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ON A SUBCLASS OF MULTIVALENT FUNCTIONS DEFINED BY AL-OBOUDI OPERATOR

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ABSTRACT

In the present paper , a subclass of analytic and multivalent function is defined by Al-Oboudi Operator and we have obtained among other results like, Coefficient stimates , Growth and distortion theorem , external properties for the classes $T_n S_p^{\lambda}(\alpha,\beta,\xi,\gamma,\delta,A,B)$ and $T_n V^{\lambda}(\alpha,\beta,\xi,\gamma,\delta,A,B)$.

KEYWORDS : Al-Oboudi Operator, Starlike Functions, Analytic Function, Multivalent Function

1. INTRODUCTION

Let S denote the class of function of the form

$$f(z) = z^p + \sum_{n=2}^{\infty} a_{n+p-1} z^{n+p-1} \qquad (1)$$

That are analytic and multivalent in the disk $|z| < 1$ for $0 \le \alpha < 1$, $S^*(\alpha)$ and $K(\alpha)$ denotes the subfamily of S consisting of the functions Starlike of order α and Convex of order α respectively .

The subfamily T of S consists of functions of the form

$$f(z) = z^p - \sum_{n=2}^{\infty} a_{n+p-1} z^{n+p-1} \quad a_n \ge 0 \ for\ n = 2,3,4,......z \in U \qquad (2)$$

Silverman [6] investigated function in the classes $T^*(\alpha) = T \cap S^*(\alpha)$ and $C(\alpha) = T \cap K(\alpha)$

Let $n \in N$ and $\lambda \ge 0$ denote by D_λ^n the Al-Oboudi Operator [3] defined by

$$D_\lambda^n : A \to A$$

$$D_\lambda^0 f(z) = f(z)$$

$$D_\lambda^1 f(z) = (1-\lambda) f(z) + \lambda z f'(z) = D_\lambda f(z)$$

$$D_\lambda^n f(z) = D_\lambda[D_\lambda^{n-1} f(z)]$$

Note that for f(z) is given by (1),

$$D_\lambda^n f(z) = z^p + \sum^{\infty} [1 + (n+p-2)\lambda]^\beta a_{n+p-1} z^{n+p-1}$$

When $\lambda = 1$

D_λ^n is the Salagean differential operator D_λ^n: $A \to A$, $n \in N$ defined as

$$D^0 f(z) = f(z)$$

$$D^1 f(z) = Df(z) = zf'(z)$$

$$D^n f(z) = D\,[D^{n-1} f(z)]$$

Definition 1 : Let$\beta, \lambda \in R, \beta \geq 0, \lambda \geq 0$ and

$$f(z) = z^p + \sum_{n=2}^{\infty} a_{n+p-1} z^{n+p-1}$$

We denote by D_λ^β the linear operator defined by D_λ^β: $A \to A$

$$D_\lambda^\beta f(z) = z^p + \sum_{n=2}^{\infty} [1 + (n+p-2)\lambda]^\beta a_{n+p-1} z^{n+p-1}$$

Remark1.1 If $f(z) \in T$,

$$f(z) = z^p - \sum_{n=2}^{\infty} a_{n+p-1} z^{n+p-1}$$

, $a_{n+p-1} \geq 0, n+p-2 = 2, 3, 4 ... z \in U$

then

$$D_\lambda^\beta f(z) = z^p + \sum_{n=2}^{\infty} [1 + (n+p-2)\lambda]^\beta a_{n+p-1} z^{n+p-1}$$

In this paper using the operator D_λ^n we introduce the classes $T_n S_p^\lambda(\alpha, \beta, \xi, \gamma, \delta, A, B)$) and $T_n V^\lambda(\alpha, \beta, \xi, \gamma, \delta, A, B)$ and obtained coefficient estimates for these classes.When the functions have negative coefficient. We also obtain growth, and distortion theorems, closure theorem for function in these classes .

Deffinition 2 : We say that a function f(z) ∊ T is in the class $T_n S_p^\lambda(\alpha, \beta, \xi, \gamma, \delta, A, B)$ if and only if

$$\left| \frac{\frac{D_\lambda^{\beta+1} f(z)}{D_\lambda^\beta} - 1}{(B-A)\xi(\frac{D_\lambda^{\beta+1} f(z)}{D_\lambda^\beta} - 1) + A\gamma(\frac{D_\lambda^{\beta+1} f(z)}{D_\lambda^\beta} - 1)} \right| < \delta$$

Where $|z| < 1$, $0 < \delta \leq 1, 1/2 \leq \xi \leq 1, \lambda \geq 0, 0 \leq \alpha \leq 1/2\xi, 1/2 < \gamma \leq 1, \beta \geq 0, 0 < B \leq 1, -1 \leq A < B \leq 1$.

Definition 3 : A function f f(z) ∊ T is said to belong to the class $T_n S_p^\lambda(\alpha, \beta, \xi, \gamma, \delta, A, B)$ if and only if

$$\left| \frac{\frac{D_\lambda^{\beta+2} f(z)}{D_\lambda^{\beta+1}} - 1}{(B-A)\xi(\frac{D_\lambda^{\beta+2} f(z)}{D_\lambda^{\beta+1}} - 1) + A\gamma(\frac{D_\lambda^{\beta+2} f(z)}{D_\lambda^{\beta+1}} - 1)} \right| < \delta$$

Where $|z| < 1$, $0 < \delta \leq 1, 1/2 \leq \xi \leq 1, \lambda \geq 0, 0 \leq \alpha \leq 1/2\xi, 1/2 < \gamma \leq 1, \beta \geq 0, 0 < B \leq 1, -1 \leq A < B \leq 1$.

If we replace $\beta = 0$, $\lambda = 1$ we obtain the corresponding results of S.M.Khairnar and Meena More [4]. If we replace $\beta = 0$, $\lambda = 1$ and $\gamma = 1$ we obtain the results of Aghalary and Kulkarni [2] and Silvarman and Silvia [7] . If we replace $\beta = 0$, $\gamma = 1$ and $\xi = 1$ we obtain the corresponding results of [9] .

2. MAIN RESULTS COEFFICIENT ESTIMATES

Theorem 2.1 : A function

$$f(z) = z^p - \sum_{n=2}^{\infty} a_{n+p-1} z^{n+p-1}$$

($a_n \geq 0$) is in $T_n S_p^{\lambda}(\alpha, \beta, \xi, \gamma, \delta, A, B)$ if and only if

$$\sum_{n=2}^{\infty} [1+(n+p-2)\lambda]^{\beta} a_{n+p-2} [(n+p-2)\lambda\{1+A\delta\gamma+(B-A)\delta\xi\}+(B-A)\delta\xi(1-\alpha)] \leq (B-A)\delta\xi(1-\alpha)[1+(p-1)\lambda]$$

Proof : Suppose

$$\sum_{n=2}^{\infty} [1+(n+p-2)\lambda]^{\beta} a_{n+p-2} [(n+p-2)\lambda\{1+A\delta\gamma+(B-A)\delta\xi\}+(B-A)\delta\xi(1-\alpha)] \leq (B-A)\delta\xi(1-\alpha)[1+(p-1)\lambda]$$

We have

$$\left|D_{\lambda}^{\beta+1} f(z) - D_{\lambda}^{\beta} f(z)\right| - \delta \left|(B-A)\xi[D_{\lambda}^{\beta+1} f(z) - \alpha D_{\lambda}^{\beta} f(z)] + A\gamma[D_{\lambda}^{\beta+1} f(z) - D_{\lambda}^{\beta} f(z)]\right| < 0$$

With the provision,

$$\begin{aligned} &\Bigg| z^p (1+(p-1)\lambda) - \sum_{n=2}^{\infty} [1+(n+p-2)\lambda]^{\beta+1} a_{n+p-1} z^{n+p-1} - z^p[(1+(p-1)\lambda \\ &+ \sum_{n=2}^{\infty} [1+(n+p-2)\lambda]^{\beta} a_{n+p-1} z^{n+p-1} \Bigg| - \delta \,|(B-A)\xi[z^p (1+(p-1)\lambda) \\ &- \sum_{n=2}^{\infty} [1+(n+p-2)\lambda]^{\beta+1} a_{n+p-1} z^{n+p-1} - \alpha z^p[(1+(p-1)\lambda] \\ &+\alpha \sum_{n=2}^{\infty} [1+(n+p-2)\lambda]^{\beta} a_{n+p-1} z^{n+p-1}] + A\gamma[z^p (1+(p-1)\lambda) \\ &- \sum_{n=2}^{\infty} [1+(n+p-2)\lambda]^{\beta+1} a_{n+p-1} z^{n+p-1} - z^p[(1+(p-1)\lambda] \\ &+ \sum_{n=2}^{\infty} [1+(n+p-2)\lambda]^{\beta} a_{n+p-1} z^{n+p-1}] \Bigg| < 0 \end{aligned}$$

$$\begin{aligned} \Rightarrow \sum_{n=2}^{\infty} [1+(n+p-2)\lambda]^{\beta} a_{n+p-1} [&-1+(1+(n+p-2)\lambda)+(B-A)\xi\delta(1+(n+p-2)\lambda) \\ &-(B-A)\delta\alpha\xi + A\delta\gamma(1+(n+p-2)\lambda) - A\gamma\delta] < [(B-A)\delta\xi(1-\alpha)(1+(p-1)\lambda)] \end{aligned}$$

$$\Rightarrow \sum_{n=2}^{\infty} [1+(n+p-2)\lambda]^{\beta} a_{n+p-1} [(n+p-2)\lambda\,\{1+A\gamma\delta+(B-A)\delta\xi\}+(B-A)\delta\xi(1-\alpha)] \leq [(B-A)\delta\xi(1-\alpha)(1+(p-1)\lambda]$$

$\Rightarrow$ For $|z| = r < 1$ It is bounded above by

$$\sum_{n=2}^{\infty}[1+(n+p-2)\lambda]^{\beta}a_{n+p-1}r^{n+p-1}[(n+p-2)\lambda\{1+A\delta\gamma+(B-A)\delta\xi\}+(B-A)\delta\xi(1-\alpha)]$$
$$\leq (B-A)\delta\xi(1-\alpha)[1+(p-1)\lambda]$$

Hence

$f(z)\in T_nS_p^{\lambda}(\alpha,\beta,\xi,\gamma,\delta,A,B)$

Now we prove the converse result

Let

$$\left|\frac{\frac{D_\lambda^{\beta+1}f(z)}{D_\lambda^{\beta}}-1}{(B-A)\xi\left(\frac{D_\lambda^{\beta+1}f(z)}{D_\lambda^{\beta}}-1\right)+A\gamma\left(\frac{D_\lambda^{\beta+1}f(z)}{D_\lambda^{\beta}}-1\right)}\right|<\delta$$

$$\Rightarrow\left|\frac{\frac{z^p(1+(p-1)\lambda)-\sum_{n=2}^{\infty}[1+(n+p-2)\lambda]^{\beta+1}a_{n+p-1}z^{n+p-1}}{z^p(1+(p-1)\lambda)-\sum_{n=2}^{\infty}[1+(n+p-2)\lambda]^{\beta}a_{n+p-1}z^{n+p-1}}-1}{\begin{array}{l}(B-A)\xi\left[\frac{z^p(1+(p-1)\lambda)-\sum_{n=2}^{\infty}[1+(n+p-2)\lambda]^{\beta+1}a_{n+p-1}z^{n+p-1}}{z^p(1+(p-1)\lambda)-\sum_{n=2}^{\infty}[1+(n+p-2)\lambda]^{\beta}a_{n+p-1}z^{n+p-1}}-\alpha\right]\\+A\gamma\left[\frac{z^p(1+(p-1)\lambda)-\sum_{n=2}^{\infty}[1+(n+p-2)\lambda]^{\beta+1}a_{n+p-1}z^{n+p-1}}{z^p(1+(p-1)\lambda)-\sum_{n=2}^{\infty}[1+(n+p-2)\lambda]^{\beta}a_{n+p-1}z^{n+p-1}}-1\right]\end{array}}\right|<\delta$$

$$\Rightarrow\left|\frac{[-(n+p-2)\lambda]\sum_{n=2}^{\infty}[1+(n+p-2)\lambda]^{\beta}a_{n+p-1}z^{n+p-1}}{\begin{array}{l}(B-A)\xi z^p[1+(p-1)\lambda](1-\alpha)-\sum_{n=2}^{\infty}[1+(n+p-2)\lambda]^{\beta}(B-A)\xi a_{n+p-1}z^{n+p-1}[1\\+(n+p-2)\lambda-\alpha+A\gamma+A\gamma\lambda(n+p-2)-A\gamma]\end{array}}\right|<\delta$$

As $|Ref(z)|\leq|z|$ for all z We have

$$\Rightarrow Re\left|\frac{[-(n+p-2)\lambda]\sum_{n=2}^{\infty}[1+(n+p-2)\lambda]^{\beta}a_{n+p-1}z^{n+p-1}}{\begin{array}{l}(B-A)\xi z^p[1+(p-1)\lambda](1-\alpha)-\sum_{n=2}^{\infty}[1+(n+p-2)\lambda]^{\beta}a_{n+p-1}z^{n+p-1}[(B-A)\xi\\(1-\alpha)+(n+p-2)\lambda\{(B-A)\xi+A\gamma\}]\end{array}}\right|<\delta$$

We close value of *z* on real axis such that $\left(\frac{D_\lambda^{\beta+1}}{D_\lambda^{\beta}}\right)$ is real and clearing the denominator of above expression and allowing z→1 through real values .

We obtain

$$\Rightarrow[-(n+p-2)\lambda]\sum_{n=2}^{\infty}[1+(n+p-2)\lambda]^{\beta}a_{n+p-1}\leq(B-A)\xi\delta[1+(p-1)\lambda](1-\alpha)$$
$$-\sum_{n=2}^{\infty}[1+(n+p-2)\lambda]^{\beta}a_{n+p-1}\delta[(B-A)\xi(1-\alpha)+(n+p-2)\lambda\{(B-A)\xi+A\gamma\}]$$

$$\Rightarrow [-(n+p-2)\lambda]\sum_{n=2}^{\infty}[1+(n+p-2)\lambda]^{\beta}a_{n+p-1}+\sum_{n=2}^{\infty}[1+(n+p-2)\lambda]^{\beta}a_{n+p-1}\delta[(B-A)\xi(1-\alpha)$$

$$+(n+p-2)\lambda\{(B-A)\xi+A\gamma\}]\leq (B-A)\xi\delta[1+(p-1)\lambda](1-\alpha)$$

$$\Rightarrow \sum_{n=2}^{\infty}[1+(n+p-2)\lambda]^{\beta}a_{n+p-1}[(n+p-2)\lambda\{1+A\delta\gamma+(B-A)\delta\xi\}+(B-A)\delta\xi(1-\alpha)]$$

$$-[(B-A)\delta\xi(1-\alpha)\{1+(p-1)\lambda\}]\leq 0$$

Remark- 2.1 : If $f(z)\in T_nS_p^{\lambda}(\alpha,\beta,\xi,\gamma,\delta,A,B)$

Then

$$a_{n+p-1}\leq\left[\frac{[(B-A)\delta\xi(1-\alpha)\{1+(p-1)\lambda\}]}{[1+(n+p-2)\lambda]^{\beta}[(n+p-2)\lambda\{1+A\delta\gamma+(B-A)\delta\xi\}+(B-A)\delta\xi(1-\alpha)]}\right]$$

and equality holds for

$$f(z)=z^p-\left[\frac{[(B-A)\delta\xi(1-\alpha)\{1+(p-1)\lambda\}]}{[1+(n+p-2)\lambda]^{\beta}[(n+p-2)\lambda\{1+A\delta\gamma+(B-A)\delta\xi\}+(B-A)\delta\xi(1-\alpha)]}\right]z^{n+p-1}$$

Corollary 2.1 :

If $f(z)\in T_nS_p^1\ (\alpha,\beta,\xi,\gamma,\delta,-1,1)\ (In\ particular\ if\ A=-1,\ B=1)$

Then We get ,

$$a_{n+p-1}\leq\left[\frac{[2\delta\xi(1-\alpha)p]}{[1+(n+p-2)\lambda]^{\beta}[(n+p-2)\lambda\{1-\delta\gamma+2\delta\xi\}+2\delta\xi(1-\alpha)]}\right]$$

and equality holds for

$$f(z)=z^p-\left[\frac{[2\delta\xi(1-\alpha)p]}{[1+(n+p-2)\lambda]^{\beta}[(n+p-2)\lambda\{1-\delta\gamma+2\delta\xi\}+2\delta\xi(1-\alpha)]}\ z^{n+p-1}\right]$$

This corollary is due to [11] .

Corollary 2.2:

If $f(z)\in T_nS_p^1\ (\alpha,0,\xi,\gamma,\delta,-1,1)\ (In\ particular\ if\ \beta=0,\ \lambda=1\ A=-1,\ B=1)$

Then We get ,

$$a_{n+p-1}\leq\left[\frac{[2\delta\xi(1-\alpha)p]}{[(n+p-2)\{1-\delta\gamma+2\delta\xi\}+2\delta\xi(1-\alpha)]}\right]$$

$$a_{n+p-1}\leq\left[\frac{[2\delta\xi(1-\alpha)p]}{[(n+p-2)-\delta\{2\alpha\xi-2\xi(n+p-1)+\gamma(n+p-1)-\gamma\}]}\right]$$

and equality holds for

$$f(z)=z^p-\left[\frac{[2\delta\xi(1-\alpha)p]}{[(n+p-2)-\delta\{2\alpha\xi-2\xi(n+p-1)+\gamma(n+p-1)-\gamma\}]}\ z^{n+p-1}\right]$$

This corollary is due to [4] .

Corollary 2.3 :

If $f(z)\in T_nS_p^1\ (\alpha,0,\xi,1,\delta,-1,1)\ (In\ particular\ if\ \beta=0,\ \lambda=1,\gamma=1,\ A=-1,\ B=1)$

Then We get ,

$$a_{n+p-1}\leq\left[\frac{[2\delta\xi(1-\alpha)p]}{[(n+p-2)-\delta\{2\alpha\xi-2\xi(n+p-1)+(n+p-1)-1\}]}\right]$$

and equality holds for

$$f(z) = z^p - \left[\frac{[2\delta\xi(1-\alpha)p]}{[(n+p-2)-\delta\{2\alpha\xi-2\xi(n+p-1)+(n+p-2)\}]}\right] z^{n+p-1}$$

This corollary is due to [2] and [7].

Corollary 2.4 :

If $f(z) \in T_n S_p^1\ (\alpha, 0, 1, 1, \delta, -1, 1)$ $(In\ particular\ if\ \beta = 0,\ \lambda = 1, \gamma = 1, \xi = 1,\ A = -1,\ B = 1)$

Then We get ,

$$a_{n+p-1} \leq \left[\frac{[2\delta(1-\alpha)p]}{[(n+p-2)-\delta\{2\alpha-(n+p-1)-1\}]}\right]$$

and equality holds for

$$f(z) = z^p - \left[\frac{[2\delta(1-\alpha)p]}{[(n+p-2)-\delta\{2\alpha-(n+p-1)-1\}]}\right] z^{n+p-1}$$

This corollary is due to [9].

Corollary 2.5:

If $f(z) \in T_n S_n^1\ (\alpha, 0, 1, 1, 1, -1, 1)$ $(In\ particular\ if\ \beta = 0,\ \lambda = 1, \gamma = 1, \xi = 1, \delta = 1,\ A = -1,\ B = 1)$

Then We get ,

$$a_{n+p-1} \leq \left[\frac{[2(1-\alpha)p]}{[(n+p-2)-\{2\alpha-(n+p-1)-1\}]}\right]$$

$$a_{n+p-1} \leq \left[\frac{[2(1-\alpha)p]}{[n+p-2-2\alpha+n+p]}\right]$$

$$a_{n+p-1} \leq \left[\frac{[2(1-\alpha)p]}{[n+p-1-\alpha]}\right]$$

and equality holds for

$$f(z) = z^p - \left[\frac{[2(1-\alpha)p]}{[n+p-1-\alpha]} z^{n+p-1}\right]$$

Theorem 2.2 :

A function $f(z) = z^p - \sum_{n=2}^{\infty} a_{n+p-1} z^{n+p-1}$, ($a_n \geq 0$) is in $T_n S_p^\lambda(\alpha, \beta, \xi, \gamma, \delta, A, B)$

if and only if

$$\sum_{n=2}^{\infty} [1+(n+p-2)\lambda]^{\beta+1} a_{n+p-2}[(n+p-2)\lambda\{1+A\delta\gamma+(B-A)\delta\xi\}+(B-A)\delta\xi(1-\alpha)] \leq (B-A)\delta\xi(1-\alpha)[1+(p-1)\lambda]$$

Proof : The proof of this theorem is analogous to that of theorem 1 , because a function $f(z) \in T_n V^\lambda(\alpha, \beta, \xi, \gamma, \delta, A, B)$ if and only if $zf'(z) \in T_n S_p^\lambda(\alpha, \beta, \xi, \gamma, \delta, A, B)$ So it is enough that replacing β with β+ 1 in theorem 2.1.

Remark 2.2 : If $f(z) \in T_n V^\lambda(\alpha, \beta, \xi, \gamma, \delta, A, B)$ then

$$a_{n+p-1} \leq \left[\frac{[(B-A)\delta\xi(1-\alpha)]}{[1+(n+p-2)\lambda]^{\beta+1}[(n+p-2)\lambda\{1+A\gamma+(B-A)\delta\xi\}+(B-A)\delta\xi(1-\alpha)]}\right]$$

$$f(z)=z^p-\left[\frac{[(B-A)\delta\xi(1-\alpha)]}{[1+(n+p-2)\lambda]^{\beta+1}[(n+p-2)\lambda\{1+A\gamma+(B-A)\delta\xi\}+(B-A)\delta\xi(1-\alpha)]}\right]z^{n+p-1}$$

Corollary 2.6 :

If $f(z)\in T_nV^{\lambda}\ (\alpha,\beta,\xi,\gamma,\delta,-1,1)$ $(In\ particular\ if\ A=-1,\ B=1)$

Then We get ,

$$a_{n+p-1}\leq\left[\frac{[2\delta\xi(1-\alpha)]p}{[1+(n+p-2)\lambda]^{\beta+1}[(n+p-2)\lambda\{1-\delta\gamma+2\delta\xi\}+2\delta\xi(1-\alpha)]}\right]$$

and equality holds for

$$f(z)=z^p-\left[\frac{[2\delta\xi(1-\alpha)]p}{[1+(n+p-2)\lambda]^{\beta+1}[(n+p-2)\lambda\{1-\delta\gamma+2\delta\xi\}+2\delta\xi(1-\alpha)]}\right]z^{n+p-1}$$

This corollary is due to [11] .

Corollary 2.7:

If $f(z)\in T_nV^{1}\ (\alpha,0,\xi,\gamma,\delta,-1,1)$ $(In\ particular\ if\ \beta=0,\ \lambda=1\ A=-1,\ B=1)$

Then We get ,

$$a_{n+p-1}\leq\left[\frac{[2\delta\xi(1-\alpha)]p}{(n+p-1)[(n+p-2)-\delta\{2\alpha\xi-2\xi(n+p-1)+\gamma(n+p-1)-\gamma\}]}\right]$$

$$f(z)=z^p-\left[\frac{[2\delta\xi(1-\alpha)]p}{(n+p-1)[(n+p-2)-\delta\{2\alpha\xi-2\xi(n+p-1)+\gamma(n+p-1)-\gamma\}]}\right]z^{n+p-1}$$

This corollary is due to [4] .

Corollary 2.8:

If $f(z)\in T_nV^{1}\ (\alpha,0,\xi,1,\delta,-1,1)$ $(In\ particular\ if\ \beta=0,\ \lambda=1,\gamma=1,\ A=-1,\ B=1)$

Then We get ,

$$a_{n+p-1}\leq\left[\frac{[2\delta\xi(1-\alpha)]p}{(n+p-1)[(n+p-2)-\delta\{2\alpha\xi-2\xi(n+p-1)+(n+p-1)-1\}]}\right]$$

and equality holds for

$$f(z)=z^p-\left[\frac{[2\delta\xi(1-\alpha)]p}{(n+p-1)[(n+p-2)-\delta\{2\alpha\xi-2\xi(n+p-1)+(n+p-2)\}]}\right]z^{n+p-1}$$

This corollary is due to [2] and [7] .

Corollary 2.9 :

If $f(z)\in T_nV^{1}\ (\alpha,0,1,1,\delta,-1,1)$ $(In\ particular\ if\ \beta=0,\ \lambda=1,\gamma=1,\xi=1,\ A=-1,\ B=1)$

Then We get ,

$$a_{n+p-1}\leq\left[\frac{[2\delta(1-\alpha)]p}{(n+p-1)[(n+p-2)-\delta\{2\alpha-(n+p-1)-1\}]}\right]$$

and equality holds for

$$f(z)=z^p-\left[\frac{[2\delta(1-\alpha)]p}{(n+p-1)[(n+p-2)-\delta\{2\alpha-(n+p-1)-1\}]}\right]z^{n+p-1}$$

This corollary is due to [9].

Corollary 2.10 :
If $f(z) \in T_n V^1\ (\alpha, 0, 1, 1, 1, -1, 1)\ (In\ particular\ if\ \beta = 0,\ \lambda = 1, \gamma = 1, \xi = 1, \delta = 1,\ A = -1,\ B = 1)$
Then We get ,

$$a_{n+p-1} \le \left[\frac{[2(1-\alpha)]p}{(n+p-1)[(n+p-2) - \{2\alpha - (n+p-1) - 1\}]}\right]$$

$$a_{n+p-1} \le \left[\frac{[2(1-\alpha)]p}{(n+p-1)[n+p-2-2\alpha+n+p]}\right]$$

$$a_{n+p-1} \le \left[\frac{[(1-\alpha)]p}{(n+p-1)[n+p-1-\alpha]}\right]$$

and equality holds for

$$f(z) = z^p - \left[\frac{[(1-\alpha)]p}{(n+p-1)[n+p-1-\alpha]}\right] z^{n+p-1}$$

3 . GROWTH AND DISTORTION THEOREM

Theorem 3.1 :

If $f(z) \in T_n S_p^\lambda(\alpha, \beta, \xi, \gamma, \delta, A, B)$

Then

$$r^p - r^{p+1}\left[\frac{(B-A)\delta\xi(1-\alpha)}{(1+p\lambda)^\beta[p\lambda + (B-A)\xi\delta p\lambda + Ap\gamma\delta\lambda + (B-A)\xi\delta(1-\alpha)]}\right] \le |f(z)|$$
$$\le r^p + r^{p+1}\left[\frac{(B-A)\delta\xi(1-\alpha)}{(1+p\lambda)^\beta[p\lambda + (B-A)\xi\delta p\lambda + Ap\gamma\delta\lambda + (B-A)\xi\delta(1-\alpha)]}\right]$$

equality holds for

$$f(z) = z^p - \left[\frac{(B-A)\delta\xi(1-\alpha)}{(1+p\lambda)^\beta[p\lambda + (B-A)\xi\delta p\lambda + Ap\gamma\delta\lambda + (B-A)\xi\delta(1-\alpha)]}\right] z^{p+1}$$

Proof : By theorem 2.1 We have
$f(z) \in T_n S_p^\lambda(\alpha, \beta, \xi, \gamma, \delta, A, B)$ if and only if

$$\sum_{n=2}^{\infty}[1 + (n+p-2)\lambda]^\beta a_{n+p-2}[(n+p-2)\lambda\{1 + A\delta\gamma + (B-A)\delta\xi\} + (B-A)\delta\xi(1-\alpha)]$$
$$\le (B-A)\delta\xi(1-\alpha)[1 + (p-1)\lambda]$$

Let

$$t = 1 - \left[\frac{(B-A)\delta\xi(1-\alpha)}{\lambda - (B-A)\delta\xi\lambda + A\gamma\delta\lambda}\right]$$

$f(z) \in T_n S_p^\lambda(\alpha, \beta, \xi, \gamma, \delta, A, B)$ if and only if

$$\sum_{n=2}^{\infty}[1 + (n+p-2)\lambda]^\beta (n+p-1-t)\ a_{n+p-1} \le (1-t) \qquad (3)$$

Where $n + p - 1 = 2$

$$(1+p\lambda)^\beta(1+p-t)\sum_{n=2}^{\infty} a_{n+p-1} \le \sum_{n=2}^{\infty}[1 + (n+p-2)]^\beta a_{n+p-1}(n+p-1-t) \le (1-t)$$

$$|f(z)| \leq r^p + \sum_{n=2}^{\infty} a_{n+p-1} z^{n+p-1} \leq r^p + r^{p+1} \sum_{n=2}^{\infty} a_{n+p-1} \leq r^p + r^{p+1} \left[\frac{(1-t)}{(1+p\lambda)^{\beta}(1+p-t)} \right]$$

similarly

$$|f(z)| \geq r^p - \sum_{n=2}^{\infty} a_{n+p-1} z^{n+p-1} \geq r^p - r^{p+1} \sum_{n=2}^{\infty} a_{n+p-1} \geq r^p - r^{p+1} \left[\frac{(1-t)}{(1+p\lambda)^{\beta}(1+p-t)} \right]$$

so

$$r^p - r^{p+1} \left[\frac{(1-t)}{(1+p\lambda)^{\beta}(1+p-t)} \right] \leq |f(z)| \leq r^p + r^{p+1} \left[\frac{(1-t)}{(1+p\lambda)^{\beta}(1+p-t)} \right] \qquad (4)$$

$\Rightarrow \; Note:$

$$\left[\frac{(1-t)}{(1+p\lambda)^{\beta}(1+p-t)} \right] = \left[\frac{1-1+\dfrac{(B-A)\delta\xi(1-\alpha)}{\lambda+(B-A)\xi\delta\lambda+A\gamma\delta\lambda}}{(1+p\lambda)^{\beta}(1+p-t)} \right]$$

$$= \left[\frac{(B-A)\delta\xi(1-\alpha)}{(1+p\lambda)^{\beta}(1+p-t)[\lambda+(B-A)\xi\delta\lambda+A\gamma\delta\lambda]} \right]$$

$$= \left[\frac{(B-A)\delta\xi(1-\alpha)}{(1+p\lambda)^{\beta}[p+\dfrac{(B-A)\delta\xi(1-\alpha)}{\lambda+(B-A)\xi\delta\lambda+A\gamma\delta\lambda}][\lambda+(B-A)\xi\delta\lambda+A\gamma\delta\lambda]} \right]$$

$$= \left[\frac{(B-A)\delta\xi(1-\alpha)}{(1+p\lambda)^{\beta}[p\{\lambda+(B-A)\xi\delta\lambda+A\gamma\delta\lambda\}+(B-A)\delta\xi(1-\alpha)]} \right]$$

$$= \left[\frac{(B-A)\delta\xi(1-\alpha)}{(1+p\lambda)^{\beta}[p\lambda+(B-A)p\delta\lambda\xi+Ap\gamma\delta\lambda+(B-A)\delta\xi(1-\alpha)]} \right]$$

By using the above value in equation [2.4] We get
Hence the result

$$r^p - r^{p+1} \left[\frac{(B-A)\delta\xi(1-\alpha)}{(1+p\lambda)^{\beta}[p\lambda+(B-A)\xi\delta p\lambda+Ap\gamma\delta\lambda+(B-A)\xi\delta(1-\alpha)]} \right] \leq |f(z)|$$
$$\leq r^p + r^{p+1} \left[\frac{(B-A)\delta\xi(1-\alpha)}{(1+p\lambda)^{\beta}[p\lambda+(B-A)\xi\delta p\lambda+Ap\gamma\delta\lambda+(B-A)\xi\delta(1-\alpha)]} \right]$$

Corollary 3.1 :

If $f(z) \in T_n S_p^1(\alpha, 0, \xi, \gamma, \delta, A, B)$ $(i.e. replacing \beta = 0, \lambda = 1)$

Then We get

$$r^p - r^{p+1}\left[\frac{(B-A)\delta\xi(1-\alpha)}{p+(B-A)\delta\xi(p+1)+\delta\left\{Ap\gamma-(B-A)\xi\alpha\right\}}\right] \le |f(z)|$$
$$\le r^p + r^{p+1}\left[\frac{(B-A)\delta\xi(1-\alpha)}{p+(B-A)\delta\xi(p+1)+\delta\left\{Ap\gamma-(B-A)\xi\alpha\right\}}\right]$$

and equality holds

$$f(z) = z^p - \left[\frac{(B-A)\delta\xi(1-\alpha)}{p+(B-A)\delta\xi(p+1)+\delta\left\{Ap\gamma-(B-A)\xi\alpha\right\}}\right] z^{n+p-1}$$

This corollary is due to [4] .

Corollary 3.2 :

If $f(z) \in T_n S_p^1(\alpha, 0, \xi, 1, \delta, A, B)$ $(i.e. replacing \beta = 0, \lambda = 1, \gamma = 1)$

then we get

$$r^p - r^{p+1}\left[\frac{(B-A)\delta\xi(1-\alpha)}{p+(B-A)\delta\xi(p+1)+\delta\left\{Ap-(B-A)\xi\alpha\right\}}\right] \le |f(z)|$$
$$\le r^p + r^{p+1}\left[\frac{(B-A)\delta\xi(1-\alpha)}{p+(B-A)\delta\xi(p+1)+\delta\left\{Ap-(B-A)\xi\alpha\right\}}\right]$$

and equality holds

$$f(z) = z^p - \left[\frac{(B-A)\delta\xi(1-\alpha)}{p+(B-A)\delta\xi(p+1)+\delta\left\{Ap-(B-A)\xi\alpha\right\}}\right] z^{n+p-1}$$

This corollary is due to [2] and [7] .

Corollary 3.3 :

If $f(z) \in T_n S_p^1(\alpha, 0, 1, 1, \delta, A, B)$ $(i.e. replacing \beta = 0, \lambda = 1, \gamma = 1, \xi = 1)$

then we get

$$r^p - r^{p+1}\left[\frac{(B-A)\delta(1-\alpha)}{p+(B-A)\delta(p+1)+\delta\left\{Ap-(B-A)\alpha\right\}}\right] \le |f(z)|$$
$$\le r^p + r^{p+1}\left[\frac{(B-A)\delta(1-\alpha)}{p+(B-A)\delta(p+1)+\delta\left\{Ap-(B-A)\alpha\right\}}\right]$$

and equality holds for

$$f(z) = z^p - \left[\frac{(B-A)\delta(1-\alpha)}{p+(B-A)\delta(p+1)+\delta\left\{Ap-(B-A)\alpha\right\}}\right] z^{n+p-1}$$

This corollary is due to [9].

Corollary 3.4 :

If $f(z) \in T_n V^\lambda(\alpha, \beta, \xi, \gamma, \delta, A, B)$

then

$$r^p - r^{p+1}\left[\frac{(B-A)\delta\xi(1-\alpha)}{(1+p\lambda)^{\beta+1}[p\lambda+(B-A)\xi\delta p\lambda+Ap\gamma\delta\lambda+(B-A)\xi\delta(1-\alpha)]}\right] \le |f(z)|$$
$$\le r^p + r^{p+1}\left[\frac{(B-A)\delta\xi(1-\alpha)}{(1+p\lambda)^{\beta+1}[p\lambda+(B-A)\xi\delta p\lambda+Ap\gamma\delta\lambda+(B-A)\xi\delta(1-\alpha)]}\right]$$

Proof : The proof of this theorem is analogous to that of theorem 3.1 , because a function $f(z) \in T_nV^{\lambda}(\alpha,\beta,\xi,\gamma,\delta,A,B)$ if and if only $zf'(z) \in T_nS_p^{\lambda}(\alpha,\beta,\xi,\gamma,\delta,A,B)$ So it is enough that replacing βwith β+ 1 in theorem 3.1.

Corollary 3.4 : If $f(z) \in T_nV^1(\alpha,0,\xi,\gamma,\delta,A,B)$ $(i.e.replacing\beta=0,\lambda=1)$ then we get

$$r^p - r^{p+1}\left[\frac{(B-A)\delta\xi(1-\alpha)}{(1+p)[p+(B-A)\delta\xi(p+1)+\delta\{Ap\gamma-(B-A)\xi\alpha\}]}\right] \le |f(z)|$$
$$\le r^p + r^{p+1}\left[\frac{(B-A)\delta\xi(1-\alpha)}{(1+p)[p+(B-A)\delta\xi(p+1)+\delta\{Ap\gamma-(B-A)\xi\alpha\}]}\right]$$

and equality holds for

$$f(z) = z^p - \left[\frac{(B-A)\delta\xi(1-\alpha)}{(1+p)[p+(B-A)\delta\xi(p+1)+\delta\{Ap\gamma-(B-A)\xi\alpha\}]}\right] z^{n+p-1}$$

This corollary is due to [4] .

Corollary 3.5 :

If $f(z) \in T_nV^1(\alpha,0,\xi,1,\delta,A,B)$ $(i.e.replacing\beta=0,\lambda=1,\gamma=1)$

Then We get

$$r^p - r^{p+1}\left[\frac{(B-A)\delta\xi(1-\alpha)}{(1+p)[p+(B-A)\delta\xi(p+1)+\delta\{Ap-(B-A)\xi\alpha\}]}\right] \le |f(z)|$$
$$r^p + r^{p+1}\left[\frac{(B-A)\delta\xi(1-\alpha)}{(1+p)[p+(B-A)\delta\xi(p+1)+\delta\{Ap-(B-A)\xi\alpha\}]}\right]$$

and equality holds for

$$f(z) = z^p - \left[\frac{(B-A)\delta\xi(1-\alpha)}{(1+p)[p+(B-A)\delta\xi(p+1)+\delta\{Ap-(B-A)\xi\alpha\}]}\right] z^{n+p-1}$$

This corollary is due to [2] and [7] .

Corollary 3.6 :

If $f(z) \in T_nV^1(\alpha,0,1,1,\delta,A,B)$ $(i.e.replacing\beta=0,\lambda=1,\gamma=1,\xi=1)$

Then We get

$$r^p - r^{p+1}\left[\frac{(B-A)\delta(1-\alpha)}{(1+p)[p+(B-A)\delta(p+1)+\delta\{Ap-(B-A)\alpha\}]}\right] \le |f(z)|$$
$$r^p + r^{p+1}\left[\frac{(B-A)\delta(1-\alpha)}{(1+p)[p+(B-A)\delta(p+1)+\delta\{Ap-(B-A)\alpha\}]}\right]$$

and equality holds for

$$f(z) = z^p - \left[\frac{(B-A)\delta(1-\alpha)}{(1+p)[p+(B-A)\delta(p+1)+\delta\{Ap-(B-A)\alpha\}]}\right] z^{n+p-1}$$

This corollary is due to [9]

Theorem 3.3: If $f(z) \in T_n S_n^{\lambda}(\alpha, \beta, \xi, \gamma, \delta, A, B)$

Then

$$pr^{p-1} - r^p \left[\frac{(B-A)^2\delta\xi(1-\alpha)}{(1+p\lambda)^{\beta}\left\{p\lambda(1+A\gamma\delta)+(B-A)\delta\xi(1-\alpha+p\lambda)\right\}}\right] \le |f(z)|$$
$$\le pr^{p-1} + r^p \left[\frac{(B-A)^2\delta\xi(1-\alpha)}{(1+p\lambda)^{\beta}\left\{p\lambda(1+A\gamma\delta)+(B-A)\delta\xi(1-\alpha+p\lambda)\right\}}\right]$$

Proof : By theorem 3.1 We have

$f(z) \in T_n S_p^{\lambda}(\alpha, \beta, \xi, \gamma, \delta, A, B)$ if and only if

$\sum_{n=2}^{\infty}[1+(n+p-2)\lambda]^{\beta}(n+p-1-t)\ a_{n+p-1} \le (1-t)$

In view of theorem 3.1 We have

$$\sum_{n=2}^{\infty}(n+p-1)a_{n+p-1} = \sum_{n=2}^{\infty}(n+p-2)a_{n+p-1} + t\sum_{n=2}^{\infty}a_{n+p-1} \le \left[\frac{(B-A)(1-t)}{(1+p\lambda)^{\beta}(1+p-t)}\right] \quad (5)$$

$$\left|f'(z)\right| \le p|z|^{p-1} + \sum_{n=2}^{\infty}(n+p-1)a_{n+p-1}|z|^{n+p-2} \le pr^{p-1} + r^p\sum_{n=2}^{\infty}(n+p-1)a_{n+p-1}$$
$$\le pr^{p-1} + r^p\left[\frac{(B-A)(1-t)}{(1+p\lambda)^{\beta}(1+p-t)}\right]$$

Similarly

$$\left|f'(z)\right| \ge p|z|^{p-1} - \sum_{n=2}^{\infty}(n+p-1)a_{n+p-1}|z|^{n+p-2} \ge pr^{p-1} - r^p\sum_{n=2}^{\infty}(n+p-1)a_{n+p-1}$$
$$\ge pr^{p-1} - r^p\left[\frac{(B-A)(1-t)}{(1+p\lambda)^{\beta}(1+p-t)}\right]$$

By substituting the value of t in the above inequality We get

$$pr^{p-1} - r^p \left[\frac{(B-A)^2\delta\xi(1-\alpha)}{(1+p\lambda)^{\beta}\left\{p\lambda(1+A\gamma\delta)+(B-A)\delta\xi(1-\alpha+p\lambda)\right\}}\right] \le \left|f'(z)\right|$$
$$\le pr^{p-1} + r^p \left[\frac{(B-A)^2\delta\xi(1-\alpha)}{(1+p\lambda)^{\beta}\left\{p\lambda(1+A\gamma\delta)+(B-A)\delta\xi(1-\alpha+p\lambda)\right\}}\right]$$

Corollary 3.7 :

If $f(z) \in T_n S_p^{1}(\alpha, 0, \xi, \gamma, \delta, A, B)$ $(i.e.replacing \beta = 0, \lambda = 1)$

Then We get

$$pr^{p-1} - r^p \left[\frac{(B-A)^2\delta\xi(1-\alpha)}{\left\{p(1+A\gamma\delta)+(B-A)\delta\xi(1-\alpha+p)\right\}}\right] \le \left|f'(z)\right|$$
$$\le pr^{p-1} + r^p \left[\frac{(B-A)^2\delta\xi(1-\alpha)}{\left\{p\lambda(1+A\gamma\delta)+(B-A)\delta\xi(1-\alpha+p)\right\}}\right]$$

This corollary is due to [4].

Corollary 3.8 :

If $f(z) \in T_n S_p^{1}(\alpha, 0, \xi, 1, \delta, A, B)$ $(i.e.replacing \beta = 0, \lambda = 1, \gamma = 1)$

Then We get

$$pr^{p-1} - r^p \left[\frac{(B-A)^2 \delta \xi (1-\alpha)}{\{p(1+A\delta) + (B-A)\delta\xi(1-\alpha+p)\}} \right] \le \left| f'(z) \right| \le pr^{p-1} + r^p \left[\frac{(B-A)^2 \delta \xi (1-\alpha)}{\{p\lambda(1+A\delta) + (B-A)\delta\xi(1-\alpha+p)\}} \right]$$

This corollary is due to [2] and [7].

Corollary 3.9 : If $f(z) \in T_n S_p^1(\alpha, 0, 1, 1, \delta, A, B)$ $(i.e. replacing \beta = 0, \lambda = 1, \gamma = 1, \xi = 1)$

Then We get

$$pr^{p-1} - r^p \left[\frac{(B-A)^2 \delta (1-\alpha)}{\{p(1+A\delta) + (B-A)\delta(1-\alpha+p)\}} \right] \le \left| f'(z) \right| \le pr^{p-1} + r^p \left[\frac{(B-A)^2 \delta (1-\alpha)}{\{p\lambda(1+A\delta) + (B-A)\delta(1-\alpha+p)\}} \right]$$

This corollary is due to [9]

Theorem 3.4 :

If $f(z) \in T_n V^\lambda(\alpha, \beta, \xi, \gamma, \delta, A, B)$

Then

$$pr^{p-1} - r^p \left[\frac{(B-A)^2 \delta \xi (1-\alpha)}{(1+p\lambda)^{\beta+1} \{p\lambda(1+A\gamma\delta) + (B-A)\delta\xi(1-\alpha+p\lambda)\}} \right] \le |f(z)| \le pr^{p-1} + r^p \left[\frac{(B-A)^2 \delta \xi (1-\alpha)}{(1+p\lambda)^{\beta+1} \{p\lambda(1+A\gamma\delta) + (B-A)\delta\xi(1-\alpha+p\lambda)\}} \right]$$

Proof : The proof of this theorem is analogous to that of theorem 3.3 , because a function $f(z) \in T_n V^\lambda(\alpha, \beta, \xi, \gamma, \delta, A, B)$ if and only if $zf'(z) \in T_n S_p^\lambda(\alpha, \beta, \xi, \gamma, \delta, A, B)$. So it is enough that replacing β with $\beta + 1$ in theorem 3.3 .

Corollary 3.10 :

If $f(z) \in T_n V^1(\alpha, 0, \xi, \gamma, \delta, A, B)$ $(i.e. replacing \beta = 0, \lambda = 1)$

Then We get

$$pr^{p-1} - r^p \left[\frac{(B-A)^2 \delta \xi (1-\alpha)}{(1+p) \{p(1+A\gamma\delta) + (B-A)\delta\xi(1-\alpha+p)\}} \right] \le \left| f'(z) \right| \le pr^{p-1} + r^p \left[\frac{(B-A)^2 \delta \xi (1-\alpha)}{(1+p) \{p\lambda(1+A\gamma\delta) + (B-A)\delta\xi(1-\alpha+p)\}} \right]$$

This corollary is due to [4].

Corollary 3.11 :

If $f(z) \in T_n V^1(\alpha, 0, \xi, 1, \delta, A, B)$ $(i.e. replacing \beta = 0, \lambda = 1, \gamma = 1)$

Then We get

$$pr^{p-1} - r^p \left[\frac{(B-A)^2 \delta \xi (1-\alpha)}{(1+p) \{p(1+A\delta) + (B-A)\delta\xi(1-\alpha+p)\}} \right] \le \left| f'(z) \right| \le pr^{p-1} + r^p \left[\frac{(B-A)^2 \delta \xi (1-\alpha)}{(1+p) \{p\lambda(1+A\delta) + (B-A)\delta\xi(1-\alpha+p)\}} \right]$$

This corollary is due to [2] and [7]

Corollary 3.12 :

If $f(z) \in T_n V^1(\alpha, 0, 1, 1, \delta, A, B)$ $(i.e. replacing \beta = 0, \lambda = 1, \gamma = 1, \xi = 1)$

Then We get

$$pr^{p-1} - r^p \left[\frac{(B-A)^2\delta(1-\alpha)}{(1+p)\{p(1+A\delta)+(B-A)\delta(1-\alpha+p)\}}\right] \le \left|f'(z)\right|$$
$$\le pr^{p-1} + r^p \left[\frac{(B-A)^2\delta(1-\alpha)}{(1+p)\{p\lambda(1+A\delta)+(B-A)\delta(1-\alpha+p)\}}\right]$$

This corollary is due to [9].

4 . CLOSURE THEOREM

Theorem 4.1 :

Let $f_1(z) = z^p$ and

$$f_{n+p-1}(z) = \left[\frac{[(B-A)\delta\xi(1-\alpha)]}{[1+(n+p-2)\lambda]^\beta[(n+p-2)\lambda\{1+A\delta\gamma+(B-A)\delta\xi\}+(B-A)\delta\xi(1-\alpha)]}\right] z^{n+p-1}$$

for $n + p - 1 = \ 2, 3, 4, \ldots$

Then $f(z) \in T_n V^\lambda(\alpha, \beta, \xi, \delta, \gamma, A, B)$ if and only if $f(z)$ can be expressed in the form $f(z) = \sum_{n=1}^{\infty} \lambda_{n+p-1} z^{n+p-1}$, Where $\lambda_{n+p-1} \ge 0$ and $\sum_{n=1}^{\infty} \lambda_{n+p-1} = 1$

Proof : Let

$$f(z) = \sum_{n=1}^{\infty} \lambda_{n+p-1} z^{n+p-1} \quad , \quad \lambda_{n+p-1} \ge 0 \quad , \quad n+p-1 = 2, 3, 4, \ldots \quad with \sum_{n=1}^{\infty} \lambda_{n+p-1} = 1$$

We have

$$f(z) = \sum_{n=1}^{\infty} \lambda_{n+p-1} f_{n+p-1}(z) = \lambda_1 f_1(z) + \sum_{n=1}^{\infty} \lambda_{n+p-1} f_{n+p-1}(z)$$

$$f(z) = z^p - \sum_{n=2}^{\infty} \lambda_{n+p-1} \left[\frac{(B-A)\delta\xi(1-\alpha)}{\begin{array}{l}[1+(n+p-2)\lambda]^\beta[(n+p-2)\lambda\{1+A\delta\gamma+(B-A)\delta\xi\} \\ +(B-A)\delta\xi(1-\alpha)]\end{array}}\right] z^{n+p-1}$$

Then

$$\sum_{n=2}^{\infty} \lambda_{n+p-1} \left[\frac{(B-A)\delta\xi(1-\alpha)}{[1+(n+p-2)\lambda]^\beta[(n+p-2)\lambda\{1+A\delta\gamma+(B-A)\delta\xi\}+(B-A)\delta\xi(1-\alpha)]}\right]$$
$$\left[\frac{[1+(n+p-2)\lambda]^\beta[(n+p-2)\lambda\{1+A\delta\gamma+(B-A)\delta\xi\}+(B-A)\delta\xi(1-\alpha)]}{(B-A)\delta\xi(1-\alpha)}\right]$$

$$= \sum_{n=2}^{\infty} \lambda_{n+p-1} = 1 - \lambda \le 1$$

Then hence $f(z) \in T_n V^\lambda(\alpha, \beta, \xi, \gamma, \delta, A, B)$

Conversely suppose

$$f(z) \in T_n V^{\lambda}(\alpha, \beta, \xi, \gamma, \delta, A, B)$$

Then Remark of theorem 2.1 gives us

$$a_{n+p-1} \leq \left[\frac{(B-A)\delta\xi(1-\alpha)\{1+(p-1)\lambda\}}{[1+(n+p-2)\lambda]^{\beta}[(n+p-2)\lambda\{1+A\delta\gamma+(B-A)\delta\xi\}+(B-A)\delta\xi(1-\alpha)]}\right]$$

we take

$$\lambda_{n+p-1} = \left[\frac{[1+(n+p-2)\lambda]^{\beta}[(n+p-2)\lambda\{1+A\delta\gamma+(B-A)\delta\xi\}+(B-A)\delta\xi(1-\alpha)]}{(B-A)\delta\xi(1-\alpha)\{1+(p-1)\lambda\}}\right] a_{n+p-1}$$

and

$$\lambda = 1 - \sum_{n=2}^{\infty} \lambda_{n+p-1}$$

then

$$f(z) = \sum_{n=2}^{\infty} \lambda_{n+p-1} f_{n+p-1}(z)$$

Corollary 4.1 : If $f_1(z) = z^p$ *and*

$$f_{n+p-1}(z) = z^p - \left[\frac{(B-A)\delta\xi(1-\alpha)\{1+(p-1)\lambda\}}{[(n+p-2)-\delta\{(B-A)\alpha\xi - A\gamma(n+p-2)-(B-A)\xi(n+p-1)\}]}\right] z^{n+p-1}$$

then

$$f(z) \in T_n V^1(\alpha, 0, \xi, \gamma, \delta, A, B)$$

If and only if f(z) can be expressed in the form

$$f(z) = \sum_{n=1}^{\infty} \lambda_{n+p-1} z^{n+p-1} \quad , \quad Where \ \ \lambda_{n+p-1} \geq 0 \ \ and \ \ \sum_{n=1}^{\infty} \lambda_{n+p-1} = 1$$

for $(n+p-1) = 1,2,3,4,\ldots$

This corollary is due to [4].

Corollary 4.2 : If $f_1(z) = z^p$ and

$$f_{n+p-1}(z) = z^p - \left[\frac{(B-A)\delta\xi(1-\alpha)p}{[(n+p-2)-\delta\{(B-A)\alpha\xi - A(n+p-2)-(B-A)\xi(n+p-1)\}]}\right] z^{n+p-1}$$

Then

$$f(z) \in T_n V^1(\alpha, 0, \xi, 1, \delta, A, B)$$

If and only if f(z) can be expressed in the form

$$f(z) = \sum_{n=1}^{\infty} \lambda_{n+p-1} z^{n+p-1} \quad , \quad Where \ \ \lambda_{n+p-1} \geq 0 \ \ and \ \ \sum_{n=1}^{\infty} \lambda_{n+p-1} = 1$$

for $(n + p - 1) = 1,2,3,4,\ldots$

This corollary is due to [2] and [7].

Corollary 4.3 : If $f1(z) = z^p$ and

$$f_{n+p-1}(z) = z^p - \left[\frac{(B-A)\delta(1-\alpha)p}{[(n+p-2)-\delta\{(B-A)\alpha - B(n+p-1)+A\}]}\right] z^{n+p-1}$$

then

$$f(z) \in T_n V^1(\alpha, 0, 1, 1, \delta, A, B)$$

If and only if f(z) can be expressed in the form

$$f(z)=\sum_{n=1}^{\infty}\lambda_{n+p-1}z^{n+p-1} \quad , \quad Where \;\; \lambda_{n+p-1}\geq 0 \;\; and \; \sum_{n=1}^{\infty}\lambda_{n+p-1}=1$$

for (n + p − 1) = 1,2,3,4,...

This corollary is due to [9].

Corollary 4.4 : If $f1(z) = z^p$ and

$$f_{n+p-1}(z)=z^p-\left[\frac{(B-A)p}{[(n+p-2)+B(n+p-1)-A]}\right]z^{n+p-1}$$

$\Rightarrow$

$$f_{n+p-1}(z)=z^p-\left[\frac{(B-A)p}{[(B+1)(n+p-1)-(A+1)]}\right]z^{n+p-1}$$

Then

$$f(z)\in T_nV^1(0,0,1,1,1,A,B)$$

If and only if *f(z)* can be expressed in the form

$$f(z)=\sum_{n=1}^{\infty}\lambda_{n+p-1}z^{n+p-1} \quad , \quad Where \;\; \lambda_{n+p-1}\geq 0 \;\; and \; \sum_{n=1}^{\infty}\lambda_{n+p-1}=1$$

for (n + p − 1) = 1,2,3,4,...

5 . CONCLUSIONS

In this paper making use Al-Oboudi Operator two new subclasses of analytic and multivalent functions are introduced for the functions with negative coefficient . Many subclasses which are already studied by various researchers are obtained as special cases of our two new subclasses . We have obtained varies properties such as coefficient estimates , growth distortion theorems ,Further new subclasses may be possible from the two classes introduced in this paper .

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