} = N $\cap$ (m + $\frac{1}{2}$, ∞) are U-open subsets of N which contain m and n respectively.

Thus N is totally disconnected.

(ii) The proof that Z is totally disconnected is similar.

(iii) Let a, b $\in$ Q with a < b. Then there exists an irrational number x such that a < x < b. Then, A $\cup$ B, where A = {y $\in$ Q **:** y < x } and

B = {y $\in$ Q **:** y > x } is a disconnection of Q. Then a $\in$ A, b $\in$ B, and

A = Q $\cap$ (-∞, x), B = Q $\cap$ (x, ∞). So that A and B are U-open in Q. Hence Q is totally disconnected.

(iv) We can prove similarly that $Q^{'}$ is totally disconnected.

Example 2.7 Every discrete U-space consisting of more than one element is totally disconnected. This is obvious**.**

Theorem 2.14 The U-components of totally disconnected U-spaces consists of exactly one element.

Proof: Let X be a totally disconnected U-space. It is enough to prove that every U-subspace of X with two distinct elements is disconnected. Let x, y $\in$ X and x $\neq$ y. Since X is totally disconnected, there exist X = A $\cup$ B such that x $\in$ A and y $\in$ B. Thus {x, y} = (A $\cap$ {x, y}) $\cup$ (B $\cap$ {x, y}).

Hence {x, y} is disconnected.

Definition 2.4 A U-space X is said to be l**ocally connected** if for every x $\in$ X, and for every neighborhood G of x, there is a connected U-open set V of X, such that x $\in$ V $\subseteq$ G. X is a locally connected U-space if and only if X is locally connected U-space at each of its points.

Our Theorems 2.15-2.22 are generalizations of theorems in ([5], P-123-131)

Theorem 2.15 Every U-open subspace of a locally connected U-space is locally connected.

Proof: Let X be a locally connected U-space and G be a U-open subspace of X. Let H be a U-open set containing a point x of G. Since G is U-open, so H is a U-open set of X. Since X is locally connected, there exists a connected U-open set V in X which contains x and is contained in H. Also V is a U-open set of G.

Hence G is locally connected.

Theorem 2.16 The image of a locally connected U-space under a mapping which is both U-continuous and U-open is locally connected.

Proof: Let X be a locally connected U-space and Y be a U-space. Let f : X $\rightarrow$ Y be U-continuous, U-open and onto mapping. Let y $\in$ Y and G be a U-open set of Y containing y. For each x $\in$ X, y = f(x) and f^{-1}(G) is U-open set of X containing x.

Since X is locally connected, there exists a connected U-open set V of f^{-1}(G) containing x. i.e.

$x \in V \subseteq f^{-1}(G)$. Since f is U-open and U-continuous.
f(V) is a connected U-open set of Y and f(x) = y = f(V). Since $V \subseteq f^{-1}(G)$, f(V) $\subseteq$ G.
Hence f(X) = Y is locally connected.

Theorem 2.17 The product space of two locally connected U-spaces is locally connected.

Proof: Let X and Y be locally connected U-space. We shall show that $X \times Y$ is locally connected.

Let (x, y) $\in X \times Y$ and G be a U-open set of $X \times Y$ containing (x, y). Since projection mapping $\pi_X : X \times Y \rightarrow X$ is U-open, π_X (G) is a U-open set containing x. Since X is locally connected, so there exists a connected U-open set V_1 of X containing x of π_X (G).

Again π_Y(G) is a U-open set and there exists a connected U-open set V_2 of Y containing y of a locally connected U-open set π_Y(G). Therefore $V_1 \times V_2$ is a connected U-open set of $X \times Y$ containing (x, y) and $V_1 \times V_2 \subseteq$ G.
Hence $X \times Y$ is locally connected.

Theorem 2.18 A U-space X is locally connected if and only if for each U- component of every U-open set of X is U-open.

Proof: Let X be a locally connected U-space and let G be a U-open set in X. According to the above Theorem 2.15 G is locally connected. Let C be a component of G and let $a \in C$. Since G is a locally connected U-space, there exists connected U-open set V of G containing a. i.e. $V \subseteq G$. Since C is a component and $a \in C$, $V \subseteq C$. Therefore C is U-open in X.

Conversely, suppose that U-components of U-open sets in X are open. Suppose $x \in X$ and a neighborhood G of x. Let C be the U-component of G containing x. C is U-open in X by hypothesis. So C is a connected U-open set of G containing x.
Hence X is locally connected.

Definition 2.5 Let X be a U-space and let f : $[0,1] \rightarrow X$ be a U-continuous mapping.
If f(0) = x, f(1) = y, then f is called **path** from x to y.

Definition 2.6 Any U-space X is called **path connected U-space** if there is a path in X from x to y.

Definition 2.7 [5](p. 131)
A U- space X is said to be **locally path connected U-space** at x if for every open set G of x have a open subset V which is path connected U-space containing x.
If X is locally path connected at each of its points, then it is said to be **locally path connected U-space.**

Example 2.8 Each interval and each ray in the usual U-space R are connected, locally connected, path-connected and locally path-connected U-spaces.
Each of the subspaces [-1, 0) $\cup$ (0, 1] and [1, 2] $\cup$ [3, 4] of R is neither connected nor path-connected but each is both locally connected and locally path-connected.

Example 2.9 Let **C = ([**0,1] $\times\{0\}) \cup (\{\frac{1}{n} : n \in Z\} \times [0,1]) \cup (\{0\} \times [0,1])$ be a U-subspace of R^2 and let D = C - $\{0\} \times (0,1)$ be a U-space. Here C is the union of connected U-subset I_α , where I_α = [0,1] $\times\{0\}$, $\left\{\frac{1}{n}\right\} \times [0,1]$ or $\{0\} \times [0,1]$. Since each I_α is connected and $I_\alpha \cap \left(\bigcup_{\alpha \neq \beta} I_\beta\right) \neq \Phi$. Therefore C is a connected U- space and also D is a connected U-space and $\overline{D}$ = C. If p is any point on $\{0\} \times [0,1]$, then for any open sphere $S_\in$ (p) with centered at p there exist a U-open set $G \subset S_\in$ (p) such that G is disconnected. Therefore, C is not locally connected U-space. If we consider p is (0,1), then similarly we can show that D is locally disconnected.

Theorem 2.19 Every path connected U-space is connected.

Proof: Suppose X be a path connected U-space and $x_o \in X$. Then for any $x \in X$ there is a path from x_o to x. That means there exists a U-continuous mapping f : $I \rightarrow X$ such that f(0) = x_o , f(1) = x_o. Since I is a connected U-

space, f(I) is connected U-subset of X. Therefore x_o and x are contained in same component of X. Since for any $x \in X$ true that X has only one component.
Therefore X is connected.

Theorem 2.20 The image of a U-continuous mapping of path connected U-space is path connected.

Proof: Let X be a path connected U- space and Y be a U-space. Let $\phi : X \to Y$ be a onto U-continuous mapping. We shall show that Y is a path connected U-space. Let y_1 and y_2 are two points of Y. Then there exists $x_1, x_2 \in X$ such that $\phi(x_1) = y_1$ and $\phi(x_2) = y_2$. Since X is a path connected U-space, there exists a U-continuous mapping $f : I \to X$ such that $f(0) = x_1$ and $f(I) = x_2$. Then $\phi(f(0)) = y_1$, $\phi(f(I)) = y_2$.
So $\phi f : I \to Y$ is a U-continuous mapping i.e. Y is path connected.

Theorem 2.21 The product space of any finite number of path connected U-spaces is path connected.

Proof: Let X_1, X_2, X_3, ---------, X_n be path connected U-spaces and
$X = X_1 \times X_2 \times X_3 \times \text{--------} \times X_n$. Suppose $x, y \in X$, then $x = (x_1, x_2, x_3, \text{-------}, x_n)$ and $y = (y_1, y_2, y_3, \text{------}, y_n)$, $x_i, y_i \in X_i$. Since each X_i path connected, there exists a U-continuous mapping $f_i : [0, 1] \to X_i$ such that $f_i(0) = x_i$, $f_i(1) = y_i$.
If $f : [0, 1] \to X$ defined by $f(a) = (f_1(t), f_2(t), \text{------}, f_n(t))$, then $f(0) = (x_1, x_2, x_3, \text{-------}, x_n) = x$ and $f(1) = (y_1, y_2, y_3, \text{--------}, y_n) = y$.
We shall show that f is U-continuous. Let G be a U-open set of X, then $\pi_i(G) = G_i$, G_i is a U-open set of X_i, where $\pi_i(G): X \to X_i$ is a projection mapping. Since f_i is U-continuous, $f_i^{-1}(G_i)$ is a U- open set of
[0, 1]. Now $f^{-1}(G) = f_1^{-1}(G_1) \cap f_2^{-1}(G_2) \cap \text{--------} \cap f_n^{-1}(G_n)$.
Therefore $f^{-1}(G)$ is U-open. i.e. f is U-continuous.

Remark 2.1 The closure of a path connected subsets of U-space may not be path connected.

Example 2.10 Let $S = \{(x, \sin\frac{1}{x}) : 0 < x \leq 1\}$ be a subset of the product U-space $R \times R$. Then it is a path connected U-space but the closure $\overline{S}$ is not a path connected U-space.
Here, we see that $\overline{S}$ is connected but not path connected.

Definition 2.8 Let X be a U-space and a and b be two separate point of X. A finite sequence of subsets A_1, A_2, A_3, ------, A_n of X is called **simple chain** from a to b if a only belongs to A_1 and b only belongs to A_n and $A_i \cap A_j \neq \Phi$ iff $|i - j| \leq 1$.

Theorem 2.22. If a and b are two separate points of connected U-space X and $\{U_\alpha\}$ is a U-open cover of X, then there exists a simple chain of U_α from a to b.

Proof: Let $\{U_\alpha\}$ be a U-open cover of X and let Y be a collection of points y of X and there exists a simple chain of U_α from a to y. Then Y is U-open.

Because if $y \in Y$ and U_1, U_2, U_3, -----, U_n ($U_i \in \{U_\alpha\}$) from a to y is a simple chain, then U_1, U_2, U_3, ------, U_n or U_1, U_2, U_3, ------, U_{n-1} from a to y' for each $y' \in U_n$ is a simple chain. So, $y' \in Y$ and $U_n \subseteq Y$. Therefore Y is a U-open set.

Now we shall show that Y is closed. Suppose y be a limit point of Y. Then there is a point y' ($y' \neq y$) of Y in each U-open set U containing y.

Therefore the exists a simple chain $U_1, U_2, U_3, \ldots\ldots, U_n$ from a to y. Now we can consider y' is a point of Y in which n is the smallest.
Since $U_n \cap U \neq \Phi$, $U_1, U_2, U_3, \ldots, U_n$, U from a to y will be simple chain if for each i < n, $U_i \cap U = \Phi$. Because if $U_i \cap U \neq \Phi$ ($i_0 < n$) and let $y'' \in U_{i_0} \cap U$. Then $y'' \in Y$ and $U_1, U_2, U_3, \ldots, U_{i_0}$ from a to y'' is a simple chain. Since $i_0 < n$ which is contradictory to the smallest n. So $U_1, U_2, U_3, \ldots, U_n$, U from a to y is a simple chain. i.e. $y \in Y$.

Hence theorem is proved.

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