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RESEARCH ARTICLE

RULED SURFACE PAIR GENERATED BY DARBOUX VECTORS OF A CURVE AND ITS NATURAL LIFT IN IR^3

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ABSTRACT

Abstract: In this study,firstly, the darboux vector $\overline{W}$ of the natural lift $\overline{\alpha}$ of the curve α are calculated in terms of those of α in IR^3. Secondly, we obtained striction lines and distribution parameters of ruled surface pair generated by Darboux Vectors of the curve α and its natural lift $\overline{\alpha}$. Finally, for α and $\overline{\alpha}$ those notions are compared with each other.

MSC:14J26,53A04,

Key Words: Natural Lift, Ruled Surface, Striction Line, Distribution Parameter

INTRODUCTION AND PRELIMINARY NOTE

The concepts of the natural lift curve and geodesic sprays have first been given by Thorpe in [13]. Thorpe provied the natural lift $\overline{\alpha}$ of the curve α is an integral curve of the geodesic spray iff α is an geodesic on M . Çalışkan at al. studied the natural lift curves of the spherical indicatries of tangent, principal normal, binormal vectors and fixed centrode of a curve in [12]. They gave some interesting results about the original curve, depending on the assumption that the natural lift curve should be the integral curve of the geodesic spray on the tangent bundle $T\left(S^2\right)$. Some properties of $\overline{M}$ -vector field Z defined on a hypersurface M of $\overline{M}$ were studied by Agashe in [1]. $\overline{M}$ -integral curve of Z and $\overline{M}$ -geodesic spray are defined by Çalışkan and Sivridağ. They gave the main theorem: The natural lift $\overline{\alpha}$ of the curve α (in $\overline{M}$) is an $\overline{M}$ -integral curve of the geodesic spray Z iff α is an $\overline{M}$ -geodesic in [5]. Bilici et al. have proposed the natural lift curves and the geodesic sprays for the spherical indicatrices of the the involute evolute curve couple in Euclidean 3-space. They gave some interesting results about the evolute curve, depending on the assumption that the natural lift curve of the spherical indicatrices of the involute should be the integral curve on the tangent bundle $T\left(S^2\right)$ in [3]. In differential geometry, especially the theory of space curve, the Darboux vector is the areal velocity vector of the Frenet frame of a spacere curve. It is named after Gaston Darboux who discovered it. In term of the Frenet-Serret apparatus, the darboux vector W can be expressed as $W = \tau T + \kappa B$, details are given in Lambert et al. in [8].

Let $\alpha : I \to \mathrm{IR}^3$ be a parametrized curve. We denote by $\{T(s), N(s), B(s)\}$ the moving Frenet frame along the curve α , where T, N and B are the tangent, the principal normal and the binormal vector fields of the curve α , respectively.

Let α be a reguler curve in IR^3. Then

$$T = \frac{\alpha'}{\left\|\alpha'\right\|},\ N = B \times T,\ B = \frac{\alpha' \times \alpha''}{\left\|\alpha' \times \alpha''\right\|},$$

$$\kappa = \frac{\alpha' \times \alpha''}{\left\|\alpha'\right\|^3},\ \tau = \frac{\det\left(\alpha', \alpha'', \alpha'''\right)}{\left\|\alpha' \times \alpha''\right\|^2},[9].$$

If α is a unit speed curve,then

$$T = \alpha',\ N = \frac{\alpha''}{\left\|\alpha''\right\|},\ B = T \times N,$$

$$\kappa = \left\langle T', N \right\rangle,\ \tau = \left\langle N', B \right\rangle,[9].$$

Let α be a unit speed space curve with curvature κ and torsion τ. Let Frenet vector fields of α be $\{T, N, B\}$. Then, Frenet formulas are given by

$$T' = \kappa N,\ N' = -\kappa T + \tau B,\ B' = -\tau N,[10].$$

For any unit speed curve $\alpha : I \to \mathrm{IR}^3$, we call $W(s) = \tau T(s) + \kappa B(s)$ the Darboux vector field of α , [10].

A ruled surface is generated by a one-parameter family of straight lines and it possesses a parametric representation

$$X(s, v) = \alpha(s) + v e(s),[9,11].$$

where $\alpha(s)$ represents a space curve which is called the base curve and e is a unit vector representing the direction of a straight line.

The striction point on a ruled surface X is the foot of the common normal between two consecutive generators (or ruling). The set of striction points defines the striction curve given as

$$\beta(s)=\alpha(s)-\frac{\langle \alpha^{'},e^{'}\rangle}{\langle e^{'},e^{'}\rangle}e(s).[9,11].$$

The distribution parameter of the ruled surface X is defined by

$$P_e=\frac{\det\left(\alpha^{'},e,e^{'}\right)}{\left\|e^{'}\right\|^2}[9,11].$$

The ruled surface is developable if and only if $P_e=0,[9,11]$.

Ruled Surface Pair Generated By a Curve and Its Natural Lift in IR^3

Let M be a hypersurface in IR^3 and let $\alpha : I\to M$ be a parametrized curve. α is called an integral curve of X if

$$\frac{d}{ds}\left(\alpha(s)\right)=X\left(\alpha(s)\right)\ (\text{for all } t\in I),[10].$$

where X is a smooth tangent vector field on M . We have

$$TM=\bigcup_{P\in M}T_PM=\chi(M)$$

where T_PM is the tangent space of M at P and $\chi(M)$ is the space of vector fields on M .

For any parametrized curve $\alpha : I\to M$, $\overline{\alpha} : I\to TM$ given by

$$\overline{\alpha}(s)=\left(\alpha(s),\alpha^{'}(s)\right)=\alpha^{'}(s)|_{\alpha(s)},[13]$$

is called the natural lift of α on TM. Thus, we can write

$$\frac{d\overline{\alpha}}{ds}=\frac{d}{ds}\left(\alpha^{'}(s)|_{\alpha(s)}\right)=D_{\alpha^{'}(s)}\alpha^{'}(s)$$

where D is the Levi-Civita connection on IR^3 .

Corollary 1: *Let α be a unit speed space curve and $\overline{\alpha}$ be the natural lift of α Then*

$$\overline{T}(s)=N(s),\ \overline{N}(s)=-\frac{\kappa(s)}{\|W\|}T(s)+\frac{\tau(s)}{\|W\|}B(s)\ ,\ \overline{B}(s)=\frac{\tau(s)}{\|W\|}T(s)-\frac{\kappa(s)}{\|W\|}B(s)\ ,[6].$$

Corollary 2: *Let α be a unit speed space curve and the natural lift $\overline{\alpha}$ of the curve α be a space curve with curvature $\overline{\kappa}$ and torsion $\overline{\tau}$. Then*

$$\overline{\kappa}(s)=\frac{\|W\|}{\kappa(s)},\overline{\tau}(s)=\frac{\kappa(s)\tau^{'}(s)-\kappa^{'}(s)\tau(s)}{\kappa(s)\|W\|^2}.$$

Corollary 3: *Let α be a unit speed space curve and the natural lift $\overline{\alpha}$ of the curve α be a space curve,then*

$$\overline{W}=\frac{\tau(s)}{\kappa(s)}T+\left(\frac{\kappa(s)\tau^{'}(s)-\kappa^{'}(s)\tau(s)}{\kappa(s)\|W\|^2}\right)N+B$$

Corollary 4: *Let X and $\overline{X}$ be two ruled surfaces which is given by $X(s,v)=\alpha(s)+vC(s)$, $\overline{X}(s,v)=\overline{\alpha}(s)+v\overline{C}(s)$. The striction curves of X and $\overline{X}$ are given by $\beta(s)=\alpha(s)-\lambda C(s)$ and $\overline{\beta}(s)=\overline{\alpha}(s)-\mu\overline{C}(s)$,respectively. Then we obtain*

$$\lambda=\frac{\tau'(s)}{\left[\kappa'(s)\right]^2+\left[\tau'(s)\right]^2}\|W\|,\ \mu=\frac{\|\overline{W}\|\kappa(s)\sigma'(s)}{\left[\frac{\kappa(s)\tau'(s)-\kappa'(s)\tau(s)}{\kappa(s)^2}\right]^2+\left[\sigma'(s)\right]^2+\left[\sigma(s)\tau(s)\right]^2}$$

where $\sigma(s)=\dfrac{\kappa(s)\tau'(s)-\kappa'(s)\tau(s)}{\kappa(s)\|W\|^2}$.

Corollary 5: *Let X and $\overline{X}$ be two ruled surfaces which is given by $X(s,v)=\alpha(s)+vC(s)$, $\overline{X}(s,v)=\overline{\alpha}(s)+v\overline{C}(s)$. The distribution parameters of the ruled surfaces X and $\overline{X}$ are defined by* $P_C=\dfrac{\det\left(\alpha',C,C'\right)}{\|C'\|^2}$ *and* $\overline{P}_{\overline{C}}=\dfrac{\det\left(\overline{\alpha}',\overline{C},\overline{C}'\right)}{\|\overline{C}'\|^2}$ *. Then we have*

$$P_C=0,$$

$$\overline{P}_{\overline{C}}=\frac{\kappa(s)\left(\left[\frac{\kappa(s)\tau'(s)-\kappa'(s)\tau(s)}{\kappa(s)^2}-\kappa(s)\sigma(s)\right]-\kappa(s)\sigma(s)\right)-\sigma(s)\tau(s)^2}{\left[\frac{\kappa(s)\tau'(s)-\kappa'(s)\tau(s)}{\kappa(s)^2}-\kappa(s)\sigma(s)\right]^2+\left[\sigma'(s)\right]^2+\left[\sigma(s)\tau(s)\right]^2}$$

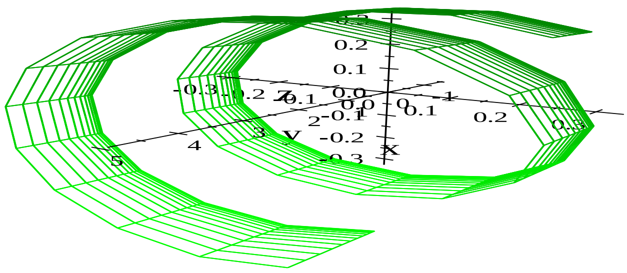
Example 1: *Let* $\alpha(s)=\left(\frac{\sqrt{6}}{3}s,\frac{1}{3}\cos\left(\sqrt{3}s\right),\frac{1}{3}\sin\left(\sqrt{3}s\right)\right)$ *be a unit speed with;*

$$T(s)=\left(\frac{\sqrt{6}}{3},-\frac{\sqrt{3}}{3}\sin\left(\sqrt{3}s\right),\frac{\sqrt{3}}{3}\cos\left(\sqrt{3}s\right)\right),$$

$$N(s)=\left(0,-\cos\left(\sqrt{3}s\right),-\sin\left(\sqrt{3}s\right)\right),$$

$$B(s)=\left(\frac{\sqrt{3}}{3},\frac{\sqrt{6}}{3}\sin\left(\sqrt{3}s\right),-\frac{\sqrt{6}}{3}\cos\left(\sqrt{3}s\right)\right)\text{ and }\kappa=1,\tau=\sqrt{2}$$

$$C(s)=(1,0,0)$$



$$X_C(s,t)=\left(\frac{\sqrt{6}}{3}s+\cosh t,\frac{1}{3}\cos\left(\sqrt{3}s\right),\frac{1}{3}\sin\left(\sqrt{3}s\right)\right)$$

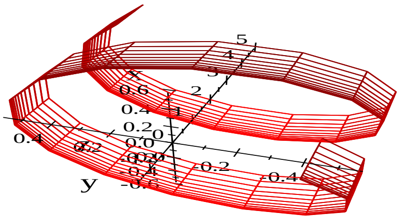
$\bar{\alpha}(s)=\left(\frac{\sqrt{6}}{3},-\frac{\sqrt{3}}{3}\sin\left(\sqrt{3}s\right),\frac{\sqrt{3}}{3}\cos\left(\sqrt{3}s\right)\right)$ be the natural lift $\bar{\alpha}$ of the curve α with;

$$\bar{T}(s)=\left(0,-\cos\left(\sqrt{3}s\right),-\sin\left(\sqrt{3}s\right)\right),$$

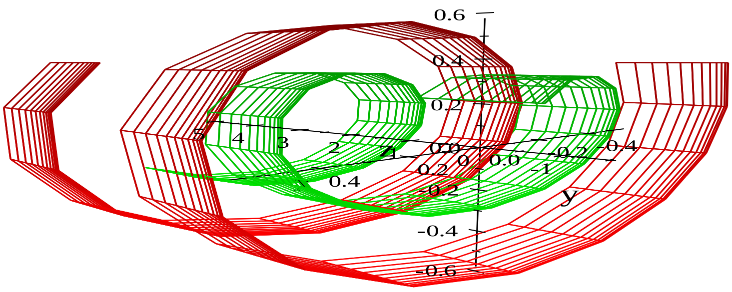
$$\bar{N}(s)=\left(0,\frac{\sqrt{6}+\sqrt{3}}{3}\sin\left(\sqrt{3}s\right),-\frac{\sqrt{6}+\sqrt{3}}{3}\cos\left(\sqrt{3}s\right)\right),$$

$$\bar{B}(s)=\left(\frac{\sqrt{3}}{3},-\frac{2\sqrt{6}}{3}\sin\left(\sqrt{3}s\right),\frac{2\sqrt{6}}{3}\cos\left(\sqrt{3}s\right)\right) \text{ and } \bar{\kappa}=1,\bar{\tau}=0$$

$$\bar{C}(s)=(1,0,0)$$



$$\bar{X}_{\bar{C}}(s,t)=\left(\frac{\sqrt{6}}{3}s+t,-\frac{\sqrt{3}}{3}\sin\left(\sqrt{3}s\right),\frac{\sqrt{3}}{3}\cos\left(\sqrt{3}s\right)\right)$$



$X_C(s,t)$ *and* $\bar{X}_{\bar{C}}(s,t)$

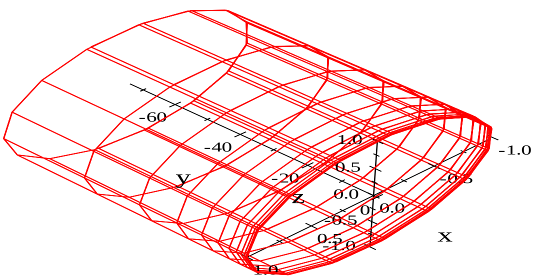
Example 2: *Let* $\alpha(s)=(\cos(s),0,\sin(s))$ *be a unit speed with;*

$$T(s)=(-\sin(s),0,\cos(s)),$$

$$N(s)=(-\cos(s),0,-\sin(s)),$$

$$B(s)=(0,-1,0) \text{ and } \kappa=1,\tau=0$$

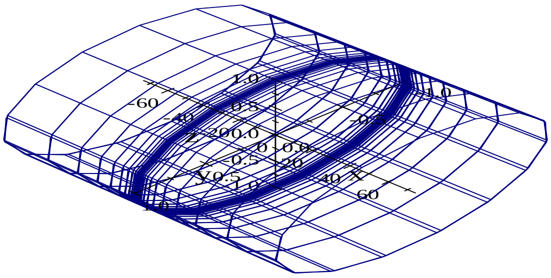
$$C(s)=(0,-1,0)$$



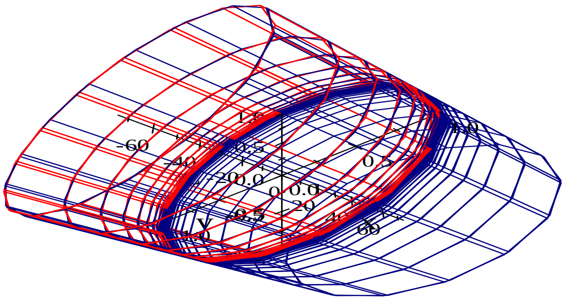
$$X_C(s,t)=(\cos(s),-\cosh t,\sin(s))$$

$\bar{\alpha}(s)=(-\sin(s),0,\cos(s))$ be the natural lift $\bar{\alpha}$ of the curve α with;

$$\bar{T}(s)=\left(0,-\cos\left(\sqrt{3}s\right),-\sin\left(\sqrt{3}s\right)\right),$$
$$\bar{N}(s)=(-\cos(s),0,-\sin(s)),$$
$$\bar{B}(s)=(0,-1,0) \text{ and } \bar{\kappa}=1,\bar{\tau}=0$$
$$\bar{C}(s)=(0,-1,0)$$



$$\bar{X}_{\bar{C}}(s,t)=(-\sin(s),-\sinh t,\cos(s))$$



$$X_C(s,t)\, and\, \bar{X}_{\bar{C}}(s,t)$$

References

[1]. Agashe, N. S., Curves associated with an $\overline{M}$ -vector field on a hypersurfaceM of a Riemmanian manifold M, Tensor, N.S., 28 (1974), 117-122.

[2]. Akutagawa, K., Nishikawa, S.,The Gauss Map and Spacelike Surfacewith Prescribed Mean Curvature in Minkowski3-Space,Töhoko Math., J.,42, 67-82,(1990)

[3]. Bilici M., Çalışkan M. and Aydemir İ., The natural lift curves and the geodesic sprays for the spherical indicatrices of the pair of evolute-involute curves, International Journal of Applied Mathematics, Vol.11,No.4(2002),415-420,

[4]. Çalışkan, M., Sivridağ, A.İ., Hacsalihoğlu, H.H, Some Characterizationsfor the natural lift curves and the geodesic spray, Communications, Fac. Sci.Univ. Ankara Ser. A Math. 33 (1984), Num. 28,235-242.

[5]. Çalışkan, M., Ergün, E.,On The $\overline{M}$ -Integral Curves and $\overline{M}$ -Geodesic Sprays In Minkowski 3-Space International Journal of Contemp. Math. Sciences,Vol. 6, no. 39, (2011), 1935-1939.

[6]. E.Ergün,M.Çalışkan, Ruled Surface Pair Generated by a Curve and its Natural Lift In IR^3 , Pure Mathematical Sciences Vol.1,No.2 (2012),75-80.

[7]. E.Ergün,M.Çalışkan, Ruled Surface Pair Generated by a Curve and its Natural Lift In IR_1^3, J. Math. Comput. 2 (2012), No. 5; 1387-1400.

[8]. Lambert MS, Mariam TT, Susan FH (2010).Darboux Vector. VDM Publishing House.
[9]. M.P. Do Carmo, Differential Geometry of Curve and Surface, Prentice-Hall Inc. Englewood Cliffs, New Jersey,1976.
[10]. O'Neill, B. Elementary Differential Geometry, Academic Press, New York and London,
[11]. (1967).
[12]. Orbay K. Kasap E. Aydemir İ. Mannheim Offsets of Ruled Surfaces, Mathematical Problems in EngineringVolume 2009, Article ID 160917, 9 pages, doi:10.1155/2009/160917.
[13]. Sivridağ A.İ. Çalışkan M. On the $\overline{M}$ -Integral Curves and $\overline{M}$ -Geodesic Sprays Erc.Uni. Fen Bil. Derg. 7, 2, (1991), 1283-1287.
[14]. Thorpe, J.A., Elementary Topics In Differential Geometry,Springer-Verlag,New York, Heidelberg-Berlin, (1979).