Vol.3.Issue.2.2015

http://www.bomsr.com

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

ISSN:2348-0580

RESEARCH ARTICLE

Ω-FUZZY SUBBIGROUP OF A BIGROUP

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ABSTRACT

In this paper, we made an attempt to study the algebraic nature of Ω-fuzzy subbigroup of a bigroup.

2000 AMS Subject classification: 03F55, 06D72, 08A72.

KEY WORDS: Bigroup, fuzzy subset, fuzzy subbigroup, Ω-fuzzy subbigroup

INTRODUCTION

In 1965, the fuzzy subset was introduced by L.A.Zadeh[10], after that several researchers explored on the generalization of the concept of fuzzy sets. The notion of fuzzy subgroups was introduced by Azriel Rosenfeld[2] and Gyu Ihn Chae, Young Sik Park and Chul Hwan Park [3] have introduced and defined a new algebraic structure called Ω-bifuzzy subsemigroup. After that A.Solairaju, R.Nagarajan[7, 8, 9] and K.Arjunan, Selvak Kumaraen [6] extend the theory to many algebraic structure. In this paper, we introduce the some theorems in Ω-fuzzy subbigroup of a bigroup.

1.PRELIMINARIES:

1.1 Definition: A set $(G, +, \bullet)$ with two binary operations $+$ and $\bullet$ is called a bigroup if there exist two proper subsets G_1 and G_2 of G such that (i) $G = G_1 \cup G_2$ (ii) $(G_1, +)$ is a group (iii) $(G_2, \bullet)$ is a group.

1.2 Definition: A non-empty subset H of a bigroup $(G, +, \bullet)$ is called a sub-bigroup if H itself is a bigroup under the operations $+$ and $\bullet$ defined on G.

1.3 Definition: Let X be a non–empty set. A **fuzzy subset A** of X is a function $A: X \rightarrow [0, 1]$.

1.4 Definition: Let $G = (G_1 \cup G_2, +, \bullet)$ be a bigroup. Then a fuzzy set A of G is said to be a fuzzy subbigroup of G if there exist two fuzzy subsets A_1 of G_1 and A_2 of G_2 such that (i) $A = A_1 \cup A_2$ (ii) A_1 is a fuzzy subgroup of $(G_1, +)$ (iii) A_2 is a fuzzy subgroup of $(G_2, \bullet)$.

1.5 Definition: Let G = ($G_1 \cup G_2$, + , $\bullet$) be a bigroup and Ω be a nonempty set. The Ω-fuzzy subset A: $G\times\Omega\to[0, 1]$ of G is said to be a Ω-fuzzy subbigroup of G if there exist two Ω-fuzzy subsets A_1: $G_1\times\Omega\to[0, 1]$ of G_1 and A_2 : $G_2\times\Omega\to[0, 1]$ of G_2 such that (i) A = $A_1\cup A_2$ (ii) A_1 is a Ω-fuzzy subgroup of (G_1, +) (iii) A_2 is a Ω-fuzzy subgroup of (G_2 , $\bullet$).

2. PROPERTIES:

2.1 Theorem: If A = M$\cup$N is a Ω-fuzzy subbigroup of a bigroup G = E$\cup$F, then $\mu_M(-x, q) = \mu_M(x, q)$, $\mu_M(x, q) \le \mu_M(e, q)$, $\mu_N(x^{-1}, q) = \mu_N(x, q)$, $\mu_N(x, q) \le \mu_N(e', q)$ for all x, e in E and x, e' in F and q in Ω.

Proof: Let x, e in E and x, e' in F and q in Ω. Now $\mu_M(x, q) = \mu_M((-(-x)), q) \ge \mu_M(-x, q) \ge \mu_M(x, q)$. Therefore $\mu_M(-x, q) = \mu_M(x, q)$ for all x in E and q in Ω. And $\mu_M(e, q) = \mu_M(x-x, q) \ge \min \{ \mu_M(x, q), \mu_M(x, q) \}= \mu_M(x, q)$. Therefore $\mu_M(e, q) \ge \mu_M(x, q)$ for all x, e in E and q in Ω. Also $\mu_N(x, q) = \mu_N((x^{-1})^{-1}, q) \ge \mu_N(x^{-1}, q) \ge \mu_N(x, q)$. Therefore $\mu_N(x^{-1}, q) = \mu_N(x, q)$ for all x in F and q in Ω. And $\mu_N(e', q)= \mu_N(xx^{-1}, q) \ge \min\{ \mu_N(x, q), \mu_N(x^{-1}, q) \}= \mu_N(x, q)$. Therefore $\mu_N(e', q) \ge \mu_N(x, q)$ for all x, e' in F and q in Ω.

2.2 Theorem: If A = M$\cup$N is a Ω-fuzzy subbigroup of a bigroup G = E$\cup$F, then (i) $\mu_M(x-y, q) = \mu_M(e, q)$ gives $\mu_M(x, q) = \mu_M(y, q)$ for all x, y and e in E and q in Ω
(ii) $\mu_N(xy^{-1}, q) = \mu_N(e', q)$ gives $\mu_N(x, q) = \mu_N(y, q)$ for all x, y and e' in F and q in Ω.

Proof: (i) Let x, y and e in E and q in Ω. Then $\mu_M(x, q) = \mu_M(x-y+y, q) \ge \min \{ \mu_M(x-y, q), \mu_M(y, q) \} = \min \{ \mu_M(e, q), \mu_M(y, q) \}= \mu_M(y, q) = \mu_M(y-x+x, q) \ge \min \{ \mu_M(y-x, q), \mu_M(x, q) \}= \min \{ \mu_M(e, q), \mu_M(x, q) \}= \mu_M(x, q)$. Therefore $\mu_M(x, q) = \mu_M(y, q)$ for all x and y in E and q in Ω. (ii) Let x, y and e' in F and q in Ω. Then $\mu_N(x, q) = \mu_N(xy^{-1}y, q) \ge \min \{ \mu_N(xy^{-1}, q), \mu_N(y, q) \} = \min \{ \mu_N(e', q), \mu_N(y, q) \}= \mu_N(y, q) = \mu_N(yx^{-1}x, q) \ge \min \{ \mu_N(yx^{-1}, q), \mu_N(x, q) \}= \min \{ \mu_N(e', q), \mu_N(x, q) \}= \mu_N(x, q)$. Therefore $\mu_N(x, q) = \mu_N(y, q)$ for all x and y in F and q in Ω.

2.3 Theorem: If A= M$\cup$N is a Ω-fuzzy subbigroup of a bigroup G = E$\cup$F, then
(i) H_1= { x / x$\in$E and $\mu_M(x, q) = 1$ } is either empty or a subgroup of E.
(ii) H_2 = { x / x$\in$F and $\mu_N(x, q) = 1$ } is either empty or a subgroup of F.
(iii) K = $H_1 \cup H_2$ is either empty or a subbigroup of G.

Proof: If no element satisfies this condition, then H_1 and H_2 are empty. Also K = $H_1\cup H_2$ is empty. (i) If x and y in H_1, then $\mu_M(x-y, q) \ge \min \{ \mu_M(x, q), \mu_M(y, q) \}\ge \min \{ 1, 1 \} = 1$. Therefore $\mu_M(x-y, q) = 1$. We get x–y in H_1. Hence H_1 is a subgroup of G_1. (ii) If x and y in H_2, then $\mu_N(xy^{-1}, q) \ge \min \{ \mu_N(x, q), \mu_N(y, q) \} = \min \{ 1, 1 \} = 1$. Therefore $\mu_N(xy^{-1}, q) = 1$. We get xy^{-1} in H_2. Hence H_2 is a subgroup of G_2. (iii) From (i) and (ii) we get K = $H_1 \cup H_2$ is a subbigroup of G.

2.4 Theorem: If A = M$\cup$N is a Ω-fuzzy subbigroup of a bigroup G = E$\cup$F, then
(i) H_1= { x / x$\in$E and $\mu_M(x, q) = \mu_M(e, q)$ } is a subgroup of E
(ii) H_2 = { x / x$\in$F and $\mu_N(x, q) = \mu_N(e', q)$ } is a subgroup of F
(iii) K = $H_1\cup H_2$ is a subbigroup of G.

Proof: (i) Clearly e in H_1 so H_1 is a non empty. Let x and y be in H_1. Then $\mu_M(x-y, q) \ge \min \{ \mu_M(x, q), \mu_M(y, q) \}= \min \{ \mu_M(e, q), \mu_M(e, q) \}= \mu_M(e, q)$. Therefore $\mu_M(x-y, q) \ge \mu_M(e, q)$ for all x and y in H_1 and q in Ω. We get $\mu_M(x-y, q) = \mu_M(e, q)$ for all x and y in H_1 and q in Ω. Therefore x–y in H_1. Hence H_1 is a subgroup of E. (ii) Clearly e' in H_2 so H_2 is a non empty. Let x and y be in H_2. Then $\mu_N(xy^{-1}, q) \ge \min \{ \mu_N(x, q), \mu_N(y, q) \}= \min \{ \mu_N(e', q), \mu_N(e', q) \} = \mu_N(e', q)$. Therefore $\mu_N(xy^{-1}, q) \ge \mu_N(e', q)$ for all x and y in H_2 and q in Ω. We get $\mu_N(xy^{-1}, q) = \mu_N(e', q)$ for all x and y in H_2 and q in Ω. Therefore xy^{-1} in H_2. Hence H_2 is a subgroup of F. (iii) From (i) and (ii) we get K = $H_1 \cup H_2$ is a subbigroup of G.

2.5 Theorem: Let A= M$\cup$N be a Ω-fuzzy subbigroup of a bigroup G = E$\cup$F.
(i) If $\mu_M(x-y, q) = 1$, then $\mu_M(x, q) = \mu_M(y, q)$ for all x and y in E and q in Ω.

(ii) If $\mu_N(xy^{-1}, q) = 1$, then $\mu_N(x, q) = \mu_N(y, q)$ for all x and y in F and q in Ω.

Proof: (i) Let x and y belongs to E and q in Ω. Then $\mu_M(x, q) = \mu_M(x-y+y, q) \geq \min\{\mu_M(x-y, q), \mu_M(y, q)\} = \min\{1, \mu_M(y, q)\} = \mu_M(y, q) = \mu_M(-y, q) = \mu_M(-x+x-y, q) \geq \min\{\mu_M(-x, q), \mu_M(x-y, q)\} = \min\{\mu_M(-x, q), 1\} = \mu_M(-x, q) = \mu_M(x, q)$. Therefore $\mu_M(x, q) = \mu_M(y, q)$ for all x and y in E and q in Ω.

(ii) Let x and y belongs to F and q in Ω. Then $\mu_N(x, q) = \mu_N(xy^{-1}y, q) \geq \min\{\mu_N(xy^{-1}, q), \mu_N(y, q)\} = \min\{1, \mu_N(y, q)\} = \mu_N(y, q) = \mu_N(y^{-1}, q) = \mu_N(x^{-1}xy^{-1}, q) \geq \min\{\mu_N(x^{-1}, q), \mu_N(xy^{-1}, q)\} = \min\{\mu_N(x^{-1}, q), 1\} = \mu_N(x^{-1}, q) = \mu_N(x, q)$. Therefore $\mu_N(x, q) = \mu_N(y, q)$ for all x and y in F and q in Ω.

2.6 Theorem: If A= $M\cup N$ is a Ω-fuzzy subbigroup of a bigroup G = $E\cup F$, then
(i) $\mu_M(x+y, q) = \min\{\mu_M(x, q), \mu_M(y, q)\}$ for each x and y in E and q in Ω with $\mu_M(x, q) \neq \mu_M(y, q)$ (ii) $\mu_N(xy, q) = \min\{\mu_N(x, q), \mu_N(y, q)\}$ for each x and y in F and q in Ω with $\mu_N(x, q) \neq \mu_N(y, q)$.

Proof: (i) Let x and y belongs to E and q in Ω. Assume that $\mu_M(x, q) > \mu_M(y, q)$, then $\mu_M(y, q) = \mu_M(-x+x+y, q) \geq \min\{\mu_M(-x, q), \mu_M(x+y, q)\} \geq \min\{\mu_M(x, q), \mu_M(x+y, q)\} = \mu_M(x+y, q) \geq \min\{\mu_M(x, q), \mu_M(y, q)\} = \mu_M(y, q)$. Therefore $\mu_M(x+y, q) = \mu_M(y, q) = \min\{\mu_M(x, q), \mu_M(y, q)\}$ for x and y in E and q in Ω. (ii) Let x and y belongs to F and q in Ω. Assume that $\mu_N(x, q) > \mu_N(y, q)$, then $\mu_N(y, q) = \mu_N(x^{-1}xy, q) \geq \min\{\mu_N(x^{-1}, q), \mu_N(xy, q)\} \geq \min\{\mu_N(x, q), \mu_N(xy, q)\} = \mu_N(xy, q) \geq \min\{\mu_N(x, q), \mu_N(y, q)\} = \mu_N(y, q)$. Therefore $\mu_N(xy, q) = \mu_N(y, q) = \min\{\mu_N(x, q), \mu_N(y, q)\}$ for x and y in F and q in Ω.

2.7 Theorem: If A = $M\cup N$ and B = $O\cup P$ are two Ω-fuzzy subbigroups of a bigroup G = $E\cup F$, then their intersection $A\cap B$ is a Ω-fuzzy subbigroup of G.

Proof: Let $A = M\cup N = \{\langle (x, q), \mu_A(x, q)\rangle / x\in G \text{ and } q\in\Omega\}$ where $M = \{\langle (x, q), \mu_M(x, q)\rangle / x\in E \text{ and } q\in\Omega\}$ and $N = \{\langle (x, q), \mu_N(x, q)\rangle / x\in F \text{ and } q\in\Omega\}$ and $B = O\cup P = \{\langle (x, q), \mu_B(x, q)\rangle / x\in G \text{ and } q\in\Omega\}$ where $O = \{\langle (x, q), \mu_O(x, q)\rangle / x\in E \text{ and } q\in\Omega\}$ and $P = \{\langle (x, q), \mu_P(x, q)\rangle / x\in F \text{ and } q\in\Omega\}$. Let $C = A\cap B = R\cup S$ where$C = \{\langle (x, q), \mu_C(x, q)\rangle / x\in G \text{ and } q\in\Omega\}$, $R = M\cap O = \{\langle (x, q), \mu_R(x, q)\rangle / x\in E \text{ and } q\in\Omega\}$ and $S = N\cap P = \{\langle (x, q), \mu_S(x, q)\rangle / x\in F \text{ and } q\in\Omega\}$. Let x and y belong to E and q in Ω. Then $\mu_R(x-y, q) = \min\{\mu_M(x-y, q), \mu_O(x-y, q)\} \geq \min\{\min\{\mu_M(x, q), \mu_M(y, q)\}, \min\{\mu_O(x, q), \mu_O(y, q)\}\} \geq \min\{\min\{\mu_M(x, q), \mu_O(x, q)\}, \min\{\mu_M(y, q), \mu_O(y, q)\}\} = \min\{\mu_R(x, q), \mu_R(y, q)\}$. Therefore $\mu_R(x-y, q) \geq \min\{\mu_R(x, q), \mu_R(y, q)\}$ for all x and y in E and q in Ω. Let x and y belong to F and q in Ω. Then $\mu_S(xy^{-1}, q) = \min\{\mu_N(xy^{-1}, q), \mu_P(xy^{-1}, q)\} \geq \min\{\min\{\mu_N(x, q), \mu_N(y, q)\}, \min\{\mu_P(x, q), \mu_P(y, q)\}\} \geq \min\{\min\{\mu_N(x, q), \mu_P(x, q)\}, \min\{\mu_N(y, q), \mu_P(y, q)\}\} = \min\{\mu_S(x, q), \mu_S(y, q)\}$. Therefore $\mu_S(xy^{-1}, q) \geq \min\{\mu_S(x, q), \mu_S(y, q)\}$ for all x and y in F and q in Ω. Hence $A\cap B$ is a Ω-fuzzy subbigroup of G.

2.8 Theorem: The intersection of a family of Ω-fuzzy subbigroups of a bigroup G is a Ω-fuzzy subbigroup of G.

Proof: It is trivial.

2.9 Theorem: If A= $M\cup N$ is a Ω-fuzzy subbigroup of a bigroup G = $E\cup F$, then (i) $\mu_M(x+y, q) = \mu_M(y+x, q)$ if and only if $\mu_M(x, q) = \mu_M(-y+x+y, q)$ for all x and y in E and q in Ω (ii) $\mu_N(xy, q) = \mu_N(yx, q)$ if and only if $\mu_N(x, q) = \mu_N(y^{-1}xy, q)$ for all x and y in F and q in Ω.

Proof: (i) Let x and y be in E and q in Ω. Assume that $\mu_M(x+y, q) = \mu_M(y+x, q)$, then $\mu_M(-y+x+y, q) = \mu_M(-y+y+x, q) = \mu_M(e_1+x, q) = \mu_M(x, q)$. Therefore $\mu_M(x, q) = \mu_M(-y+x+y, q)$ for all x and y in E and q in Ω. Conversely, assume that $\mu_M(x, q) = \mu_M(-y+x+y, q)$, then $\mu_M(x+y, q) = \mu_M(x+y-x+x, q) = \mu_M(y+x, q)$. Therefore $\mu_M(x+y, q) = \mu_M(y+x, q)$ for all x and y in E and q in Ω. (ii) Let x and y be in F and q in Ω. Assume that $\mu_N(x+y, q) = \mu_N(y+x, q)$, then $\mu_N(y^{-1}xy, q) = \mu_N(y^{-1}yx, q) = \mu_N(e_2x, q) = \mu_N(x, q)$. Therefore $\mu_N(x, q) = \mu_N(y^{-1}xy, q)$ for all x and y in F and q in Ω. Conversely, assume that $\mu_N(x, q) = \mu_N(y^{-1}xy, q)$, then $\mu_N(xy, q) = \mu_N(xyxx^{-1}, q) = \mu_N(yx, q)$. Therefore $\mu_N(xy, q) = \mu_N(yx, q)$ for all x and y in F and q in Ω.

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