

ON SOME INTUITIONISTIC SUPRA CLOSED SETS ON INTUITIONISTIC SUPRA
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ABSTRACT

The purpose of this paper is to introduce the sets called intuitionistic supra α -closed set, intuitionistic supra semi closed set and intuitionistic supra Ω closed set on intuitionistic supra topological space. Also we investigate about the continuity and irresoluteness of these sets in the intuitionistic supra topological space.

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Keywords: ISCS, IS α CS, ISSCS, IS Ω CS, ISRCS, intuitionistic supra continuity, intuitionistic supra α continuity, intuitionistic supra semi continuity, intuitionistic supra Ω continuity, intuitionistic supra α irresolute map, intuitionistic supra semi irresolute map, intuitionistic supra Ω irresolute map.

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1. INTRODUCTION

The concept of intuitionistic set and intuitionistic topological spaces was introduced by Coker[1,3]. Zadeh[8] introduced the concept of fuzzy sets. Later Coker[2] introduced the intuitionistic fuzzy topological spaces. Supra topology was introduced by A.S.Mashhour et al.[6]. Supra α open set in supra topological space was introduced by R.Devi et al.[4]. N.Levine[5] introduced semi-open sets and semi-continuity in topological space. T.Noiri and O.R.Sayed [7] introduced Ω closed set and Ω_s -closed set in topological space.

In this paper we have introduced the set called intuitionistic supra α -closed set, intuitionistic supra semi closed set and intuitionistic supra Ω -closed set on intuitionistic supra topological space and we have discussed about the continuity and irresoluteness of these sets in the intuitionistic supra topological space. Also we have discussed about the closed mapping of these intuitionistic sets on intuitionistic supra topological space.

2.Preliminaries

Definition 2.1[1] Let X be a non-empty set, an intuitionistic set (IS in short) A is an object having the form $A = \langle X, A_1, A_2 \rangle$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \emptyset$. The set A_1 is called the set of members of A , while A_2 is called the set of non-members of A .

Definition 2.2[1] Let X be a non empty set, $A = \langle X, A_1, A_2 \rangle$ and $B = \langle X, B_1, B_2 \rangle$ be IS's on X and let $\{A_i : i \in J\}$ be an arbitrary family of IS's in X , where $A_i = \langle X, A_i^{(1)}, A_i^{(2)} \rangle$. then

$$(i) A \subseteq B \text{ iff } A_1 \subseteq B_1 \text{ and } A_2 \supseteq B_2.$$

$$(ii) A = B \text{ iff } A \subseteq B \text{ and } B \subseteq A.$$

$$(iii) \bar{A} = \langle X, A_2, A_1 \rangle.$$

$$(iv) A \cup B = \langle X, A_1 \cup B_1, A_2 \cap B_2 \rangle.$$

$$(v) A \cap B = \langle X, A_1 \cap B_1, A_2 \cup B_2 \rangle.$$

$$(vi) A - B = A \cap \bar{B}.$$

$$(vii) [A] = \langle X, A_1, (A_1)^c \rangle.$$

$$(viii) \langle \rangle A = \langle X, (A_2)^c, A_2 \rangle.$$

$$(ix) \underline{X} = \langle X, X, \emptyset \rangle.$$

$$(x) \underline{\emptyset} = \langle X, \emptyset, X \rangle.$$

Definition 2.3[3] An Intuitionistic topology on a nonempty set X is a family τ of IS's in X satisfying the following axioms:

$$(i) \underline{X}, \underline{\emptyset} \in \tau.$$

$$(ii) A_1 \cap A_2 \in \tau \text{ for any } A_1, A_2 \in \tau.$$

$$(iii) \cup A_i \in \tau \text{ for any arbitrary family } \{A_i : i \in J\} \subseteq \tau.$$

The pair (X, τ) is called intuitionistic topological space (ITS in short) and IS in τ is known as an intuitionistic open set (IOS in short) in X , the complement of IOS is called intuitionistic closed set (ICS in short).

Definition 2.4[3] Let (X, τ) be an ITS and let $A = \langle X, A_1, A_2 \rangle$ be an IS in X , then the interior and closure of A are defined by:

$$\text{cl}(A) = \bigcap \{K : K \text{ is an ICS in } X \text{ and } A \subseteq K\}.$$

$$\text{int}(A) = \bigcup \{K : K \text{ is an IOS in } X \text{ and } A \supseteq K\}.$$

Definition 2.5[3] Let (X, τ) and (Y, σ) be two ITS's and let $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be continuous iff the preimage of each IS in σ is an IS in τ .

Definition 2.6[6]

A subfamily μ of X is said to be supra topology on X if

$$i) X, \emptyset \in \mu$$

$$ii) \text{If } A_i \in \mu, \forall i \in J \text{ then } \cup A_i \in \mu$$

(X, μ) is called supra topological space.

The element of μ are called supra open sets in (X, μ) and the complement of supra open set is called supra closed sets and it is denoted by μ^c .

Definition 2.7[6]

The supra closure of a set A is denoted by $\text{cl}^\mu(A)$, and is defined as

$$\text{supra cl}(A) = \bigcap \{B : B \text{ is supra closed and } A \subseteq B\}.$$

The supra interior of a set A is denoted by $\text{int}^\mu(A)$, and is defined as $\text{supra int}(A) = \bigcup \{B : B \text{ is supra open and } A \supseteq B\}.$

Definition 2.8[6] Let (X, τ) be a topological space and μ be a supra topology on X . We call μ a supra topology associated with τ , if $\tau \subseteq \mu$.

3. Intuitionistic supra closed set

Definition 3.1 An Intuitionistic supra topology on a nonempty set X is a family τ of IS's in X satisfying the following axioms:

(i) $X, \phi \in \tau$.

(ii) $\cup A_i \in \tau$ for any arbitrary family $\{A_i : i \in J\} \subseteq \tau$.

The pair (X, τ) is called intuitionistic supra topological space (ISTS in short) and IS in τ is known as an intuitionistic supra open set (ISOS in short) in X , the complement of ISOS is called intuitionistic supra closed set (ISCS in short).

Definition 3.2 Let (X, τ) be an ISTS and let $A = \langle X, A_1, A_2 \rangle$ be an IS in X ,

then the supra closure and supra interior of A are defined by:

$$cl^\mu(A) = \cap \{K : K \text{ is an ISCS in } X \text{ and } A \subseteq K\}.$$

$$int^\mu(A) = \cup \{K : K \text{ is an ISOS in } X \text{ and } A \supseteq K\}.$$

Definition 3.3 Let (X, τ) be an intuitionistic supra topological space. An intuitionistic set A is called

(i) intuitionistic supra α -closed set (IS α CS in short) if, $cl^\mu(int^\mu(cl^\mu(A))) \subseteq A$.

(ii) intuitionistic supra semi closed set (ISSCS in short) if, $int^\mu(cl^\mu(A)) \subseteq A$.

(iii) intuitionistic supra Ω -closed set (IS Ω CS in short) if, $scl^\mu(A) \subseteq int^\mu(U)$, when-ever $A \subseteq U$, U is intuitionistic supra open set.

(iv) intuitionistic supra regular closed set (ISRCS in short) if, $A = cl^\mu(int^\mu(A))$.

The complement of intuitionistic supra α -closed set is intuitionistic supra α -open set (IS α OS in short).

The complement of intuitionistic supra semi-closed set is intuitionistic supra semi-open set (ISSOS in short).

The complement of intuitionistic supra Ω -closed set is intuitionistic supra Ω -open set (IS Ω OS in short).

The complement of intuitionistic supra regular closed set is intuitionistic supra regular open set (ISROS in short).

Theorem 3.4 Every ISRCS is ISCS.

Proof Let (X, τ) be an intuitionistic supra topological space. Let A be intuitionistic supra regular closed set in (X, τ) . Since A is ISRCS, we have $A = cl^\mu(int^\mu(A))$. Then $A = cl^\mu(int^\mu(A)) \subseteq cl^\mu(A)$. Hence A is intuitionistic supra closed set.

Converse of the above theorem need not be true. It is shown by the following example.

Example 3.5 Let $X = \{a, b, c\}$. $\tau = \{X, \phi, A_1, A_2, A_3\}$, where $A_1 = \langle X, \{b\}, \{a, c\} \rangle$,

$A_2 = \langle X, \{a\}, \{b\} \rangle$ and $A_3 = \langle X, \{a, b\}, \phi \rangle$. ISCS's are

$\{X, \phi, \langle X, \{a, c\}, \{b\} \rangle, \langle X, \{b\}, \{a\} \rangle, \langle X, \phi, \{a, b\} \rangle\}$. ISRCS's are

$\{X, \phi, \langle X, \{a, c\}, \{b\} \rangle, \langle X, \{b\}, \{a\} \rangle\}$. Here $\langle X, \phi, \{a, b\} \rangle$ is ISCS but it

is not ISRCS.

Theorem 3.6 Every ISCS is IS α CS.

Proof Let (X, τ) be an intuitionistic supra topological space. Let A be intuitionistic supra closed set in (X, τ) . Since A is ISCS, we have $cl^\mu(A) \subseteq A$. Then $int^\mu(cl^\mu(A)) \subseteq cl^\mu(A)$. Implies $cl^\mu(int^\mu(cl^\mu(A))) \subseteq cl^\mu(A)$.

Therefore $cl^\mu(int^\mu(cl^\mu(A))) \subseteq A$. Hence A is IS α CS.

Converse of the above theorem need not be true. It is shown by the following example.

Example 3.7 Let $X=\{a, b, c\}$. $\tau = \{ \underline{X}, \phi, A_1, A_2 \}$, where $A_1 = \langle X, \{a\}, \{c\} \rangle$ and $A_2 = \langle X, \{a, b\}, \phi \rangle$. ISCS's are $\{ \underline{X}, \phi, \langle X, \{c\}, \{a\} \rangle, \langle X, \phi, \{a, b\} \rangle \}$. IS α CS's are $\{ \underline{X}, \phi, \langle X, \{c\}, \{a\} \rangle, \langle X, \phi, \{a, b\} \rangle, \langle X, \{c\}, \{a, b\} \rangle, \langle X, \phi, \{a\} \rangle, \langle X, \phi, \{a, c\} \rangle \}$. Here $\langle X, \{c\}, \{a, b\} \rangle$ is IS α CS but not ISCS.

Theorem 3.8 Every ISCS is ISSCS

Proof Let (X, τ) be an intuitionistic supra topological space. Let A be ISCS, we have $cl^u(A) \subseteq A$. Then $int^u(cl^u(A)) \subseteq cl^u(A) \subseteq A$. Hence A is ISSCS.

Converse of the above theorem need not be true. It is shown by the following example.

Example 3.9 Let $X=\{a, b, c\}$. $\tau = \{ \underline{X}, \phi, A_1, A_2, A_3 \}$, where $A_1 = \langle X, \{b\}, \{a, c\} \rangle$, $A_2 = \langle X, \{a\}, \{b\} \rangle$ and $A_3 = \langle X, \{a, b\}, \phi \rangle$. ISCS's are $\{ \underline{X}, \phi, \langle X, \{a, c\}, \{b\} \rangle, \langle X, \{b\}, \{a\} \rangle, \langle X, \phi, \{a, b\} \rangle \}$. ISSCS's are $\{ \underline{X}, \phi, \langle X, \{a, c\}, \{b\} \rangle, \langle X, \{b\}, \{a\} \rangle, \langle X, \phi, \{a, b\} \rangle, \langle X, \{a\}, \{b\} \rangle, \langle X, \{b\}, \{a, c\} \rangle \}$. Here $\langle X, \{a\}, \{b\} \rangle$ is ISSCS but it is not ISCS.

Theorem 3.10 Every ISCS is ISCS.

Proof: Let (X, τ) be an intuitionistic supra topological space. Let A be ISCS in (X, τ) . Let $A \subseteq U$, U is ISOS. Since A is ISCS, $cl^u(A) \subseteq A \subseteq U$. Since every ISCS is ISSCS, then $scl^u(A) \subseteq cl^u(A) \subseteq A \subseteq U$. Since U is ISOS, we have $scl^u(A) \subseteq int^u(U)$. Hence A is ISCS.

Converse of the above theorem need not be true. It is shown by the following example.

Example 3.11 Let $X=\{a, b, c\}$. $\tau = \{ \underline{X}, \phi, A_1, A_2, A_3 \}$, where $A_1 = \langle X, \{b\}, \{a, c\} \rangle$, $A_2 = \langle X, \{a\}, \{b\} \rangle$ and $A_3 = \langle X, \{a, b\}, \phi \rangle$. ISCS's are $\{ \underline{X}, \phi, \langle X, \{a, c\}, \{b\} \rangle, \langle X, \{b\}, \{a\} \rangle, \langle X, \phi, \{a, b\} \rangle \}$. IS Ω CS's are $\{ \underline{X}, \phi, \langle X, \{a, c\}, \{b\} \rangle, \langle X, \{b\}, \{a\} \rangle, \langle X, \phi, \{a, b\} \rangle, \langle X, \{a\}, \{b\} \rangle, \langle X, \{b\}, \{a, c\} \rangle, \langle X, \{c\}, \{a\} \rangle, \langle X, \{a\}, \{b, c\} \rangle, \langle X, \{b, c\}, \{a\} \rangle, \langle X, \{a, c\}, \{b\} \rangle, \langle X, \{a, b\}, \{c\} \rangle, \langle X, \{c\}, \{b\} \rangle, \langle X, \{c\}, \phi \rangle, \langle X, \{a, c\}, \phi \rangle, \langle X, \phi, \{a\} \rangle, \langle X, \phi, \{b\} \rangle, \langle X, \phi, \{a, c\} \rangle, \langle X, \phi, \{b, c\} \rangle, \langle X, \phi, \phi \rangle, \langle X, \{c\}, \{a, b\} \rangle \}$. Here $\langle X, \{a\}, \{b\} \rangle$ is IS Ω CS but it is not ISCS.

Theorem 3.12 Every IS α CS is ISSCS.

Proof Let (X, τ) be an intuitionistic supra topological space. Let A be IS α CS in (X, τ) , then $cl^u(int^u(cl^u(A))) \subseteq A$. We have $int^u(cl^u(A)) \subseteq cl^u(int^u(cl^u(A))) \subseteq A$. Hence A is ISSCS.

Converse of the above theorem need not be true. It is shown by the following example

Example 3.13 Let $X=\{a, b, c\}$. $\tau = \{ \underline{X}, \phi, A_1, A_2, A_3 \}$, where $A_1 = \langle X, \{b\}, \{a, c\} \rangle$, $A_2 = \langle X, \{a\}, \{b\} \rangle$ and $A_3 = \langle X, \{a, b\}, \phi \rangle$. IS α CS's are $\{ \underline{X}, \phi, \langle X, \{a, c\}, \{b\} \rangle, \langle X, \{b\}, \{a\} \rangle, \langle X, \phi, \{a, b\} \rangle \}$. ISSCS's are $\{ \underline{X}, \phi, \langle X, \{a, c\}, \{b\} \rangle, \langle X, \{b\}, \{a\} \rangle, \langle X, \phi, \{a, b\} \rangle, \langle X, \{a\}, \{b\} \rangle, \langle X, \{b\}, \{a, c\} \rangle \}$. Here $\langle X, \{a\}, \{b\} \rangle$ is ISSCS but it is not IS α CS.

Theorem 3.14 Every ISSCS is ISCS.

Proof Let (X, τ) be an intuitionistic supra topological space. Let A be ISSCS in (X, τ) . Let $A \subseteq U$, U is ISOS. Since A is ISSCS, $scl^u(A) \subseteq U$. Hence A is ISCS. Converse of the above theorem need not be true. It is shown by the following example.

Example 3.15 Let $X=\{a, b, c\}$. $\tau = \{ \underline{X}, \underline{\phi}, A_1, A_2, A_3 \}$, where $A_1 = \langle X, \{b\}, \{a, c\} \rangle$, $A_2 = \langle X, \{a\}, \{b\} \rangle$ and $A_3 = \langle X, \{a, b\}, \phi \rangle$. ISCS's are $\{ \underline{X}, \underline{\phi}, \langle X, \{a, c\}, \{b\} \rangle, \langle X, \{b\}, \{a\} \rangle, \langle X, \phi, \{a, b\} \rangle \}$. ISSCS's are $\{ \underline{X}, \underline{\phi}, \langle X, \{a, c\}, \{b\} \rangle, \langle X, \{b\}, \{a\} \rangle, \langle X, \phi, \{a, b\} \rangle, \langle X, \{a\}, \{b\} \rangle, \langle X, \{b\}, \{a, c\} \rangle \}$. $\text{IS}\Omega\text{CS}$'s are $\{ \underline{X}, \underline{\phi}, \langle X, \{a, c\}, \{b\} \rangle, \langle X, \{b\}, \{a\} \rangle, \langle X, \phi, \{a, b\} \rangle, \langle X, \{a\}, \{b\} \rangle, \langle X, \{b\}, \{a, c\} \rangle, \langle X, \{c\}, \{a\} \rangle, \langle X, \{a\}, \{b, c\} \rangle, \langle X, \{b, c\}, \{a\} \rangle, \langle X, \{a, c\}, \{b\} \rangle, \langle X, \{a, b\}, \{c\} \rangle, \langle X, \{c\}, \{b\} \rangle, \langle X, \{c\}, \phi \rangle, \langle X, \{a, c\}, \phi \rangle, \langle X, \phi, \{a\} \rangle, \langle X, \phi, \{b\} \rangle, \langle X, \phi, \{a, c\} \rangle, \langle X, \phi, \{b, c\} \rangle, \langle X, \phi, \phi \rangle, \langle X, \{c\}, \{a, b\} \rangle \}$. Here $\langle X, \phi, \{b\} \rangle$ is $\text{IS}\Omega\text{CS}$ but it is not ISSCS.

Theorem 3.16 Every $\text{IS}\alpha\text{CS}$ is $\text{IS}\Omega\text{CS}$.

Proof It is obvious from theorem 3.8 and theorem 3.9.

Converse of the above theorem need not be true. It is shown by the following example.

Example 3.17 Let $X=\{a, b, c\}$. $\tau = \{ \underline{X}, \underline{\phi}, A_1, A_2, A_3 \}$, where $A_1 = \langle X, \{b\}, \{a, c\} \rangle$, $A_2 = \langle X, \{a\}, \{b\} \rangle$ and $A_3 = \langle X, \{a, b\}, \phi \rangle$. $\text{IS}\alpha\text{CS}$'s are $\{ \underline{X}, \underline{\phi}, \langle X, \{a, c\}, \{b\} \rangle, \langle X, \{b\}, \{a\} \rangle, \langle X, \phi, \{a, b\} \rangle \}$. $\text{IS}\Omega\text{CS}$'s are $\{ \underline{X}, \underline{\phi}, \langle X, \{a, c\}, \{b\} \rangle, \langle X, \{b\}, \{a\} \rangle, \langle X, \phi, \{a, b\} \rangle, \langle X, \{a\}, \{b\} \rangle, \langle X, \{b\}, \{a, c\} \rangle, \langle X, \{c\}, \{a\} \rangle, \langle X, \{a\}, \{b, c\} \rangle, \langle X, \{b, c\}, \{a\} \rangle, \langle X, \{a, c\}, \{b\} \rangle, \langle X, \{a, b\}, \{c\} \rangle, \langle X, \{c\}, \{b\} \rangle, \langle X, \{c\}, \phi \rangle, \langle X, \{a, c\}, \phi \rangle, \langle X, \phi, \{a\} \rangle, \langle X, \phi, \{b\} \rangle, \langle X, \phi, \{a, c\} \rangle, \langle X, \phi, \{b, c\} \rangle, \langle X, \phi, \phi \rangle, \langle X, \{c\}, \{a, b\} \rangle \}$. Here $\langle X, \{a\}, \{b\} \rangle$ is $\text{IS}\Omega\text{CS}$ but it is not $\text{IS}\alpha\text{CS}$.

4. Intuitionistic supra continuity

Definition 4.1 Let (X, τ) and (Y, σ) be two intuitionistic supra topological space. A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is called

- (i) intuitionistic supra continuous map if $f^{-1}(V)$ is ISCS in X for every ISCS V in Y .
- (ii) intuitionistic supra α -continuous map if $f^{-1}(V)$ is $\text{IS}\alpha\text{CS}$ in X for every ISCS V in Y .
- (iii) intuitionistic supra semi-continuous map if $f^{-1}(V)$ is ISSCS in X for every ISCS V in Y .
- (iv) intuitionistic supra Ω -continuous map if $f^{-1}(V)$ is $\text{IS}\Omega\text{CS}$ in X for every ISCS V in Y .
- (v) intuitionistic supra α -irresolute map if $f^{-1}(V)$ is $\text{IS}\alpha\text{CS}$ in X for every $\text{IS}\alpha\text{CS}$ V in Y .
- (vi) intuitionistic supra semi-irresolute map if $f^{-1}(V)$ is ISSCS in X for every ISSCS V in Y .
- (vii) intuitionistic supra Ω -irresolute map if $f^{-1}(V)$ is $\text{IS}\Omega\text{CS}$ in X for every $\text{IS}\Omega\text{CS}$ V in Y .

Theorem 4.2 Every intuitionistic supra continuous map is intuitionistic supra α -continuous map.

Proof Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic supra continuous map. Let V be ISCS in Y . Then $f^{-1}(V)$ is ISCS in X . Since every ISCS is $\text{IS}\alpha\text{CS}$, then $f^{-1}(V)$ is $\text{IS}\alpha\text{CS}$ in X . Hence f is intuitionistic supra α -continuous map.

The converse of the above theorem need not be true. It is shown by the following example.

Example 4.3 Let $X=Y=\{a, b, c\}$. $\tau = \{ \underline{X}, \underline{\phi}, A_1, A_2 \}$, where $A_1 = \langle X, \{a\}, \{c\} \rangle$ and $A_2 = \langle X, \{a, b\}, \phi \rangle$. $\sigma = \{ \underline{Y}, \underline{\phi}, B_1, B_2, B_3 \}$ where $B_1 = \langle Y, \{b\}, \{a, c\} \rangle$, $B_2 = \langle Y, \{a\}, \{b\} \rangle$, $B_3 = \langle Y, \{a, b\}, \phi \rangle$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a)=a$, $f(b)=c$ and $f(c)=b$. Here $V = \langle Y, \phi, \{a, b\} \rangle$ is ISCS in Y and $f^{-1}(V) = \langle X, \phi, \{a, c\} \rangle$ is $\text{IS}\alpha\text{CS}$ but not ISCS in X . Hence f is not intuitionistic supra continuous map.

Theorem 4.4 Every intuitionistic supra continuous map is intuitionistic supra semi-continuous map.

Proof Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be an intuitionistic supra continuous map. Let V be ISCS in Y . Then $f^{-1}(V)$ is ISCS in X . Since every ISCS is ISSCS, then $f^{-1}(V)$ is ISSCS in X . Hence f is intuitionistic supra semi-continuous map.

The converse of the above theorem need not be true. It is shown by the following example.

Example 4.5 Let $X=Y=\{a, b, c\}$. $\tau = \{ \underline{X}, \underline{\phi}, A_1, A_2 \}$, where $A_1 = \langle X, \{a\}, \{c\} \rangle$ and $A_2 = \langle X, \{a, b\}, \phi \rangle$. $\sigma = \{ \underline{Y}, \underline{\phi}, B_1, B_2, B_3 \}$ where $B_1 = \langle Y, \{b\}, \{a, c\} \rangle$, $B_2 = \langle Y, \{a\}, \{b\} \rangle$, $B_3 = \langle Y, \{a, b\}, \phi \rangle$. Let $f:(X,\tau) \rightarrow (Y,\sigma)$ defined by $f(a)=a$, $f(b)=c$ and $f(c)=b$. Here $V = \langle Y, \phi, \{a, b\} \rangle$ is ISCS in Y and $f^{-1}(V) = \langle X, \phi, \{a, c\} \rangle$ is ISSCS but not ISCS in X . Hence f is not intuitionistic supra continuous map.

Theorem 4.6 Every intuitionistic supra continuous map is intuitionistic supra Ω -continuous map.

Proof Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be an intuitionistic supra continuous map. Let V be ISCS in Y . Then $f^{-1}(V)$ is ISCS in X . Since every ISCS is IS Ω CS, then $f^{-1}(V)$ is IS Ω CS in X . Hence f is intuitionistic supra Ω -continuous map.

The converse of the above theorem need not be true. It is shown by the following example.

Example 4.7 Let $X=Y=\{a, b, c\}$. $\tau = \{ \underline{X}, \underline{\phi}, A_1, A_2 \}$, where $A_1 = \langle X, \{a\}, \{c\} \rangle$ and $A_2 = \langle X, \{a, b\}, \phi \rangle$. $\sigma = \{ \underline{Y}, \underline{\phi}, B_1, B_2, B_3 \}$ where $B_1 = \langle Y, \{b\}, \{a, c\} \rangle$, $B_2 = \langle Y, \{a\}, \{b\} \rangle$, $B_3 = \langle Y, \{a, b\}, \phi \rangle$. Let $f:(X,\tau) \rightarrow (Y,\sigma)$ defined by $f(a)=a$, $f(b)=c$ and $f(c)=b$. Here $V = \langle Y, \phi, \{a, b\} \rangle$ is ISCS in Y and $f^{-1}(V) = \langle X, \phi, \{a, c\} \rangle$ is IS Ω CS but not ISCS in X . Hence f is not intuitionistic supra continuous map.

Theorem 4.8 Every intuitionistic supra α -continuous map is intuitionistic supra semi-continuous map.

Proof Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be an intuitionistic supra α -continuous map. Let V be ISCS in Y . Then $f^{-1}(V)$ is IS α CS in X . Since every IS α CS is ISSCS, then $f^{-1}(V)$ is ISSCS in X . Hence f is intuitionistic supra semi-continuous map.

The converse of the above theorem need not be true. It is shown by the following example.

Example 4.9 Let $X=Y=\{a, b, c\}$. $\tau = \{ \underline{X}, \underline{\phi}, A_1, A_2, A_3 \}$, where $A_1 = \langle X, \{c\}, \{a, b\} \rangle$, $A_2 = \langle X, \{a\}, \{b, c\} \rangle$ and $A_3 = \langle X, \{a, c\}, \{b\} \rangle$. $\sigma = \{ \underline{Y}, \underline{\phi}, B_1, B_2 \}$, where $B_1 = \langle Y, \{a\}, \{c\} \rangle$ and $B_2 = \langle Y, \{a, b\}, \phi \rangle$. Let $f:(X,\tau) \rightarrow (Y,\sigma)$ defined by $f(a)=a$, $f(b)=c$ and $f(c)=b$. Here $V = \langle Y, \phi, \{a, b\} \rangle$ is ISCS in Y and $f^{-1}(V) = \langle X, \phi, \{a, c\} \rangle$ is ISSCS but not IS α CS in X . Hence f is not intuitionistic supra α continuous.

Theorem 4.10 Every intuitionistic supra semi-continuous map is intuitionistic supra Ω -continuous map.

Proof Let $f:(X,\tau) \rightarrow (Y,\sigma)$ be an intuitionistic supra semi-continuous map. Let V be ISCS in Y . Then $f^{-1}(V)$ is ISSCS in X . Since every ISSCS is IS Ω CS, then $f^{-1}(V)$ is IS Ω CS in X . Hence f is intuitionistic supra Ω -continuous map.

The converse of the above theorem need not be true. It is shown by the following example.

Example 4.11 Let $X=Y=\{a, b, c\}$. $\tau = \{ \underline{X}, \underline{\phi}, A_1, A_2, A_3 \}$, where $A_1 = \langle X, \{c\}, \{a, b\} \rangle$, $A_2 = \langle X, \{a\}, \{b, c\} \rangle$ and $A_3 = \langle X, \{a, c\}, \{b\} \rangle$. $\sigma = \{ \underline{Y}, \underline{\phi}, B_1, B_2, B_3 \}$, where $B_1 = \langle Y, \{a\}, \{c\} \rangle$, $B_2 = \langle Y, \{c\}, \{a\} \rangle$ and $B_3 = \langle Y, \{a, c\}, \phi \rangle$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a)=a$, $f(b)=b$ and $f(c)=c$. Here $V = \langle Y, \{a, b\}, \{c\} \rangle$ is ISCS in Y and $f^{-1}(V) = \langle X, \{a, b\}, \{c\} \rangle$ is $IS\Omega CS$ but not ISSCS in X . Hence f is not intuitionistic supra semi continuous.

Theorem 4.12 Every intuitionistic supra α -irresolute map is intuitionistic supra α -continuous map.

Proof Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic supra α -irresolute map. Let V be ISCS in Y , then V is $IS\alpha CS$ in Y , since every ISCS is $IS\alpha CS$. Then $f^{-1}(V)$ is $IS\alpha CS$ in X . Hence f is intuitionistic supra α -continuous map.

The converse of the above theorem need not be true. It is shown by the following example.

Example 4.13 Let $X=Y=\{a, b, c\}$. $\tau = \{ \underline{X}, \underline{\phi}, A_1, A_2, A_3 \}$, where $A_1 = \langle X, \{b\}, \{a, c\} \rangle$, $A_2 = \langle X, \{a\}, \{b\} \rangle$ and $A_3 = \langle X, \{a, b\}, \phi \rangle$. $\sigma = \{ \underline{Y}, \underline{\phi}, B_1, B_2 \}$, where $B_1 = \langle Y, \{a\}, \{b\} \rangle$, $B_2 = \langle Y, \{a, b\}, \phi \rangle$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a)=a$, $f(b)=b$ and $f(c)=c$. Here f is intuitionistic supra α -continuous but not intuitionistic supra α -irresolute map, since $V = \langle Y, \phi, \{a, b\} \rangle$ is $IS\alpha CS$ in Y but $f^{-1}(V) = \langle X, \phi, \{a, c\} \rangle$ is not $IS\alpha CS$ in X .

Theorem 4.14 Every intuitionistic supra semi-irresolute map is intuitionistic supra semi-continuous map.

Proof Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic supra semi-irresolute map. Let V be ISCS in Y . Since every ISCS is ISSCS, then V is ISSCS. Then $f^{-1}(V)$ is ISSCS in X . Hence f is intuitionistic supra semi-continuous map.

The converse of the above theorem need not be true. It is shown by the following example.

Example 4.15 Let $X=Y=\{a, b, c\}$. $\tau = \{ \underline{X}, \underline{\phi}, A_1, A_2, A_3 \}$, where $A_1 = \langle X, \{b\}, \{a, c\} \rangle$, $A_2 = \langle X, \{a\}, \{b\} \rangle$ and $A_3 = \langle X, \{a, b\}, \phi \rangle$. $\sigma = \{ \underline{Y}, \underline{\phi}, B_1, B_2 \}$, where $B_1 = \langle Y, \{a\}, \{b\} \rangle$, $B_2 = \langle Y, \{a, b\}, \phi \rangle$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a)=a$, $f(b)=b$ and $f(c)=c$. Here f is intuitionistic supra semi-continuous but not intuitionistic supra semi-irresolute map, since $V = \langle Y, \phi, \{a\} \rangle$ is ISSCS in Y but $f^{-1}(V) = \langle X, \phi, \{a\} \rangle$ is not ISSCS in X .

Theorem 4.16 Every intuitionistic supra Ω -irresolute map is intuitionistic supra Ω -continuous map.

Proof Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic supra Ω -irresolute map. Let V be ISCS in Y . Since every ISCS is $IS\Omega CS$, then V is $IS\Omega CS$. Then $f^{-1}(V)$ is $IS\Omega CS$ in X . Hence f is intuitionistic supra Ω -continuous map.

The converse of the above theorem need not be true. It is shown by the following example.

Example 4.17 Let $X=Y=\{a, b, c\}$. $\tau = \{ \underline{X}, \underline{\phi}, A_1, A_2 \}$, where $A_1 = \langle X, \{a\}, \{b\} \rangle$, $A_2 = \langle X, \{a, b\}, \phi \rangle$. $\sigma = \{ \underline{Y}, \underline{\phi}, B_1, B_2, B_3 \}$, where $B_1 = \langle Y, \{b\}, \{a, c\} \rangle$, $B_2 = \langle Y, \{a\}, \{b\} \rangle$ and $B_3 = \langle Y, \{a, b\}, \phi \rangle$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a)=a$, $f(b)=b$ and $f(c)=c$. Here f is intuitionistic supra Ω -continuous but not intuitionistic supra Ω -irresolute map, since $V = \langle Y, \phi, \{b\} \rangle$ is $IS\Omega CS$ in Y but $f^{-1}(V) = \langle X, \phi, \{b\} \rangle$ is not $IS\Omega CS$ in X .

5. Intuitionistic supra closed map

Definition 5.1 Let (X, τ) and (Y, σ) be two intuitionistic supra topological space. A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is called

(i) intuitionistic supra closed map if the $f(V)$ is ISCS in Y for every ISCS V in X . (ii) intuitionistic supra α -closed map if the $f(V)$ is IS α CS in Y for every ISCS V in X .

(iii) intuitionistic supra semi-closed map if the $f(V)$ is ISSCS in Y for every ISCS V in X .

(iv) intuitionistic supra Ω -closed map if the $f(V)$ is IS Ω CS in Y for every ISCS V in X .

Theorem 5.2 Every intuitionistic supra closed map is intuitionistic supra α -closed map (resp. semi-closed map, Ω -closed map).

Proof Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic supra closed map. Let V be ISCS in X . Then $f(V)$ is ISCS in Y . Since every ISCS is IS α CS (resp. ISSCS, IS Ω CS), then $f(V)$ is IS α CS (resp. ISSCS, IS Ω CS) in Y . Hence f is intuitionistic supra α -closed map (resp. semi-closed map, Ω -closed map).

The converse of the above theorem need not be true. It is shown by the following example.

Example 5.3 Let $X=Y=\{a, b, c\}$. $\tau = \{ \underline{X}, \underline{\phi}, A_1, A_2 \}$, where $A_1 = \langle X, \{a\}, \{c\} \rangle$, $A_2 = \langle X, \{a, b\}, \phi \rangle$. $\sigma = \{ \underline{Y}, \underline{\phi}, B_1, B_2 \}$, where $B_1 = \langle Y, \{a\}, \{b\} \rangle$, and $B_2 = \langle Y, \{a, b\}, \phi \rangle$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a)=a$, $f(b)=c$ and $f(c)=b$. Here f is intuitionistic supra α -closed map (resp. semi-closed map, Ω -closed map) but not intuitionistic supra closed map, since $V = \langle X, \phi, \{a, b\} \rangle$ is ISCS in X but $f(V) = \langle Y, \phi, \{a, c\} \rangle$ is not ISCS in Y .

Theorem 5.4 Every intuitionistic supra α -closed map is intuitionistic semi-closed map (resp. Ω -closed map).

Proof Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic supra α -closed map. Let V be ISCS in X . Then $f(V)$ is IS α CS in Y . Since every IS α CS is ISSCS (resp. IS Ω CS), then $f(V)$ is ISSCS (resp. IS Ω CS) in Y . Hence f is intuitionistic supra semi-closed map (resp. Ω -closed map).

The converse of the above theorem need not be true. It is shown by the following example.

Example 5.5 Let $X=Y=\{a, b, c\}$. $\tau = \{ \underline{X}, \underline{\phi}, A_1, A_2 \}$, where $A_1 = \langle X, \{a\}, \{b\} \rangle$, $A_2 = \langle X, \{a, b\}, \phi \rangle$. $\sigma = \{ \underline{Y}, \underline{\phi}, B_1, B_2, B_3 \}$, where $B_1 = \langle Y, \{b\}, \{a, c\} \rangle$, $B_2 = \langle Y, \{a\}, \{b\} \rangle$ and $B_3 = \langle Y, \{a, b\}, \phi \rangle$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a)=b$, $f(b)=a$ and $f(c)=c$. Here f is intuitionistic supra semi-closed map (resp. Ω -closed map) but not intuitionistic supra α -closed map, since $V = \langle X, \{b\}, \{a\} \rangle$ is ISCS in X but $f(V) = \langle Y, \{a\}, \{b\} \rangle$ is not IS α CS in Y .

Theorem 5.6 Every intuitionistic supra semi-closed map is intuitionistic Ω -closed map.

Proof Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an intuitionistic supra semi-closed map. Let V be ISCS in X . Then $f(V)$ is ISSCS in Y . Since every ISSCS is IS Ω CS, then $f(V)$ is IS Ω CS in Y . Hence f is intuitionistic supra Ω -closed map.

The converse of the above theorem need not be true. It is shown by the following example.

Example 5.7 Let $X=Y=\{a, b, c\}$. $\tau = \{X, \phi, A_1, A_2\}$, where $A_1 = \langle X, \{a\}, \{c\} \rangle$, $A_2 = \langle X, \{a, b\}, \phi \rangle$. $\sigma = \{Y, \phi, B_1, B_2, B_3\}$, where $B_1 = \langle Y, \{b\}, \{a, c\} \rangle$, $B_2 = \langle Y, \{a\}, \{b\} \rangle$ and $B_3 = \langle Y, \{a, b\}, \phi \rangle$. Let $f:(X, \tau) \rightarrow (Y, \sigma)$ defined by $f(a)=b$, $f(b)=a$ and $f(c)=c$. Here f is intuitionistic supra Ω -closed map but not intuitionistic supra semi-closed map, since $V = \langle X, \{c\}, \{a\} \rangle$ is ISCS in X but $f(V) = \langle Y, \{c\}, \{b\} \rangle$ is not ISSCS in Y .

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