Vol.3.Issue.3.2015 (Jul-Sept)

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

http://www.bomsr.com

A Peer Reviewed International Research Journal

ISSN:2348-0580

RESEARCH ARTICLE

NOTE ON QUASI NANO B-NORMAL SPACES

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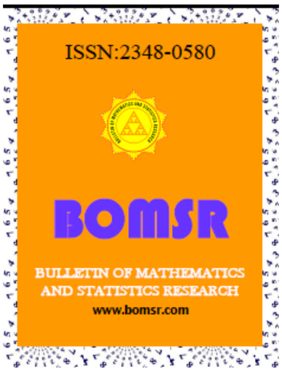


ABSTRACT
The main aim of this paper is to introduce a weaker form of nano b-normality called quasi nano b-normality. We obtain some properties and various preservation theorems of this normality under generalized nano continuous mappings.

Mathematics Subject Classification: 54D10, 54D15, 54C08, 54C10

Key words: quasi nano b-normal space, nano πb-normal space, nano pre πgb-closed map, almost nano πgb-closed map and nano pre πgb-continuous maps.

INTRODUCTION

In 1971 Vigilino [20] defined a new topological property called semi normal spaces. The notions of p-normal spaces, s-normal spaces were introduced by Paul and Bhattacharya [18].Then Signal and Arya [19] introduced the class of almost normal spaces and proved that a space is normal if and only if it is both a semi-normal space and an almost normal space. In recent years, many others have studied several forms of normality [9, 10, 15, 17]. Levine [11] initiated the investigation of g-closed sets in topological spaces, since then many modifications of g-closed sets were defined and investigated by number of topologists [4, 5, 6, 7]. The notion of nano topology was introduced by Lellis Thivagar [12], which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it.

The purpose of the present paper is to introduce a new class of functions called pre πgb-closed functions and pre πgb-continuous functions. We also obtain several preservation theorems of quasi nano b-normal spaces and some of the characterizations of quasi nano πb-normal spaces under nano pre πgb-closed functions.

Preliminaries

Throughout this paper $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ mean nano topological spaces on which no separation axioms are assumed unless explicitly stated. The nano closure of A and the nano interior of A are denoted by Ncl(A) and Nint(A) respectively. A subset A of a space U is said to be nano regularly open or a nano open domain if it is the nano interior of its own nano closure. A set A is said to be a nano regularly closed or a nano closed domain if its complement is an nano open domain.

Definition 2.1[22]: Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space.

Let X $\subseteq$ U

1.The lower approximation of X with respect to R is the set of all objects, which can be for certainly classified as X with respect to R and it is denoted by $L_R(X)$. That is

$L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x∈U.

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is

$U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}$

3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is

$B_R(X) = U_R(X) – L_R(X)$.

Definition 2.2[12]: Let U be non-empty, finite universe of objects and R be an equivalence relation on U. Let X $\subseteq$ U. Let $\tau_R(X)$ = {U, ϕ, $L_R(X)$, $U_R(X)$, $B_R(X)$}. Then $\tau_R(X)$ is a topology on U, called as the nano topology with respect to X. Elements of the nano topology are known as the nano-open sets in U and $(U, \tau_R(X))$ is called the nano topological space. $[\tau_R(X)]^c$ is called as the dual nano topology of $\tau_R(X)$. Elements of $[\tau_R(X)]^c$ are called as nano closed sets.

Definition 2.3[12]: Let $(U, \tau_R(X))$ be a nano topological space and A $\subseteq$ U. Then A is said to be

(i) nano semi-open if A $\subseteq$ Ncl(Nint(A))
(ii) nano pre-open if A $\subseteq$ Nint(Ncl(A))
(iii) nano α -open if A $\subseteq$ Nint(Ncl(Nint(A)))
(iv) nano semi pre-open if A $\subseteq$ Ncl(Nint(Ncl(A)))
(v) nano b-open if A $\subseteq$ Ncl(NintA) $\cup$ Nint(NclA).

NSO(U, X), NPO(U, X), NαO(U, X), NSPO(U, X) and NBO(U, X) respectively denote the families of all nano semi-open, nano pre-open, nano α-open, nano semi pre-open and nano b- open subsets of U.

Let $(U, \tau_R(X))$ be a nano topological space and A $\subseteq$ U. A is said to be nano semi closed, nano pre-closed, nano α-closed, nano semi pre closed and nano b-closed if its complement is respectively nano semi-open, nano pre-open, nano α-open, nano semi pre open and nano regular open.

Definition 2.4[6]: A subset A of a nano topological space $(U, \tau_R(X))$ is called nano generalized b-closed (briefly, nano gb-closed), if Nbcl(A) $\subseteq$ G whenever A $\subseteq$ G and G is nano open in U.

Definition 2.5[13]: A function $f:\left(U,\tau_R(X)\right)\to\left(V,\tau_{R'}(Y)\right)$ is called a nano continuous if the inverse image of every nano closed set in $\left(V,\tau_{R'}(X)\right)$ is nano closed in $\left(U,\tau_R(X)\right)$.

Definition 2.6[7]: A function $f:\left(U,\tau_R(X)\right)\to\left(V,\tau_{R'}(Y)\right)$ is called a nano gb-irresolute if the inverse image of every nano gb-closed set in $\left(V,\tau_{R'}(X)\right)$ is nano gb-closed in $\left(U,\tau_R(X)\right)$.

Definition 2.7: A space X is said to be p-normal [19] (resp.s-normal [14]) if for any pair of disjoint closed sets A and B, there exist disjoint preopen (resp. semi open) sets U and V such that A $\subset$U and B $\subset$ V.

Definition 2.8: A nano topological space $\left(U,\tau_R(X)\right)$ is said to nano b-normal if for any pair of disjoint nano closed sets A and B, there exist disjoint nano b-open sets M and N such that A$\subset$M and B$\subset$N.

3. Nano pre πgb-closed functions

Definition 3.1: A function f : U → V is said to be nano b-closed (resp. nano gb-closed , nano π-closed, nano πgs-closed) if for each nano closed subset F of U, f(F) is a nano b-closed (resp. nano gb-closed, nano π-closed, nano πgb-closed) subset of V .

Definition 3.2: A function f : U → V is said to be nano pre b-closed (resp. nano pre gb-closed , nano pre πgb-closed) if for each nano b-closed subset F of U, f(F) is a nano b-closed (resp. nano gb-closed, nano πgb-closed) subset of V.

Definition 3.3: A function f : U → V is said to be nano regular gb-closed (resp. nano regular πgb-closed) if for each nano b-regular subset F of U, f(F) is a nano gb-closed (resp. nano πgb-closed) subset of V .

Definition 3.4: A function f : U → V is said to be almost nano gb-closed (resp. almost nano πgb-closed) if for each nano regular-closed subset F of U, f(F) is a nano gb-closed (resp. nano πgb-closed) subset of V .

Definition 3.5: A function f : U → V is said to be nano π-irresolute (resp. almost nano continuous) if for each nano π-closed subset (resp. nano regular-open) F of V, $f^{-1}(F)$ is nano π-closed (resp. nano open) subset of U.

Remark 3.6: Let $f:\left(U,\tau_R(X)\right)\to\left(V,\tau_{R'}(Y)\right)$be a function. Then

(i) Every nano pre-b-closed function is nano b-closed
(ii) Every nano pre-b-closed function is nano pre-gb-closed
(iii) Every nano pre-gb-closed function is nano pre- πgb -closed
(iv) Every nano pre- πgb -closed function is nano regular πgb –closed
(v) Every nano gb-closed function is nano πgb -closed
(vi) Every nano gb-closed function is nano regular πgb –closed
(vii) Every nano πgb -closed function is almost nano πgb -closed
(viii) Every almost nano gb-closed function is almost nano πgb –closed

None of the above implications is reversible which can be seen from the following examples.

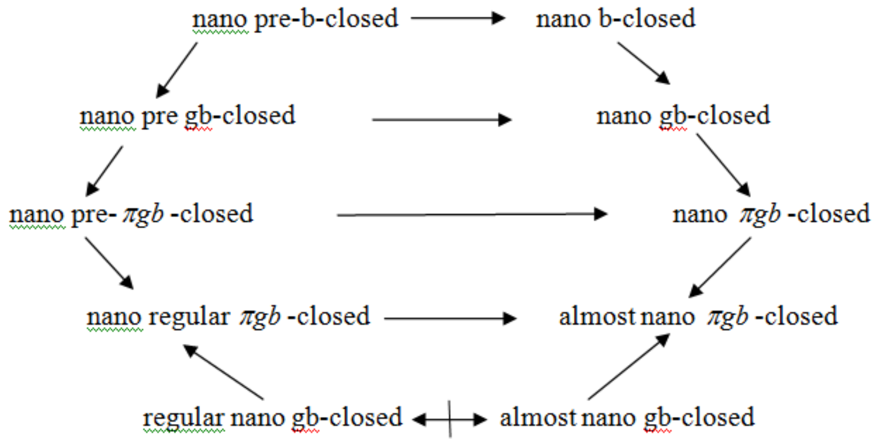
Example 3.7: Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X = {a, b}. Then the nano topology is defined as $\tau_R(X)$ = {U, ϕ, {a}, {b, d}, {a, b, d}} . Let V = {x, y, z, w} with V/R' = {{x}, {y, w}, {z}} and Y = {y, w}. Then $\tau_{R'}(Y)$ = {V,ϕ, {y, w}}. Define $f:\left(U,\tau_R(X)\right)\to\left(V,\tau_{R'}(Y)\right)$ to be f(a) = x, f(b)= x, f(c) = y, f(d) = w. Then f is nano gb-closed map but not nano b-closed because f({b, c, d}) = {x, y, w} is not nano b-closed in V.

Example 3.8: Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X = {a, b}. Then the nano topology is defined as $\tau_R(X)$ = {U, ϕ, {a}, {b, d}, {a, b, d}} . Let V = {x, y, z, w} with V/R' = {{x}, {y, w}, {z}} and Y = {y, w}. Then $\tau_{R'}(Y)$ = {V, ϕ, {y, w}}. Define $f:(U,\tau_R(X))\rightarrow(V,\tau_{R'}(Y))$ to be f(a) = y, f(b)= w, f(c) = z, f(d) = x. Then f is nano pre-gb-closed map but not nano pre-b-closed because f({a, b, c}) = {y, z, w} is not nano b-closed in V.

Example 3.9: Let U = V = {a, b, c, d} with U/R = {{a}, {b, d}, {c}} = V/R' and X = Y = {b, d}. Then $\tau_R(X)$ = $\tau_{R'}(Y)$ = {V, ϕ, {b, d}}. Define $f:U\rightarrow V$ as f(a) = b, f(b) = a, f(c) = c and f(d) = d. Then the function f is nano b-closed but not nano pre-b-closed because f({a, c, d}) = {b, c, d} is not nano b-closed in V.

Example 3.10: Let U = V = {a, b, c, d} with U/R = {{a}, {b, c}, {d}} = V/R' and X = Y = {a, c}. Then $\tau_R(X)$ = $\tau_{R'}(Y)$ = {V, ϕ, {a}, {a, b, c}, {b, c}}. Define $f:U\rightarrow V$ as f(a) = c, f(b) = a, f(c) = b and f(d) = d. Then the function f is nano gb-closed but not nano pre-gb-closed because f({b, c}) = {a, b} is not nano gb-closed in V.

By definitions we obtain the following relationships between the classes of functions defined.



Theorem 3.11: A function f : U → V is a nano pre πgb-closed(resp. nano regular πgb-closed) if and only if for each subset B of V and each M $\in$ NBO(U) (resp. M $\in$ NBR(U)) containing $f^{-1}(B)$, there exists a nano πgb-open subset N of V such that B $\subseteq$ N and $f^{-1}(N)\subseteq$ M.

Proof: Necessity. Suppose that f is nano pre πgb-closed (resp. nano regular πgb-closed). Let B be a subset of V and M $\in$ NBO(U) (resp. M$\in$ NBR(X)) containing $f^{-1}(B)$. If N = V \ f(U \ M), then N is a nano πgb-open subset of V such that B $\subseteq$ N and $f^{-1}(N)\subseteq$ M.

Sufficiency. Let F be any nano b-closed (resp. nano b-regular) subset of U. Then $f^{-1}(V\setminus f(F))\subseteq$ U \ F and U \ F$\in$ NBO(U) (resp. U \ F $\in$ NBR(U)). There exists a nano πgb-open subset N of V such that V \ f(F) $\subseteq$ N and $f^{-1}(N)\subseteq$ U \ F. Therefore, we have f(F) $\supseteq$ V \ N and F$\subseteq$ $f^{-1}(V\setminus N)$. So, we obtain f(F) = V \ N and f(F) is nano πgb-closed in V . This shows that f is nano pre πgb-closed (resp. nano regular πgb-closed).

Corollary 3.12: If a function f : U → V is nano pre πgb-closed (resp. nano regular πgb-closed), then for each nano π-closed K of V and each M$\in$ NBO(U) (resp. M$\in$NBR(U)) containing $f^{-1}(K)$, there exists N$\in$ NBO(V) containing K such that $f^{-1}(N)\subseteq$ M.

Theorem 3.13: If f : U → V is nano π-irresolute and nano pre πgb-closed, then f(A) is nano πgb-closed in V for every nano πgb-closed subset A of U.

Proof: Let A be any nano πgb-closed subset of U and M a nano π-open subset of V containing f(A). Since $f^{-1}(M)$ is a nano π-open subset of U containing A and A is a nano πgb-closed set, NbCl(A) $\subseteq f^{-1}(M)$. Since f is nano pre πgb-closed and NbCl(A) $\in$NbCl(U), we have NbCl(f(A)) $\subseteq$ NbCl(f(NbCl(A))) $\subseteq$ M. Therefore, f(A) is a nano πgb-closed in V.

Definition 3.14: A function f : U → V is said to be nano pre gb-continuous (resp. nano pre πgb-continuous) if $f^{-1}(K)$ is nano gb-closed (resp. nano πgb-closed) in U for every nano b-closed subset K of V.

Theorem 3.15: If f : U → V is nano π-closed nano pre πgb-continuous, then $f^{-1}(K)$ is nano πgb-closed in U for every nano πgb-closed subset K of V.

Proof: Let K be a nano πgb -closed set of V and M a nano π -open set of U containing $f^{-1}(K)$. Put N = V \ f(U \ M), then N is nano π -open in V, K$\subset$N, and $f^{-1}(N) \subset$M. Therefore we have NbCl(K) $\subset$N and hence $f^{-1}(K) \subset f^{-1}(\mathcal{N}bCl(K)) \subset f^{-1}(N) \subset$M. Since f is nano pre πgb -continuous, $f^{-1}(\mathcal{N}bCl(K))$ is nano πgb -closed in U and hence $\mathcal{N}bCl(f^{-1}(K)) \subset \mathcal{N}bCl(f^{-1}(\mathcal{N}bCl(K))) \subset$M. This shows that $f^{-1}(K)$ is nano πgb -closed in U.

Theorem 3.16: If f : U → V is nano open nano pre πgb-continuous bijection, then $f^{-1}(K)$ is nano πgb-closed in U for every nano πgb-closed subset K of V.

Proof: Let K be a nano πgb -closed set of V and M be a nano π -open set of U containing $f^{-1}(K)$. Since f is nano open surjective, K = $f(f^{-1}(K)) \subset$f(M) and f(M) is nano π -open. Therefore, NbCl(K) $\subset$f(M). Since f is injective, $f^{-1}(K) \subset f^{-1}(\mathcal{N}bCl(K)) \subset f^{-1}(f(M))$ = M. Since f is nano pre-πgb continuous, $f^{-1}(\mathcal{N}bCl(K))$ is nano πgb -closed in U. And hence $\mathcal{N}bCl(f^{-1}(K)) \subset \mathcal{N}bCl(f^{-1}(\mathcal{N}bCl(K))) \subset$M. Therefore $f^{-1}(K)$ is nano πgb -closed in U.

Corollary 3.17: Let f : U → V and g: V → W be functions. Then the composition gof : U → W is nano pre πgb-closed if f and g satisfy one of the following conditions:

(a) f is nano pre πgb-closed and g is nano π-irresolute and nano pre πgb-closed

(b) f is nano pre b-closed and g is nano pre πgb-closed

(c) f is nano b-closed and g is nano πgb-closed

Proof: (a) The proof follows immediately from Theorem 3.13, while (b) and (c) are obvious.

Theorem 3.18: Let f : U → V and g: V → W be functions such that the composition g ∘ f : U → W is g: V → W nano pre πgb-closed if f and g satisfy one of the following conditions.

(a) f is an nano irresolute surjection, then g is nano pre πgb-closed

(b) g is a nano π-closed, nano pre πgb-continuous injection, then f is nano pre πgb-closed

(c) g is a nano π-closed, nano irresolute bijection, then f is nano pre πgb-closed.

Proof: (a) Let K $\in$ NbCl(U). Since f is an nano irresolute surjection, $f^{-1}(K)$ $\in$ NbCl(U) and (g ∘ f)$(f^{-1}(K))$ = g(K).Therefore g(K) is nano πgb-closed in W and hence g is nano pre πgb-closed.

(b) The proof follows from Theorem 3.16.

(c) Let F be a nano b-closed subset of U and M be a nano π-open subset of V containing f(F). Since g ∘ f is nano πgb-closed, (g ∘ f)(F) is a nano πgb-closed subset of W. Put N = V \ M. Then N is a nano π-closed in V . Since g is a nano π-closed bijection, g(N) is nano π-closed in W and g(N) = W \ g(M) $\subseteq$ W

\ (g ∘ f)(F) and hence g(N) ∩ (g ∘ f)(F) = ϕ . Since (g ∘ f)(F) is a nano πgb-closed subset of W, NbCl((g ∘ f)(F)) $\subseteq$ W \ g(N). Since g is an nano irresolute bijection, NbCl(f(F)) = NbCl ((g^{-1} ∘ (g ∘ f))(F)) $\subseteq$ g^{-1}(NbCl((g ∘ f)(F))) $\subseteq$ g^{-1} (W \ g(N)) = V \ N = M.

4. Quasi nano b-normal spaces

Definition 4.1: A nano topological space $(U,\tau_R(X))$ is said to be quasi nano b-normal if for every pair of disjoint nano π-closed subsets A and B of U there exist disjoint nano b-open subsets M and N of U such that A$\subseteq$M and B$\subseteq$N.

Theorem 4.2: For a nano topological space $(U,\tau_R(X))$ the following are equivalent:

(1) U is quasi nano b-normal,

(2) For every pair of nano π-open subsets M and N of U whose union is U, there exist nano b-closed subsets G and H of U such that G$\subseteq$M, H$\subseteq$N and G $\cup$ H = U,

(3) For any nano π-closed set A and each nano π-open set B such that A$\subseteq$B, there exists a nano b-open set M such that A $\subset$M $\subset$ NbCl(M)$\subset$B.

(4) For every pair of disjoint nano π-closed subsets A and B of U, there exist nano b-open subsets M and N of U such that A$\subseteq$M, B$\subseteq$N and NbCl(M) $\cap$ NbCl(N) = ϕ.

Proof: (1) $\Rightarrow$ (2): Let M and N be any nano π-open subsets of a quasi nano b-normal space U such that M$\cup$N = U. Then, U \ M and U \ N are nano π-closed subsets of U. By quasi nano b-normality of U, there exist disjoint nano b-open subsets M_1 and such N_1 of U such that U \ M $\subseteq M_1$ and U \ N $\subseteq N_1$.

Let G = U \ M_1, H = U \ N_1. Then G and H are nano b-closed subsets of U such that G$\subseteq$M and H$\subseteq$N and G $\cup$H = U.

(2) $\Rightarrow$(3) Let A be a nano π-closed set and B be a nano π-open set of U such that A$\subseteq$B. Then U \ A and B are nano π-open subsets of U whose union is U. Then by (2), there exist nano b-closed sets G and H of U such that G $\subseteq$ U \ A and H $\subset$B and G $\cup$H = U. Then A $\subset$U \ G and U \ B $\subset$ U \ H and (U \ G) $\cap$ (U \ H) = ϕ. Let M = U \ G and N = U \ H. Then M and N are disjoint nano b-open sets such that A $\subseteq$M $\subseteq$U \ N $\subset$B.

As U \ N is nano b-closed set, we have NbCl(M) $\subseteq$ U \ N and A $\subseteq$M $\subseteq$ NbCl(M) $\subseteq$B.

(3)$\Rightarrow$ (4): Let A and B be any two disjoint nano π-closed sets of U. Then A$\subseteq$U \ B where U \ B is nano π-open. Then by (3), there exists a nano b-open set M of U such that A $\subseteq$ M $\subseteq$NbCl(M) $\subseteq$U \ B. N = U \ NbC(M). Then N is nano b-open subset of U. Thus, we obtain A$\subseteq$M and B $\subseteq$ N and NbCl(M) $\cap$NbCl(N) = ϕ.

(4) $\Rightarrow$ (1). It is obvious.

Proposition 4.3: Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a function, then:

(i) The image of nano b-open subset under n nano open nano continuous function is nano b-open.

(ii) The inverse image of nano b-open (resp. nano b-closed) subset under a nano open nano continuous function is nano b-open.

(iii) The image of nano b-closed subset under an nano open and a nano closed nano continuous surjective function is nano b-closed.

Theorem 4.4: The image of a quasi nano b-normal space under a nano open nano continuous injective function is quasi nano b-normal.

Proof: Let U be a quasi nano b-normal space and let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a nano open nano continuous injective function. We have to prove that f(U) is quasi nano b-normal. Let A and B be two disjoint nano π-closed sets in f(U). Since the inverse image of a nano π-closed set under a nano open , nano continuous function is nano π-closed, we have $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint nano π-closed sets in U. Since U is quasi nano b-normal, there exists nano b-open sets M and N of U such that $f^{-1}(A)\subseteq$M, $f^{-1}(B)\subseteq$N and M$\cap$N = ϕ. Since f is nano open nano continuous injective function, we have A$\subseteq$f(M), B$\subseteq$f(N) and f(M) $\cap$f(N) = ϕ. By the Preposition 4.3, f(M) and f(N) are disjoint nano b-open sets in f(U) such that A$\subseteq$f(M) and B$\subseteq$f(N). Hence, f(U) is quasi nano b-normal space.

Theorem 4.5: For a space $(U,\tau_R(X))$, the following are equivalent:

(a) U is quasi nano b-normal.

(b) for any disjoint nano π-closed subsets A and B of U, there exist disjoint nano gb-open subsets M and N of U such that A$\subseteq$M and B$\subseteq$N.

(c) for any nano π-closed set A and any nano π-open set B such that A $\subseteq$B, there exists a nano gb-open subset M of U such that A $\subseteq$M $\subseteq$ NbCl(M) $\subseteq$B.

(d) for any disjoint nano π-closed subsets A and B of U, there exist disjoint nano πgb-open subsets M and N of U such that A$\subseteq$M and B$\subseteq$N.

(e) For every nano π-closed set A and every nano π-open set B such that A $\subseteq$B, there exists a nano πgb-open subset M of U such that A $\subseteq$M $\subseteq$ NbCl(M) $\subseteq$B.

Proof: (a)$\Rightarrow$ (b). Let U be a quasi b-normal space. Let A and B be any disjoint nano π-closed subsets of U. By quasi nano b-normality of U, there exist disjoint nano b-open subsets M and N of U such that A $\subseteq$M and B $\subseteq$N. Thus, M and N are disjoint nano gb-open subsets of U such that A $\subseteq$M and B $\subseteq$N.

(b) $\Rightarrow$ (c). Suppose (b) holds. Let A be a nano π-closed and B be a nano π-open subset of U such that A$\subseteq$B. Thus, A and U \ B are disjoint nano π-closed subsets of U. By (b), there exists disjoint nano gb-open subsets M and N of U such that A$\subseteq$M and U \ B$\subseteq$N. By the definition of nano gb-open sets, we have U \ B$\subseteq$Nbint(N) and M $\cap$ Nbint(N) = ϕ. Therefore, we obtain NbCl(M)$\cap$ Nbint(N) = ϕ and hence A $\subseteq$M $\subseteq$ NbCl(M) $\subseteq$B.

(c) $\Rightarrow$ (d). Let A and B be any disjoint nano π-closed subsets of U. Then A$\subseteq$ U \ B and U \ B is nano π-open and hence there exists a nano gb-open subset G of U such that A $\subseteq$G $\subseteq$ NbC(G) $\subseteq$U \ B. Since every nano gb-open set is nano πgb-open, G is nano πgb-open and U \ Nbcl(G) is nano b-open and so U \ Nbcl(G) is nano πgb-open. Now put N = U \ Nbcl(G). Then G and N are disjoint nano πgb-open subsets of U such that A$\subseteq$G and B$\subseteq$N.

(d) $\Rightarrow$(e) The proof is similar to that of (b) $\Rightarrow$ (c).

(e) $\Rightarrow$ (a) Let A and B be any disjoint nano π-closed subsets of U. Then A$\subseteq$ U \ B and U \ B is nano π-open and hence there exists a nano πgb-open subset G of U such that A $\subseteq$G $\subseteq$ NbCl(G) $\subseteq$U \ B. Put M = Nbint(G) and N = U \ Nbcl(G). Then M and N are disjoint nano b-open subsets of U such that A$\subseteq$G and B$\subseteq$N. Therefore, U is quasi nano b-normal.

Definition 4.6: A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is said to be

(a) almost nano continuous (resp. nano rc-continuous) if $f^{-1}(F)$ is a nano closed (resp. nano closed domain) set in U for each nano closed domain subset F of V.

(b) almost nano closed (resp. nano rc-preserving) function if f(F) is nano closed (resp. nano closed domain) set in V for each nano closed domain subset F of U.

(c) weakly nano open if for each nano open subset F of U, f(F) $\subseteq$ N$\operatorname{int}(f(\overline{F}))$

(d) nano R-map (resp. completely nano continuous) if $f^{-1}(F)$ is nano open domain in U for every nano open domain (resp. nano open) subset F of V.

(e) nano π-continuous (resp. nano rc-continuous) if $f^{-1}(A)$ is nano π-closed (resp. nano closed domain) set in U for each nano closed set (resp. nano closed domain) set A in V.

(f) almost nano b-irresolute if for each u in U and each nano b-neighbourhood N of f(u) in V, N$b-cl(f^{-1}(N))$ is a nano b-neighbourhood of u in U.

(g) nano Mb-closed (nano Mb-open), if f(A) is nano b-closed (resp. nano b-open) set in V for each nano b-closed (resp. nano b-open) set A in U.

Lemma 4.7: If a function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is weakly nano open nano continuous function, then f is nano Mb-open and nano R-map.

Definition 4.8: A function $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is almost nano πgb-closed if for each nano regular closed subset F of U, f(F) is nano πgb-closed subset of V.

Proposition 4.9: A surjection $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ is almost nano πgb-closed if and only if for each subset G of V and each M$\in$NRO(U) containing $f^{-1}(G)$, there exists a nano πgb-open subset N of V such that G$\subseteq$N and $f^{-1}(N)\subseteq$M.

Proof: Necessity. Suppose that f is almost nano πgb-closed. Let G be a subset of V and M$\in$NRO(U) containing $f^{-1}(G)$. If N = V \ f(U \ M), then N is a nano πgb-open set of V such that G$\subseteq$N and $f^{-1}(N)\subseteq$M.

Sufficiency. Let F be any nano regular closed set of U. Then U \ F$\in$ NRO(U) and $f^{-1}(V\setminus f(F))\subseteq$U\ F. There exists a nano πgb-open set N of V such that V \ f(F) $\subseteq$N and $f^{-1}(N)$ $\subseteq$U \ F. Therefore, we have V \ N $\subseteq$f(F) and F $\subseteq$ $f^{-1}(V\setminus N)$. Hence we obtain f(F) = V \ N and f(F) is nano πgb-closed in V. This shows that f is almost nano πgb-closed.

Theorem 4.10: If $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a nano continuous, almost nano πgb-closed surjection. If U is nano normal, then V is quasi nano b-normal.

Proof: Let A and B be any disjoint nano π-closed subsets of V. Since f is nano continuous $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint nano closed subsets of U. By the normality of U, there exist disjoint nano open subsets M and N of U such that $f^{-1}(A)\subseteq$M and $f^{-1}(B)$ $\subseteq$N. Put G = Nint(Ncl(M)) and H = Nint(Ncl(N)). Then G and H are disjoint nano regular open subsets of U such that $f^{-1}(A)\subseteq$G and $f^{-1}(B)$ $\subseteq$H. By Proposition 4.3, there exists nano πgb-open subsets K and L of V such that A $\subseteq$K and B $\subseteq$L, $f^{-1}(K)$ $\subseteq$ G and $f^{-1}(L)$ $\subseteq$H. Since G $\cap$H = ϕ and f is surjective , K$\cap$L = ϕ. It follows from Theorem 4.5 (d) that V is quasi nano b-normal.

Theorem 4.11: If $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ be a nano π-irresolute, almost nano πgb-closed surjection. If U is quasi nano normal, then V is quasi nano b-normal.

Proof: Let A and B be any disjoint nano π-closed subsets of V. Since f is nano π-irresolute $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint nano π -closed subsets of U. By the quasi-normality of U, there exist disjoint nano open subsets M and N of U such that $f^{-1}(A) \subseteq$M and $f^{-1}(B)$ $\subseteq$N. Put G = Nint(Ncl(M)) and H = Nint(Ncl(N)). Then $f^{-1}(A) \subseteq$M$\subseteq$G, $f^{-1}(B) \subseteq$N$\subseteq$H, G $\cap$ H = ϕ and G and H are disjoint nano regular open subsets of U. Since f is almost nano πgb-closed, by **Proposition 4.3,** there exists nano πgb-open subsets K and L of V such that A $\subseteq$K and B $\subseteq$L, $f^{-1}(K) \subseteq$ G and $f^{-1}(L)$ $\subseteq$H. Since G $\cap$ H = ϕ and f is surjective we have, K $\cap$ L = ϕ. Hence V is quasi nano b-normal.

Theorem 4.12: Let f : X → Y be a π-irresolute pre πgb-closed surjection. If X is quasi-b-normal, then Y is quasi-b-normal.

Proof: Let P_1 and P_2 be any disjoint nano closed sets of V. Since f is nano π-irresolute, $f^{-1}(P_1)$ and $f^{-1}(P_2)$ are disjoint nano π-closed sets of U. Since U is quasi nano b-normal, there exist disjoint M_1, $M_2 \in$NBO(U) such that $f^{-1}(P_i) \subset M_i$ for i = 1, 2. Put N_i = V- f(U-M_i), then N_i is nano πgb-open in V, $P_i \subset N_i$ and $f^{-1}(N_i) \subset M_i$ for i = 1, 2. Since $M_1 \cap M_2$ = ϕ and f is surjective; we have $N_1 \cap N_2$ = ϕ. This shows that V is quasi nano b-normal.

Theorem 4.13: Let f : X → Y be a nano π-closed, nano pre πgs-continuous injection. If V is quasi nano -b-normal, then U is quasi nano-b-normal.

Proof: Let Q_1 and Q_2 be any disjoint nano π-closed sets of U. Since f is nano π-closed injection, f(Q_1)and f(Q_2) are disjoint nano π-closed sets of V. Since V is quasi nano b-normal, there exist disjoint N_1, $N_2 \in$NBO(V) such that f(Q_i)$\subset N_i$ for i = 1, 2. Since f is nano pre πgb continuous $f^{-1}(N_1)$ and $f^{-1}(N_2)$ are nano πgb -open sets of U and $Q_i \subset f^{-1}(N_i)$ for i = 1,2. Now put M_i = Nb-int$(f^{-1}(N_i))$ for i = 1, 2. Then $M_i \in$NBO(U), $Q_i \subset M_i$ and $M_1 \cap M_2$ = ϕ. This shows that U is nano gb-normal.

Theorem 4.14: If $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is a nano Mb-open nano rc-continuous and almost nano b-irresolute function from a quasi nano b-normal space onto a space V, then V is quasi nano b-normal.

Proof: Let A be nano π-closed and B be a nano π-open subsets of V such that A$\subseteq$B. Then by nano rc-continuity of f, $f^{-1}(A)$ is nano π-closed and $f^{-1}(B)$ is nano π-open subsets of U such that $f^{-1}(A) \subseteq f^{-1}(B)$. Since U is quasi nano b-normal, then by the Theorem 4.2, there exists a nano b-open subset N of U such that $f^{-1}(A) \subseteq$N$\subseteq$Nbcl(N) $\subseteq f^{-1}(B)$. Since f is nano Mb-open and an almost nano b-irresolute surjection, it follows that f(N) is nano b-open subset of V and A$\subseteq$f(N) $\subseteq$Nbcl(f(N)) $\subseteq$B. Hence by Theorem 4.2, V is quasi nano b-normal.

Theorem 4.15: If $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is a nano weakly nano open nano π-continuous almost nano b-irresolute surjection and U is quasi nano b-normal, then V is quasi nano b-normal.

Proof: Let A be nano π-closed and B be a nano π-open subsets of V such that A$\subseteq$B. Then by nano π-continuity of f we have, $f^{-1}(A)$ is nano π-closed and $f^{-1}(B)$ is nano π-open subsets of U such that $f^{-1}(A) \subseteq f^{-1}(B)$. Since U is quasi nano b-normal, then by the Theorem 4.2, there exists a nano b-open subset M of U such that $f^{-1}(A) \subseteq$M$\subseteq$Nbcl(M) $\subseteq f^{-1}(B)$. Then $f(f^{-1}(A)) \subseteq$f(M)$\subseteq$ f(Nbcl(M)) $\subseteq f(f^{-1}(B))$. Since f is a weakly nano open nano continuous and an almost nano b-irresolute surjection, then by the Lemma 4.7, we have f is nano Mb-open and nano

R-map. Thus we have, f(M) is a nano b-open subset of V such that A$\subseteq$f(N) $\subseteq$Nbcl(f(N)) $\subseteq$B. Hence by Theorem 4.2, V is quasi nano b-normal.

Theorem 4.16: Let f : U → V be an almost nano continuous, nano regular πgb-closed surjection. If U is nano b-normal, then V is quasi nano-b-normal.

Proof: Let A and B be disjoint nano π-closed subsets of V. Since f is almost nano continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint nano closed subsets of U. By the nano b-normality of U, there exist disjoint nano b-open sets M and N such that $f^{-1}(A) \subseteq$ M and $f^{-1}(B) \subseteq$ N. Put G = NbCl(M) and H = NbCl(N). Then $f^{-1}(A) \subseteq$ M ⊂ G, $f^{-1}(B) \subseteq$ N $\subseteq$ H, G ∩ H = ϕ and G, H$\in$ NBR(U). Since f is nano regular πgb-closed, there exist nano πgb-open sets K and L such that A$\subseteq$ K, B$\subseteq$ L, $f^{-1}(K) \subseteq$G and $f^{-1}(L) \subseteq$ H. Therefore, K ∩ L = ϕ. This shows that V is quasi nano-b-normal.

The following Theorem can be proved easily by using arguments similar to those in the Theorem 4.14 and Theorem 4.15.

Theorem 4.17: The following statements are true:

(a) If $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is nano rc-continuous, nano Mb-closed map from a quasi nano b-normal space U onto a space V, then V is quasi nano b-normal.

(b) If $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is nano R-map nano pre gb-closed surjection and U is quasi nano b-normal, then V is quasi nano b-normal.

(c) If $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is completely nano continuous nano pre gb-closed surjection and U is quasi nano b-normal, then V is nano b-normal.

(d) If $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is almost nano continuous nano pre gb-closed surjection and U is nano b-normal, then V is quasi nano b-normal.

(e) If $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is nano π-continuous weakly nano open nano pre gb-closed surjection and U is quasi nano b-normal, then V is nano b-normal.

(f) If $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is nano pre-gb continuous nano rc-preserving injection and V is quasi nano b-normal, then U is quasi nano b-normal.

(g) If $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is nano pre-gb continuous almost nano closed injection and V is nano b-normal, then V is quasi nano b-normal.

5. Characterizations of nano πb-normality and preservation theorems

Definition 5.1: A nano topological space $(U,\tau_R(X))$ is said to be nano πb-normal if for any pair of disjoint nano closed sets A and B of U, one of which is nano π-closed, there exist disjoint nano b-open sets M and N of U such that A$\subset$M and B$\subset$N.

Theorem 5.2: For a nano topological space $(U,\tau_R(X))$ the following are equivalent:

(1) U is nano πb-normal,

(2) For every pair of nano open sets M and N, one of which is nano π-open, whose union is U, there exist nano b-closed sets G and H such that G$\subseteq$M, H$\subseteq$N and G $\cup$ H = U,

(3) For every nano closed set A and every nano π-open set B such that A$\subseteq$B, there exists a nano b-open set N such that A $\subset$N $\subset$ NbCl(M)$\subset$B.

Proof: (1) $\Rightarrow$ (2): Let M and N be a pair of nano open sets in a nano πb-normal space U such that N is nano π-open and M$\cup$N = U. Then, U \ M, U \ N are nano closed sets in U such that U \ N is nano π-closed and (U \ M) $\cap$ (U \ N) = ϕ. Since U is nano πb-normal there exist nano b-open sets M_1 and N_1 such that U \ M $\subseteq$$M_1$ and U \ N $\subseteq$$N_1$.

Let G = U \ M_1, H = U \ N_1. Then G and H are nano b-closed sets of U such that G $\subseteq$ M and H $\subseteq$ N and G $\cup$ H = U.

(2) $\Rightarrow$(3) Let A be a nano closed set and B be a nano π-open set of U such that A $\subseteq$ B. Then A $\cap$ (U \ B) = ϕ. Thus, (U \ A) $\cup$ B = U, where U \ A is nano open. Then by (2), there exist nano b-closed sets G and H of U such that G $\subseteq$ U \ A and H $\subseteq$ B and G $\cup$ H = U. Thus we obtain that A $\subseteq$ U \ G and U \ B $\subseteq$ U \ H. Let N = U \ G then N is nano b-open set of U. Therefore we have A $\subseteq$ N $\subseteq$ NbCl(M) $\subset$ B.

(3)$\Rightarrow$ (1): Let A and B be any disjoint nano closed sets of U such that B is nano π-closed. Since A $\cap$ B = ϕ, then A $\subseteq$ U \ B and U \ B is nano π-open. Then by (3), there exists a nano b-open set N of U such that A $\subseteq$ N $\subseteq$ NbCl(N) $\subseteq$ U \ B. Put G = N and H = U \ NbCl(N). Then G and H are disjoint nano b-open subsets of U such that A $\subseteq$ G and B $\subseteq$ H. Hence U is nano πb-normal.

Theorem 5.3: The image of a nano πb-normal space under an nano open nano continuous injective function is nano πb-normal.

Proof: Let U be a nano πb-normal space and let $f:(U,\tau_R(X))\rightarrow(V,\tau_{R'}(Y))$ be a nano open nano continuous injective function. We need to show that f(U) is nano πb-normal. Let A and B be two disjoint nano closed sets in f(U) such that A is nano π-closed. Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint nano closed sets in U such that $f^{-1}(A)$ is nano π-closed. Since U is nano πb-normal, there exists nano b-open sets M and N such that $f^{-1}(A)\subseteq$ M, $f^{-1}(B)\subseteq$ N and M $\cap$ N = ϕ. Since f is nano open nano continuous injective function, we have A $\subseteq$ f(M), B $\subseteq$ f(N) and f(M) $\cap$ f(N) = ϕ. By the Preposition 4.3, f(M) and f(N) are disjoint nano b-open sets in f(U) such that A $\subseteq$ f(M) and B $\subseteq$ f(N). Hence, f(U) is nano πb-normal space.

Theorem 5.4: If $f:(U,\tau_R(X))\rightarrow(V,\tau_{R'}(Y))$ is a nano continuous nano Mb-open nano rc-continuous and almost nano b-irresolute surjection from a nano πb-normal space U onto a space V, then V is nano πb-normal.

Proof: Let A be a nano closed subset of V and B be a nano π-open subset of V such that A $\subseteq$ B. By nano continuity and nano rc-continuity of f, we obtain that $f^{-1}(A)$ is nano closed in U and $f^{-1}(B)$ is nano π-open in U such that $f^{-1}(A) \subseteq f^{-1}(B)$. Since U is nano πb-normal, then by the Theorem 5.2, there exists a nano b-open set M of U such that $f^{-1}(A)$ $\subseteq$ M $\subseteq$ Nbcl(M) $\subseteq$ $f^{-1}(B)$. Then $f(f^{-1}(A))$ $\subseteq$ f(M) $\subseteq$ f(Nbcl(M)) $\subseteq$ $f(f^{-1}(B))$. Since f is nano Mb-open almost nano b-irresolute surjection, we obtain that A $\subset$ f(M) $\subset$ NbCl(f(M)) $\subset$ B and f(M) is nano b-open set in V. Hence by Theorem 5.2, V is nano πb-normal.

Theorem 5.5: If $f:(U,\tau_R(X))\rightarrow(V,\tau_{R'}(Y))$ is a nano Mb-open nano π-continuous almost nano b-irresolute function from a nano πb-normal space U onto a space V, then V is nano πb-normal.

Proof: Let A be a nano closed subset of V and B be a nano π-open subset of V such that A $\subseteq$ B. By nano π-continuity of f, we obtain that $f^{-1}(A)$ is nano π-closed(hence nano closed) in U and $f^{-1}(B)$ is nano π-open in U such that $f^{-1}(A) \subseteq f^{-1}(B)$. Since U is nano πb-normal, there exists a nano b-open set M of U such that $f^{-1}(A)$ $\subseteq$ M $\subseteq$ NbCl(M) $\subseteq$ $f^{-1}(B)$. Since f is nano Mb-open and almost nano b-irresolute surjection, then A $\subset$ f(M) $\subset$ NbClf((M)) $\subset$ B and f(M) is nano b-open set in V. Hence by Theorem 5.2, V is nano πb-normal.

Theorem 5.6: If $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is a nano Mb-closed nano π-continuous function from a nano πb-normal space U onto a space V, then V is nano πb-normal.

Proof is similar to the Theorem 5.5.

Theorem 5.7: If $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is a nano closed nano π-continuous surjection and U is nano π-normal, then V is nano πb-normal.

Proof: Let A and B be disjoint nano closed sets in V such that A is nano π-closed. Then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint nano π-closed sets of U by nano π-continuity of f. Since U is nano π-normal, there exist disjoint nano open sets M and N such that $f^{-1}(A) \subset$M and $f^{-1}(B) \subset$N. By Theorem 6[17], there are disjoint nano α- open sets G and H in V such that A$\subset$G and B$\subset$H. Since every nano α-open set is nano b-open, G and H are disjoint nano b-open sets containing A and B respectively. Therefore, V is nano πb-normal.

Theorem 5.8: If $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is a nano closed nano π-continuous surjection and U is nano π-normal, then V is nano πb-normal.

Proof: Let A be disjoint nano closed sets in V such that A is nano π-closed. By nano π-continuity of f, $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint nano π-closed sets of U. Since U is nano π-normal, there exists disjoint nano open sets M and N of U such that $f^{-1}(A)$ $\subseteq$M and $f^{-1}(B)$ $\subseteq$N. By the Proposition 4.3, there are disjoint nano α-open sets G and H in V such that A$\subseteq$G and B$\subseteq$H. Since every nano α-open set is nano b-open, then G and H are disjoint nano b-open sets containing A and B, respectively. Therefore, V is nano πb-normal.

Theorem 5.9: If $f:(U,\tau_R(X)) \to (V,\tau_{R'}(Y))$ is a nano π-continuous, weakly nano open nano pre gb-closed surjection and U is quasi nano b-normal, then V is nano πb-normal.

Proof: Let A and B be any disjoint nano closed subsets of V such that A is nano π-closed. Since f is nano π-continuous surjection, then $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint nano π-closed subsets of U. Since U is quasi nano b-normal, then there exist disjoint nano b-open subsets M and N of U such that $f^{-1}(A) \subseteq$M and $f^{-1}(B)$ $\subseteq$N. Since f is a weakly nano open nano continuous surjection, then by the Lemma 4.7, we have f is nano Mb-open and nano R-map. Thus, f(M) and f(N) are disjoint nano b-open subsets of V such that A$\subseteq$f(M) and B$\subseteq$f(N). Hence, V is nano πb-normal.

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