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REVIEW ARTICLE

APPLICATION OF MULTIPLICATIVE MODELS FOR ANALYZING PRODUCT FAILURE DATA: A CASE STUDY

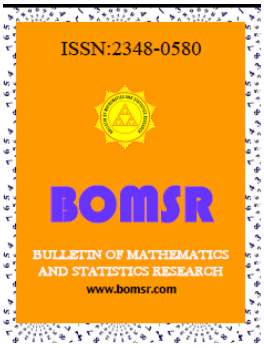
SABBA RUHI

Lecturer, Department of Mathematics

Pabna University of Science and Technology

Pabna, Bangladesh

Email: sabba.ruhi@gmail.com



ABSTRACT

Whenever purchase a viable product, we expect it to function properly at least for a reasonable time period. Customers expect purchased products to be reliable and safe. Over the last three decades there has been a heightened interest in improving quality, increasing productivity and reliability of manufactured items. Murthy, Xie and Jiang (2004) analyzed 'Throttle failure data', which consist of both failure and censored lifetimes of the throttle. They estimated the model parameters from WPP plot and selected the 2-fold Weibull multiplicative model as the best fitted model. In this study, a set of competitive models (like Weibull, Exponential, Normal, Lognormal) are applied to find out the suitable multiplicative model. We have estimated the CDF, R(t), B10 life, MTTF, etc. which are used in investigating product reliability. It discussed both the nonparametric and parametric estimation procedures including Kaplan-Meier (KM) estimate, Weibull Probability Paper (WPP) plot and Maximum Likelihood estimation methods. The Akaike Information Criterion (AIC), Anderson-Darling (AD) and Adjusted Anderson Darling (Adjusted AD) test statistics are applied to select the best fitted models.

Keywords: Reliability; Multiplicative models; Failure data; MLE

1. INTRODUCTION

Customers expect purchased products to be reliable and well performed. System, vehicle, machine, device, and so on should, with high probability, be able to perform their intended function under usual operating conditions. Whenever purchase a durable goods, we expect it to function properly at least for a reasonable period of time. So, the manufacturers of such goods bear the responsibility to inform their customers about the average lifetime of their products. Again for costly items, customers expect a minimum life span during which the item should properly without any disturbance. If so happens, the producer or his agent should give free service to bring the item in order or should replace the item. While improving product quality at first we should think about increasing product reliability. In recent years many manufacturers have started to collect and analyze field failure data to enhance the quality and reliability of their products and to increase customer satisfaction.

While solving real-world problems from many different disciplines, mathematical models have been used for a long time. But this requires building an appropriate mathematical model. According to Murthy, Xie, and Jiang (2004), lots of standard probability distributions have been used as models to model data exhibiting significant variability. Over the last three decades, several new models have been proposed that are either derived from, or some way related to the distributions like Weibull, Exponential, Normal, Lognormal etc. Meeker and Escober (1998b), discussed various models and methods for estimating reliability of a product.

Here we have analyzed the ‘Throttle failure data’. At any specific point in time, these data will include failures to date of a particular model as well as service times of all items that have not failed. Data of this type are incomplete in that not all failure times have as yet been observed. The “throttle” is an element of a vehicle. It is connected with the steering wheel, which is used to control the way of the vehicle.

The remainder of the article is organized as follows: Section 2 contains the product failure data sets which will be analyzed in this paper. Section 3 derives the Multiplicative models. Section 4 describes the model selection and parameter estimation procedures of the lifetime models of the components. Section 5 discusses the results obtained from the analysis. Finally, Section 6 concludes the article with additional implementation issues for further research.

2. Data Sets

Field failure data is better to laboratory test data in the sense that it contains precious information on the performance of a product in actual usage conditions. There are many sources of collecting product reliability data. Warranty claim data is used as an important source of field failure data which can be assembled economically and efficiently through repair service networks and therefore, a number of events have been developed for collecting and analyzing warranty claim data (e.g. Karim and Suzuki, 2005; Karim et al., 2001; Lawless, 1998; Murthy and Djamaludin, 2002; Suzuki, 1985a,; Suzuki et al., 2001)

Throttle Failure Data

The “throttle” is a component or a part of a vehicle. The data presented in table-1 is from Murthy et al. (2004), originally given in Carter (1986). This data set gives distance traveled (in thousands of kilometers) before failure or the item being suspended before failure, for a preproduction general purpose load-carrying vehicle. Hence, the independent variable is the distance moved with 1000 km being the unit of measurement.

Table-1: Throttle Failure Data

0.478	0.959	1.847+	3.904	6.711+
0.484+	1.071+	2.4	4.443+	6.835+
0.583	1.318+	2.55+	4.829	6.947+
0.626+	1.377	2.568+	5.328	7.878+
0.753	1.472+	2.639	5.562	7.884+
0.753	1.534	2.944	5.9+	10.263+
0.801	1.579+	2.981	6.122	11.019
0.834	1.61+	3.392	6.226+	12.986

0.85+	1.729+	3.393	6.331	13.103+
0.944	1.792+	3.791+	6.531	23.245+

Distance traveled before failure or censoring (denoted by +) unit, 1000 km.

3. Modeling

Jiang et al. (2001b) propose a method for estimating the model parameters, when *n* is specified, based on the WPP plot. It occupies estimating the parameters of one subpopulation based on the fit to the right (or left) asymptote. Ling and Pan (1998) consider a 2-fold multiplicative model involving three parameters Weibull distributions. The model parameters are obtained by minimizing the maximum absolute difference between the observed and the expected probability of failure. Jiang and Murthy (1995b) also used a 2-fold Weibull multiplicative model to model several different data sets. These contain failure times for transistors, shear strength of brass rivets and pull strength of welds.

Multiplicative Model

A multiplicative model involving *n* distributions is derived as follows. Let T_i denote an independent random variable with a distribution function $F_i(t)$, $1 \le i \le n$. Then the distribution function is given by

$$G(t) = \prod_{i=1}^{n} F_i(t) \tag{1}$$

The density function of the Multiplicative model is given by

$$g(t) = G(t)\sum_{i=1}^{n} \frac{f_i(t)}{F_i(t)} \tag{2}$$

And the hazard function *h*(*t*) is given by

$$h(t) = \frac{G(t)}{1-G(t)} \sum_{i=1}^{n} \frac{f_i(t)}{F_i(t)} \tag{3}$$

Special Case: Two-fold Multiplicative Model (*n* = 2)

For *n*=2, the distribution function for multiplicative model is:

$$G(t) = F_1(t).\, F_2(t) \tag{4}$$

For example, suppose, $f_1(t) \sim$ Weibull(α_1, β_1) and $f_2(t) \sim$ Weibull (α_2, β_2) distribution. Hence, the distribution function for 2-fold Weibull multiplicative model from equation (4) is:

$$G(t) = 1 - \exp\left[-\left(\frac{t}{\alpha_1}\right)^{\beta_1}\right] - \exp\left[-\left(\frac{t}{\alpha_2}\right)^{\beta_2}\right] + \exp\left\{-\left(\frac{t}{\alpha_1}\right)^{\beta_1} - \left(\frac{t}{\alpha_2}\right)^{\beta_2}\right\} \tag{5}$$

Now the probability density function $g(t)$ is

$$\begin{aligned} g(t) = {} & \frac{\beta_1}{\alpha_1}\left[\frac{t}{\alpha_1}\right]^{\beta_1 - 1} \exp\left(-\frac{t}{\alpha_1}\right)^{\beta_1} + \frac{\beta_2}{\alpha_2}\left[\frac{t}{\alpha_2}\right]^{\beta_2 - 1} \exp\left(-\frac{t}{\alpha_2}\right)^{\beta_2} - \\ & \exp\left[\left\{-\left(\frac{t}{\alpha_1}\right)^{\beta_1} - \left(\frac{t}{\alpha_2}\right)^{\beta_2}\right\}\right]\left[\frac{\beta_1}{\alpha_1}\left[\frac{t}{\alpha_1}\right]^{\beta_1 - 1} + \frac{\beta_2}{\alpha_2}\left[\frac{t}{\alpha_2}\right]^{\beta_2 - 1}\right] \end{aligned} \tag{6}$$

4. Model Selection and Parameter Estimation

In this article we have applied both the nonparametric (Kaplan-Meier estimate) and parametric (Weibull Probability paper plot and maximum likelihood method) estimation procedures.

The Akaike Information Criterion (AIC), Anderson-Darling (AD) and adjusted Anderson Darling (AD*) are applied to select the best fitted models for the data sets.

4.1 Weibull Probability Paper Plot (WPP plot)

In the early 1970s a special paper was developed using the following transformation, for plotting the data which is known as the Weibull probability paper (WPP) and the plot called the Weibull Probability Paper plot or WPP plot. The WPP plot is a special case of the probability paper plot. It is based on the Weibull transformations:

$y = \ln\{-\ln[1-F(t)]\}$ and $x = \ln(t)$

A plot of *y* versus *x* is called the Weibull probability plot.

4.2 Maximum-Likelihood estimation of lifetime model parameters

Random Censoring

For censored data the likelihood function is given by

$$L(\lambda) = \prod_{i=1}^{n} f(t_i)^{\delta_i} [R(t_i)]^{1-\delta_i}$$

Where δ_i is the failure-censoring indicator for t_i (δ_i hold the value 1 for failed items and 0 for censored).Taking log on both sides we get,

$$\ln L = \sum_{i=1}^{n} \{\delta_i \ln[f(t_i)] + (1-\delta_i)\ln[R(t_i)]\} \quad (7)$$

In the case of 2-fold Weibull multiplicative model, putting the value of CDF and pdf of the model in equation (7), we obtain the log-likelihood function of -fold Weibull multiplicative model.

The maximum likelihood estimates of the parameters are obtained by solving the partial derivative equations of the log-likelihood function with respect to $\alpha_1, \beta_1, \alpha_2$ and β_2. But the estimating equations do not give any closed form of the solutions for the parameters. For that reason, we maximize the log likelihood numerically and obtain the MLE of the parameters. For computations, computer programs are written based on R-language. It is very sensitive to initial values of parameters of these models.

Murthy, Xie and Jiang (2004) applied the WPP plot procedure over the data set and found the 2-fold Weibull multiplicative model as the best fitted model. Besides 2-fold Weibull multiplicative model, here we have taken six others multiplicative models to find out whether any other multiplicative model is suitable for this data set. Here in this paper, the value of -2log-likelihood, Akaike Information Criterion (AIC), AD and the Adjusted AD test statistics of the different seven multiplicative models are estimated for Throttle failure data. The results are displayed in Table-2:

Table-2: Results of various model selection criterions

Multiplicative Model	-2 logL	AIC	AD	Adj AD
1. Weibull-Weibull	150.01	158.01	04.29	04.47
2. Weibull-Exponential	161.71	167.71	11.42	11.88
3. Weibull-Normal	150.47	158.47	04.30	04.48
4. Weibull-Lognormal	151.43	159.43	11.57	12.04
5. Normal-Exponential	164.66	170.66	11.36	11.82
6. Normal-Lognormal	160.83	168.83	11.38	11.84
7. Lognormal-Exponential	160.02	166.02	38.82	40.38

Here, the 2-fold Weibull multiplicative model contains the smallest values for AIC, AD and AD*, among all of the seven multiplicative models, which are 158.01, 4.29 and 4.47, respectively. But the values of AIC, AD and AD* values of Weibull-Normal multiplicative model are very close to the 2-fold Weibull multiplicative model. Hence, we can say that, among all the multiplicative models, the 2-fold Weibull and Weibull-Normal multiplicative models can be selected as the best model for the data according to the value of AIC, AD and AD* test statistic.

Now, the parameters of 2-fold Weibull model, estimated by applying ML and WPP plot methods are displayed in Table-3:

Table-3: Estimated values of the parameters

Multiplicative Model	Parameters	Estimated Values by MLE	Estimated Values by WPP Plot
Weibull-Weibull	$\hat{\beta}_1$	6.2805	0.87
	$\hat{\alpha}_1$	0.7793	8.7
	$\hat{\beta}_2$	0.7625	3.93
	$\hat{\alpha}_2$	8.4003	7.23

5. Result Discussion

For the Throttle failure data set, Murthy et al.(2004) assumed that the 2-fold Weibull multiplicative model fits best. We have estimated the CDF and reliability function of 2-fold Weibull multiplicative model based on non-parametric and parametric approaches by using K-M and ML estimating methods, respectively. The CDF and R(t) are also estimated by using WPP plot [estimates are taken from Murthy et al. (2004)]. Figure-1 represents the reliability function, to see either the WPP plot or the ML method gives the best result for the data set.

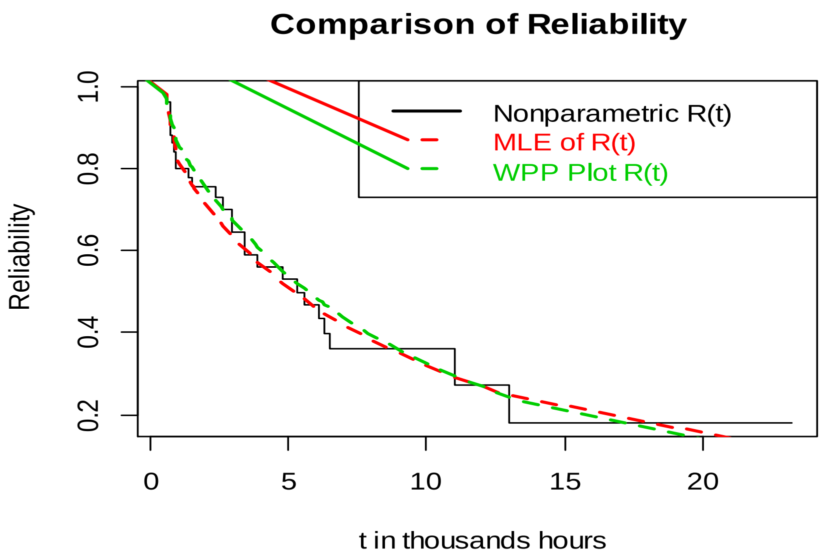


Figure-1: Comparison of reliability functions of 2-fold Weibull multiplicative model based on Kaplan-Meier estimate, WPP plot and ML method

Here, we observe that, both of the reliability functions based on WPP plot and ML method belong very closely to the reliability function based on the K-M estimate. Hence we may use both of the methods to estimate the parameters for this data set.

The CDFs and reliability functions for the seven multiplicative models are estimated by using MLE procedure. From Table-2, by comparing the value of AIC, we observe that, 2-fold Weibull, Weibull-Normal and Weibull-Lognormal multiplicative models contain the 1st, 2nd and 3rd lowest value of AIC. The results of CDFs are displayed in Figure-2 to identify the model that fits best for the data set.

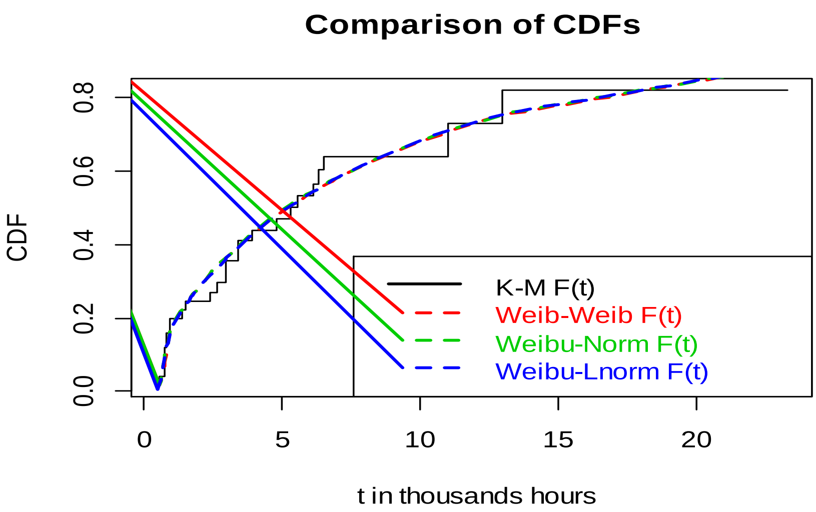


Figure-2: Comparison of CDFs of 2-fold Weibull, Weibull-Normal and Weibull-Lognormal multiplicative models based on Kaplan-Meier and ML estimate

From Table-2 we observe that the 2-fold Weibull and Weibull-Normal multiplicative models can be selected as the best model for the data according to the value of AIC, AD and AD* test statistic. But from Figure-2, we observe that, the CDFs of 2-fold Weibull, Weibull-Normal and Weibull-Lognormal multiplicative models go very close to the plot of non-parametric CDF. Hence, we may use any of these three multiplicative models for throttle failure data set.

In this article we have also estimated the B10 life, median and B75 life of the CDF based on Kaplan-Meier estimate, ML method and WPP plot for throttle failure data and displayed the result in Table-4:

Table-4: B10, median and B75 life

Estimates of	KM	ML	WPP
B10 life	0.753	0.801	0.834
B50 life or median	5.328	5.328	5.900
B75 life	11.019	12.986	12.986

From the above table, we may conclude that, 10% of the total components fail at time 0.753 for K-M procedure, at time 0.801 for MLE method and at time 0.834 for WPP Plot method. 50% of the total components fail at time 5.328 for K-M procedure, at time 5.328 for MLE method and at time 5.900 for WPP Plot method. 75% of the total components fail at time 11.019 for K-M procedure, at time 12.986 for MLE method and at time 12.986 for WPP Plot method.

Hence we may say, the WPP plot over estimate the B10 life and the median life compared with K-M and ML methods.

6. Conclusion

Both of the ML and Weibull probability paper (WPP) plot procedure provide good estimate of the parameters for this data set. Though the 2-fold Weibull multiplicative model fits well based on the values of AIC, AD and AD* test statistic, we found the 2-fold Weibull, Weibull-Normal and Weibull-Lognormal multiplicative models as the well fitted models while comparing the graphs of the CDFs based on the ML method. The WPP plot over estimate the B10 life and the median life compared with K-M and ML methods.

This article analyzed the product failure data. However, the proposed methods and models are also applicable to analyze lifetime data available in the fields, such as, Biostatistics, Medical science, Bio-informatics, etc. The article considered the first failure data of the product. If, there are repeat failures for any product, application of an approach of modeling repeated failures based on renewal function would be relevant. Finally, further investigation on the properties of the methods and models by simulation study would be useful.

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