Vol.3.Issue.3.2015 (Jul-Sept)



http://www.bomsr.com

RESEARCH ARTICLE

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal



A NEW CLASS OF FUZZY IRRESOLUTE MAPPINGS ON INTUITIONISTIC SPACES

H. JUDE IMMACULATE*, I. AROCKIARANI

Department of Mathematics, Nirmala College for women, Coimbatore, Tamilnadu, India *E-mail:juderaj.h@gmail.com



ABSTRACT

The intend of this paper is to initiate weak and strong forms of intuitionistic fuzzy irresolute mappings. Some of their characterizations and the relations between these irresolute mapping are analyzed with counter examples.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy completely dcontinuous mapping, intuitionistic fuzzy contra d-continuous mappings.

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1. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh in his classical paper [19] in 1965.After the introduction of fuzzy sets there have been a number of generalizations of this fundamental concept. The notion of intuitionistic fuzzy sets introduced by Atanassov [2] is one among them. Using the notion of intuitionistic fuzzy sets, Coker [6] introduced the notion of intuitionistic fuzzy topological spaces. The notion of intuitionistic fuzzy d-continuous mappings was introduced by I. Arockiarani et.al[1]. In this paper we propose the idea of intuitionistic fuzzy d-irresolute mapping, intuitionistic fuzzy apd-irresolute mapping and apd-closed mapping by using the notion of intuitionistic fuzzy d-closed sets and study some of its properties. This study enables us to obtain the preservation properties of these mappings. Also in this paper we present a new generalization of irresoluteness called contra d-irresoluteness which is a stronger form of intuitionistic fuzzy apdirresoluteness. The relationship between these mappings are discussed.

2. PRELIMINARIES

Definition 2.1[2]: Let X be a nonempty fixed set. An *intuitionistic fuzzy set* (IFS, for short) A is an object having the form $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle : x \in X\}$ where the function $\mu_A : X \to I$ and $\nu_A : X \to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership

(namely $v_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + v_A(x) \le 1$ for each $x \in X$.

Obviously, every fuzzy set A on a nonempty set X is an IFS having the form $A\{\langle x, \mu_A(x), 1-\mu_A(x)\rangle : x \in X\}$.

Definition 2.2[2]: Let X be a nonempty set and the IFS's A and B be in the form $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle : x \in X\}, B = \{\langle x, \mu_B(x), \nu_B(x) \rangle : x \in X\}$, and let $A = \{A_j : j \in J\}$ be an arbitrary family of IFS's in X. then we define

1. $A \subseteq B$ if and only if $\forall x \in X[\mu_A(x) \le \mu_B(x) \text{ and } \nu_A(x) \ge \nu_B(x)];$

2.
$$A = \{ \langle x, v_A(x), \mu_A(x) \rangle : x \in X \};$$

- 3. $\cap A_j = \{ \{ \langle x, \land \mu_{A_i}(x), \lor \nu_{A_i}(x) \rangle : x \in X \};$
- 4. $\cup A_i = \{ \langle x, \forall \mu_{A_i}(x), \land \nu_{A_i}(x) \rangle : x \in X \};$
- 5. $\mathbf{1}_{\sim} = \{ \langle x, 1, 0 \rangle : x \in X \} \text{ and } \mathbf{0}_{\sim} = \{ \langle x, 0, 1 \rangle : x \in X \};$

Definition 2.3[6]: Let X and Y be two nonempty sets and f: $X \rightarrow Y$ be a function

i. If $B = \{ \langle y, \mu_B(x), \nu_B(x) \rangle : y \in Y \}$, is an IFS in Y, then the preimage of B under f is denoted and defined by $f^{-1}(B) = \{ \langle x, f^{-1}(\mu_b)(x), f^{-1}(\nu_B)(x) : x \in X \};$

ii. If $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle : x \in X\}$ is an IFS in X then the image of A under f is denoted and defined by $f(A) = \{\langle y, f(\mu_A)(y), f(\nu_A)(y) \rangle : y \in Y\}$ where $f(\mu_A)(y) = \begin{cases} \sup_{x \in f^{-1}(y) \mid \mu_A(x) \\ 0 & otherwise, \end{cases}$

and

$$f(v_A)(y) = \begin{cases} inf_{x \in f^{-1}(y) v_A(x)} & \text{if } f^{-1}(y) \neq 0 \\ 1 & \text{otherwise.} \end{cases}$$

Definition 2.4 [16]: The intuitionistic fuzzy set $c(\alpha, \beta) = \langle x, c_{\alpha}, c_{1-\beta} \rangle$ where $\alpha \in (0,1]$ and $\beta \in [0,1)$ and $\alpha + \beta \le 1$ is called an intuitionistic fuzzy point (IFP for short) in X.

Corollary 2.6 [6]: Let A, A_j ($j \in J$) be IFS's in X, B, B_j ($j \in J$) be IFS's in Y and $f: X \to Y$ be a function. Then

$$\begin{array}{ll} \text{i)} & A_1 \subseteq A_2 \Longrightarrow f(A_1) \subseteq f(A_2); \\ \text{ii)} & B_1 \subseteq B_2 \Longrightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2); \end{array}$$

iii)
$$A \subseteq f^{-1}(f(A))$$
 (if f is one-to-one, then $A = f^{-1}(f(A))$;

iv)
$$f(f^{-1}(B)) \subseteq B$$
 (if f is onto, then $f(f^{-1}(B)) \subseteq B$.

v)
$$f^{-1}(1 \sim) = 1 \sim \text{and} f^{-1}(0 \sim) = 0 \sim;$$

vi) $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}.$

Definition 2.7[6]: An intuitionistic fuzzy topology (IFT, for short) on a nonempty set X is a family τ of IFS's in X satisfying the following axioms:

- (i) $\mathbf{0}_{\sim}, \mathbf{1}_{\sim} \in \tau$.
- (ii) $A_1 \cap A_2 \in \tau$ for any $A_1, A_2 \in \tau$.
- (iii) $\cup A_i \in \tau$ for any $\{A_i : j \in J\} \subseteq \tau$.

Definition 2.8[6]: The complement A of IFOS A in IFTS (X, τ) is called an *intuitionistic fuzzy closed* set (IFCS, for short).

Definition 2.9[6]: Let (X, τ) be an IFTS and A= $\langle x, \mu_A(x), \nu_A(x) \rangle$ } be an IFS in X. Then the fuzzy interior and fuzzy closure of A is are denoted and defined by

cl(A)= \cap {K:K is an IFCS in X and $A \subseteq K$ }

int(A)= \cup {G:G is an IFOS in X and $G \subseteq A$ }

Note that, for any IFS A in (X,τ) , we have $cl(\overline{A}) = \overline{int(A)}$ and $\overline{int(A)} = \overline{cl(A)}$.

Definition 2.10[7]: Let A be an IFS in an IFTS (X,τ) , then A is

- 1. An intuitionistic fuzzy semi open set (IFSOS) if $A \subseteq cl(int(A))$.
- 2. An intuitionistic fuzzy α -open set (IF α OS) if $A \subseteq int(cl(int(A)))$.
- 3. An intuitionistic fuzzy preopen set (IFPOS) if $A \subseteq int(cl(A))$.
- 4. An intuitionistic fuzzy regular open set (IFROS) if A = int(cl(A)).

An IFS A is called an *intuitionistic fuzzy semiclosed set, intuitionistic fuzzy* α -closed set, *intuitionistic fuzzy preclosed set and intuitionistic fuzzy regular closed set* (IFSCS, IF α CS, IFPCS, IFRCS), if the complement of A is an IFSOS, IF α OS, IFPOS, IFROS respectively.

Definition 2.11[1]: Let A be an IFS in an IFTS (X, τ), then A is an *intuitionistic fuzzy d- open set* (IFDOS) if $A \subseteq scl(bint(A)) \cup cl(int(A))$.

Definition 2.12: Let f be a mapping from an IFTS (X, τ) into IFTS (Y, κ) . Then f is said to be

(i) intuitionistic fuzzy continuous[5] if $f^{-1}(B) \in IFO(X)$ for every $B \in \kappa$.

- (ii) intuitionistic fuzzy semicontinuous[7] if $f^{-1}(B) \in IFSO(X)$ for every $B \in \kappa$.
- (iii) intuitionistic fuzzy α -continuous[7] if $f^{-1}(B) \in IF \alpha O(X)$ for every $B \in \kappa$.
- (iv) intuitionistic fuzzy precontinuous [7] if $f^{-1}(B) \in IFPO(X)$ for every $B \in \kappa$.

Definition 2.13[7]: An IFS A is said to be intuitionistic fuzzy dense (IFD for short) in another IFS B in an IFT (X, τ), if cl(A) = B.

Definition 2.14[1]: Let f be a mapping from an IFTS (X, τ) into IFTS (Y, κ). Then f is said to be intuitionistic fuzzy d-continuous mapping if $f^{-1}(B) \in IFDO(X)$ for every $B \in \kappa$.

Definition 2.13[10] Let $f: X \rightarrow Y$ be a mapping from an IFTS X into an IFTS Y. The mapping f is called an

1. *intuitionistic fuzzy contra continuous*, if $f^{-1}(B)$ is an *IFCS* in X, for each *IFOS* B in Y

- 2. *intuitionistic fuzzy contra semicontinuous*, if $f^{-1}(B)$ is an *IFSCS* in X, for each IFOS B in Y
- 3. *intuitionistic fuzzy contra* α *continuous*, if $f^{-1}(B)$ is an *IF* α *CS* in X, for each *IFOS* B in Y.
- 4. Intuitionistic fuzzy contra precontinuous, if $f^{-1}(B)$ is an *IFPCS* in X, for each *IFOS* B in Y.

Definition 2.14[14]: Let X and Y be two IFTSs. Let $A = \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X$ and $B = \langle y, \mu_B(y), \nu_B(y) \rangle / y \in Y$ be IFSs of X and Y respectively. Then $A \times B$ is an IFS of $X \times Y$ defined by $(A \times B)(x, y) = \langle \{(x, y), \min(\mu_A(x), \mu_B(y)), \max(\nu_A(x), \nu_B(y))) \rangle$. B

Definition 2.15[14]: Let $f_1: X_1 \to Y_1$ and $f_2: X_2 \to Y_2$. The product $f_1 \times f_2: X_1 \times X_2 \to Y_1 \times Y_2$ is defined by $(f_1 \times f_2)(x_1, x_2) = (f_1(x_1), f_2(x_2))$ for every $(x_1, x_2) \in X_1 \times X_2$.

Definition 2.16[14]: Let $c(\alpha, \beta)$ be an IFP of an IFTS $((X, \tau)$ An IFS A of X is called an intuitionistic fuzzy neighborhood (IFN for short) of $c(\alpha, \beta)$ if there exists an IFOS B in X such that $c(\alpha, \beta) \in B \subseteq A$.

3. Intuitionistic fuzzy d-irresolute mapping.

Definition 3.1: A mapping $f: X \to Y$ from an IFTS X into an IFTS Y is said to be an intuitionistic fuzzy d-irresolute mapping if $f^{-1}(B)$ is an intuitionistic fuzzy d-open set in X for every intuitionistic fuzzy d-open set in Y.

Theorem 3.2: If $f:(X,\tau) \rightarrow (Y,\sigma)$ be an intuitionistic fuzzy d-irresolute mapping then f is an intuitionistic fuzzy d-continuous mapping but not conversely.

Proof: Follows from definition.

Example 3.3: Let $X = \{a, b, c\}$, $Y = \{u, v, w\}$,

$$A = \langle x, (0.9, 0.5, 0.5), (0.1, 0.4, 0.4) \rangle$$

 $B = \langle y, (0.8, 0.4, 0.4), (0.1, 0.6, 0.6) \rangle.$

where $\tau = \{0 \sim, 1 \sim, A\}$ and $\kappa = \{0 \sim, 1 \sim, B\}$ are IFTS on X and Y respectively. Define a mapping $h: (X, \tau) \rightarrow (Y, \kappa)$ by h (a) =u, h (b) =v, h(c) =w. Clearly h is intuitionistic fuzzy d-continuous map. Infact we have $B = \langle y, (0.8, 0.4, 0.4), (0.1, 0.6, 0.6) \rangle$ is an intuitionistic fuzzy d-open set in Y, but

 $h^{-1}(B) = \langle x, (0.8, 0.4, 0.4), (0.1, 0.6, 0.6) \rangle$ is not an intuitionistic fuzzy d-open set in X. Therefore f is not an intuitionistic fuzzy d-irresolute mapping.

Theorem 3.4: If $f:(X,\tau) \rightarrow (Y,\sigma)$ be an intuitionistic fuzzy d-irresolute mapping then f is an intuitionistic fuzzy semi precontinuous mapping but not conversely.

Proof: Follows from definition.

Example 3.5: Let $X = \{a, b, c\}$, $Y = \{d, e, f\}$,

 $A = \{ \langle x, (0.3, 0.1, 0.4), (0.3, 0.35, 0.4) \rangle \}$

 $B = \{ \langle y, (0.2, 0.1, 0.3), (0.4, 0.4, 0.4) \rangle \}$

where $\tau = \{0 \sim, 1 \sim, A\}$ and $\kappa = \{0 \sim, 1 \sim, B\}$ are IFTS on X and Y respectively.

Define an intuitionistic fuzzy mapping $f:(X,\tau) \to (Y,\sigma)$ by f(a)=d, f(b)=e, f(c)=f. Then B is an IFOS in Y. $f^{-1}(B)=\{\langle x, (0.2, 0.1, 0.3), (0.4, 0.4, 0.4) \rangle x \in X\}$ is an intuitionistic fuzzy semi preopen open set in X, since $f^{-1}(B) \subseteq cl(int(cl(f^{-1}(B))))$. Hence f is intuitionistic fuzzy semi precontinuous. But f is not intuitionistic fuzzy d-irresolute since B is an intuitionistic fuzzy d-open set in Y but $f^{-1}(B)$ is not intuitionistic fuzzy d-open set in X.

Theorem 3.6: If $f: X \rightarrow Y$ be a mapping from an intuitionistic fuzzy topological space X into an intuitionistic fuzzy topological space Y. Then the following are equivalent

- a. f is an intuitionistic fuzzy d-irresolute mapping.
- b. $f^{-1}(B)$ is an intuitionistic fuzzy d-closed set in X for each intuitionistic fuzzy d-closed set B in Y.
- c. $f(dcl(A)) \subseteq dcl(f(A))$ for each intuitionistic fuzzy set B in Y.
- d. $dcl(f^{-1}(B)) \subseteq f^{-1}(dcl(B))$ for each intuitionistic fuzzy set B in Y.
- e. $f^{-1}(dint(B)) \subseteq dint(f^{-1}(B))$ for each intuitionistic fuzzy set B in Y.

Proof:

 $a \Rightarrow b$ Let B be an intuitionistic fuzzy d-closed set in Y then 1-B is an intuitionistic fuzzy d-open set in Y. Since f is intuitionistic fuzzy d-irresolute $f^{-1}(1-B) = 1 - f^{-1}(B)$ is an intuitionistic fuzzy d-open set in X. Hence $f^{-1}(B)$ is an intuitionistic fuzzy d-closed set in X.

 $b \Rightarrow c$ Let A be an intuitionistic fuzzy set in X. Then $A \subseteq f^{-1}(f(A)) \subseteq f^{-1}(dcl(f(A)))$. dcl(f(A)) is an intuitionistic fuzzy d-closed set in Y by (b) $f^{-1}(dcl(f(A)))$ is an intuitionistic fuzzy d-closed set in X. $dcl(A) \subseteq f^{-1}(dcl(A))$ and $f(dcl(A)) \subseteq f(f^{-1}(dcl(f(A)))) = dcl(f(A))$. Thus $f(dcl(A)) \subseteq dcl(f(A))$.

 $c \Rightarrow d$ For any intuitionistic fuzzy set B in Y let $f^{-1}(B) = A$ by (c) $f(dcl(f^{-1}(B))) \subseteq dcl(f(f^{-1}(B))) \subseteq dcl(B)$ and

$$dcl(f^{-1}(\mathbf{B})) \subseteq f^{-1}(f(dcl(f^{-1}(\mathbf{B})))) \subseteq f^{-1}(dcl(\mathbf{B})).$$

Thus $dcl(f^{-1}(B)) \subseteq f^{-1}(dcl(B))$.

 $d \Rightarrow e$ We know that $d \operatorname{int}(B) = \overline{dcl(\overline{B})} \Rightarrow f^{-1}(\operatorname{dint}(B)) = f^{-1}(\overline{dcl(\overline{B})}) = \overline{f^{-1}(\operatorname{dcl}(\overline{B}))} \subseteq \overline{dcl(f^{-1}(\overline{B}))}$ $\subseteq f \operatorname{din}(f^{-1}(B)).$

 $e \Rightarrow a$ Let B be any intuitionistic fuzzy d-open set in Y. Then B = dint(B). $f^{-1}(dint(B)) = f^{-1}(B) \subseteq dint(f^{-1}(B))$. But $f^{-1}(B) \supseteq dint(f^{-1}(B))$ so $f^{-1}(B) = dint(f^{-1}(B))$. Thus $f^{-1}(B)$ is an intuitionistic fuzzy d-open set in X implies that f is an intuitionistic fuzzy d-irresolute mapping.

Theorem 3.7: Let $f: X \to Y$ and $g: Y \to Z$ be intuitionistic fuzzy d-irresolute mapping where X, Y, Z are IFTS. Then $g \circ f$ is an intuitionistic fuzzy d-irresolute mapping.

Proof: Let A be an intuitionistic fuzzy d-open set in Z. Since g is an intuitionistic fuzzy d-irresolute mapping $g^{-1}(A)$ is an intuitionistic fuzzy d-open set in Y. Also since f is an intuitionistic fuzzy d-irresolute mapping $f^{-1}(g^{-1}(A))$ is an intuitionistic fuzzy d-open set in X. $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$ for each A in Z. Hence $(g \circ f)^{-1}(A)$ is an intuitionistic fuzzy d-irresolute mapping.

Theorem 3.8: Let $f: X \to Y$ and $g: Y \to Z$ be intuitionistic fuzzy d-irresolute and intuitionistic fuzzy continuous mapping respectively where X, Y, Z are IFTS. Then $g \circ f$ is an intuitionistic fuzzy d-continuous mapping.

Proof: Let A be an intuitionistic fuzzy open set in Z. Since g is an intuitionistic fuzzy d-continuous mapping $g^{-1}(A)$ is an intuitionistic fuzzy d-open set in Y. Also since f is an intuitionistic fuzzy d-irresolute mapping $f^{-1}(g^{-1}(A))$ is an intuitionistic fuzzy d-open set in X. $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$ for each A in Z. Hence $(g \circ f)^{-1}(A)$ is an intuitionistic fuzzy d-continuous mapping.

Theorem 3.9: Let X, X_1 and X_2 be intuitionistic fuzzy topological space and $p_i: X_1 \times X_2 \to X_i$ be projections of $X_1 \times X_2$ onto X. If $f: X \to X_1 \times X_2$ is an intuitionistic fuzzy d-irresolute mapping then $p_i \circ f$ is an intuitionistic fuzzy d-continuous mapping.

Proof: Since f is an intuitionistic fuzzy d-irresolute and p_i is an intuitionistic fuzzy continuous mapping by Theorem 3.8 $p_i \circ f$ is an intuitionistic fuzzy d-continuous mapping.

Theorem 3.10: A mapping $f: X \to Y$ from an intuitionistic fuzzy topological space X into an intuitionistic fuzzy topological space Y is intuitionistic fuzzy d-irresolute if and only if for each intuitionistic fuzzy point $p(\alpha, \beta)$ in X and intuitionistic fuzzy d-open set B in Y such that $f(p(\alpha, \beta)) \in B$, there exists an intuitionistic fuzzy d-open set A in X such that $p(\alpha, \beta) \in A$ and $f(A) \subseteq B$.

Proof: Let f be any intuitionistic fuzzy d-irresolute mapping $p(\alpha, \beta)$ be an intuitionistic fuzzy point in X and B be any intuitionistic fuzzy d-open set in Y such that $f(p(\alpha, \beta)) \in B$. Then $p(\alpha, \beta) \in f^{-1}(B) = dint(f^{-1}(B))$. Let $A = dint(f^{-1}(B))$. Then A is an intuitionistic fuzzy dopen set in X containing intuitionistic fuzzy point $p(\alpha, \beta)$ and $f(A) = f(dint(f^{-1}(B))) \subseteq f(f^{-1}(B)) = B$.

Conversely let B be an intuitionistic fuzzy d-open set in Y and $p(\alpha, \beta)$ be intuitionistic fuzzy point in X such that $p(\alpha, \beta) \in f^{-1}(B)$. According to our assumption there exists an intuitionistic fuzzy d-open set A in X such that $p(\alpha, \beta) \in A$ and $f(A) \subseteq B$. Hence $p(\alpha, \beta) \in A \subseteq f^{-1}(B)$ and $p(\alpha, \beta) \in A = dint(A) \subseteq dint(f^{-1}(B))$. Since $p(\alpha, \beta)$ be any arbitrary intuitionistic fuzzy point and $f^{-1}(B)$ is the union of all intuitionistic fuzzy point contained in $f^{-1}(B)$ we obtain that $f^{-1}(B) = dint(f^{-1}(B))$. So f is an intuitionistic fuzzy d-irresolute mapping.

Definition 3.11: Let (X, τ) be an intuitionistic fuzzy topological space and let A be any intuitionistic fuzzy set in X. Then A is called intuitionistic fuzzy dense set if $cl(A)=1 \sim and A$ is called nowhere intuitionistic fuzzy dense set if $int(cl(A))=0 \sim a$.

Theorem 3.12: If a function $f: X \to Y$ is an intuitionistic fuzzy d-irresolute function then $f^{-1}(A)$ is intuitionistic fuzzy d-closed set in X for any nowhere intuitionistic fuzzy dense set A in Y.

Proof: Let A be any nowhere intuitionistic fuzzy dense set in Y. Then $int(cl(A)) = 0 \sim .$ Now $1 - int(cl(A)) = 1 \sim \Rightarrow cl(1 - cl(A)) = 1 \sim \Rightarrow cl(int(1 - A)) = 1 \sim .$

Hence $1-A \subseteq cl(int(1-A)) = 1 \rightarrow a \Rightarrow 1-A \subseteq cl(int(1-A)) \cup scl(bint(1-A)) = 1 \rightarrow a$ is intuitionistic fuzzy d-open set in Y. Since f is intuitionistic fuzzy d-irresolute $f^{-1}(1-A)$ is intuitionistic fuzzy d-open set in X, hence $f^{-1}(A)$ is intuitionistic fuzzy d-closed set in X.

4. Intuitionistic fuzzy apd-irresolute, intuitionistic fuzzy apd-closed maps

Definition 4.1: An intuitionistic fuzzy set A of an intuitionistic fuzzy topological space (X, τ) is called an intuitionistic fuzzy d-generalized closed set (IFdGCS in short) if $dcl(A) \subseteq U$, whenever $A \subseteq U$ and U is intuitionistic fuzzy d-open set.

The complement of intuitionistic fuzzy d-generalized closed set is called an intuitionistic fuzzy d-generalized open set (IFdGOS).

Definition 4.2: A mapping is called an intuitionistic fuzzy d-closed mapping if f(B) is an intuitionistic fuzzy d-closed set in Y for every intuitionistic fuzzy d-closed set B in X.

Definition 4.3: A mapping $f: X \to Y$ is said to be an intuitionistic fuzzy approximately d-irresolute (intuitionistic fuzzy apd-irresolute)) if $dcl(A) \subseteq f^{-1}(B)$, whenever B is an intuitionistic fuzzy d-open set of Y, A is an IFdGCS and $A \subset f^{-1}(B)$.

Definition 4.4: A mapping $f: X \to Y$ is said to be an intuitionistic fuzzy approximately d-closed (intuitionistic fuzzy apd-closed)) if $f(B) \subseteq d$ int(A), whenever A is an intuitionistic fuzzy IFdGOS of Y,B is an IFdCS and $f(B) \subseteq A$.

Theorem 4.5: Every intuitionistic fuzzy d-irresolute mapping is an intuitionistic fuzzy apd-irresolute mapping.

Proof: Follows from definition.

The converse of the above theorem need not be true shown in the following example.

Example 4.6: Let $X = \{a, b\}$, $Y = \{d, e\}$,

$$A = \{ \langle x, (0.3, 0.4), (0.4, 0.5) \rangle \}$$

$$B = \{\langle y, (0.4, 0.7), (0.1, 0.3) \rangle\}$$

where $\tau = \{0 \sim, 1 \sim, A\}$ and $\kappa = \{0 \sim, 1 \sim, B\}$ are IFTS on X and Y respectively. Define an intuitionistic fuzzy mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = d, f(b) = e. Then B is an apd-irresolute map. Now B is an IFdOS in Y and $f^{-1}(B) = \{\langle x, (0.4, 0.7), (0.1, 0.3) \rangle\}$ is not intuitionistic fuzzy d-open set in X. Hence f is an intuitionistic fuzzy d-irresolute mapping.

Theorem 4.7: A mapping $f: X \rightarrow Y$ is

- 1. An intuitionistic fuzzy apd-irresolute if $f^{-1}(A)$ is an intuitionistic fuzzy d-closed set in X for every intuitionistic fuzzy d-open set A in Y.
- 2. An intuitionistic fuzzy apd-closed if f(B) is an intuitionistic fuzzy d-open in Y for every intuitionistic fuzzy d-closed set B in X.

Proof:

- 1. Let A be an intuitionistic fuzzy d-open set in Y and B be an IFdGCS in X such that $B \subseteq f^{-1}(A)$. Then $dcl(A) \subseteq dcl(f^{-1}(B))$. Since $f^{-1}(A)$ is an intuitionistic fuzzy d-closed set in X, $dcl(B) \subseteq f^{-1}(A)$. Thus f is an intuitionistic fuzzy apd-irresolute mapping.
- 2. Let A be an IFdGOS of Y and B be an intuitionistic fuzzy d-closed set of X such that $f(B) \subseteq A$. Then $d \operatorname{in} t(f(B)) \subseteq d \operatorname{in} t(A)$. Since f(B) is an intuitionistic fuzzy d-open set in X, we have $f(B) \subseteq \sin t(A)$. Thus f is intuitionistic fuzzy apd-closed.

The converse of the above theorem need not be true in general seen from the following example. **Example 4.8:** Let $X = \{a, b\}$, $Y = \{d, e\}$, $\tau = \{0 \sim, 1 \sim, A\}$, $\sigma = \{0 \sim, 1 \sim, B\}$

$$A = \{ \langle x, (0.3, 0.4), (0.4, 0.5) \rangle \}$$

$$B = \{\langle y, (0.4, 0.7), (0.1, 0.3) \rangle\}$$

Define an intuitionistic fuzzy mapping $f:(X,\tau) \to (Y,\sigma)$ by f(a)=d, f(b)=e. Then B is an apdirresolute map. Now B is an IFdOS in Y and $f^{-1}(B)=\{\langle x, (0.4, 0.7), (0.1, 0.3) \rangle\}$ is not intuitionistic fuzzy d-closed set in X.

Example 4.9: Let $X = \{a, b, c\}$, $Y = \{d, e, f\}$,

$$A = \{ \langle x, (0.3, 0.1, 0.4), (0.3, 0.35, 0.4) \rangle \}$$

 $B = \{ \langle y, (0.2, 0.1, 0.3), (0.4, 0.4, 0.4) \rangle \}$

where $\tau = \{0 \sim, 1 \sim, A\}$ and $\kappa = \{0 \sim, 1 \sim, B\}$ are IFTS on X and Y respectively.

Define an intuitionistic fuzzy mapping $f:(X,\tau) \rightarrow (Y,\sigma)$ by f(a)=d, f(b)=e, f(c)=f. Then B is an apd-closed map. Now B is an IFdCS in Y and $f(B)=\{\langle x, (0.2, 0.1, 0.3), (0.4, 0.4, 0.4) \rangle\}$ is not intuitionistic fuzzy d-open set in X.

Theorem 4.10: Let $f: X \rightarrow Y$ be a mapping from an intuitionistic fuzzy topological space X into an intuitionistic fuzzy topological space Y

- 1. If IFdOS and IFdCS of (X, τ) coincide, then f is intuitionistic fuzzy apd-irresolute if and only if $f^{-1}(B)$ is an intuitionistic fuzzy d-closed set of X for every intuitionistic fuzzy d-open set B of Y.
- 2. If IFdOS and IFSCS of (Y, κ) coincide, then f is intuitionistic fuzzy apd-closed if and only if f(B) is an intuitionistic fuzzy d-open set of Y for every IFdCS B of X.

Proof:

1. Assume that f is an intuitionistic fuzzy apd-irresolute mapping. Let B be any IFS of X such that $B \subseteq G$ where G is an intuitionistic fuzzy d-open set of X. Then by

hypothesis $dcl(B) \subseteq dcl(G) = G$. Therefore all subsets of X are IFdGCS (IFdGOS). So for any intuitionistic fuzzy d-open set B of Y, $f^{-1}(B)$ is an IFdGCS in X. Since f is an intuitionistic fuzzy

apd-irresolute $\mathit{dcl}(f^{-1}(B))\!\subseteq\! f^{-1}(B).$ But $f^{-1}(B)\!\subseteq\! \mathit{dcl}(f^{-1}(B))$. Therefore

 $dcl(f^{-1}(B)) = f^{-1}(B)\,$. Hence $f^{-1}(B)$ is an IFdCS in X.

The converse part follows from Theorem 4.7.

2. Assume that f is intuitionistic fuzzy apd-closed. Let B be any intuitionistic fuzzy set of X such that $G \subseteq f(B)$ where G is an IFDCS of Y. Then by hypothesis $G = d \operatorname{in} t(G) \subseteq d \operatorname{int}(f(B))$. Therefore all sets of Y are IFdGOS. Therefore for any intuitionistic fuzzy d-closed set B of X, f(B) is an IFdGOS in Y. Since f is intuitionistic fuzzy apd-closed $f(B) \subseteq d \operatorname{int}(f(B))$. But $d \operatorname{int}(f(B)) \subseteq f(B)$. Hence $f(B) = d \operatorname{int}(f(B))$. Therefore f(B) is an intuitionistic fuzzy d-open set in Y.

The converse follows from Theorem 4.7.

As a immediate consequence of Theorem 4.10 we have the following corollary.

Corollary 4.11: Let $f: X \to Y$ be a mapping from an intuitionistic fuzzy topological space X into an intuitionistic fuzzy topological space Y. If IFdOS and IFdCS of (X, τ) coincide, then f is intuitionistic fuzzy apd-irresolute if and only if f is an intuitionistic fuzzy irresolute.

Proof: Assume that f is an intuitionistic fuzzy apd-irresolute mapping. Let B be an intuitionistic fuzzy d-open set of Y. Then by Theorem 4.10 $f^{-1}(B)$ is an intuitionistic fuzzy d-closed set in X. Since intuitionistic fuzzy d-closed set and intuitionistic fuzzy d-open set of X coincide $f^{-1}(B)$ is an intuitionistic fuzzy d-open set. Hence f is an intuitionistic fuzzy irresolute mapping The converse follows from Theorem 4.3.

Definition 4.12: A mapping $f: X \to Y$ is said to be an intuitionistic fuzzy contra d-irresolute mapping if $f^{-1}(B)$ is an intuitionistic fuzzy d-closed set in X for every intuitionistic fuzzy d-open set in Y.

Definition 4.13: A mapping $f: X \rightarrow Y$ is said to be an intuitionistic fuzzy contra d-closed if f(B) is an intuitionistic fuzzy d-open set in Y for every intuitionistic fuzzy d-closed set B in X.

Remark 4.14: Intuitionistic fuzzy contra d-irresoluteness and intuitionistic fuzzy irresoluteness are independent of each other.

Example 4.15: Let $X = \{u, v\}, W = \{\langle x, (0.7, 0.3), (0.2, 0.5) \rangle\}$

where $\tau = \{0 \sim, 1 \sim, W\}$ is an IFTS. Define an intuitionistic fuzzy mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(\nu_A(\mathbf{u})) = |\mu_A(\mathbf{u}), f(\nu_A(\mathbf{v})) = |\mu_A(\nu)|$. Then f is an intuitionistic fuzzy contra d-irresolute map. Now W is an IFdOS in X and $f^{-1}(M) = \{\langle x, (0.2, 0.5), (0.7, 0.3) \rangle\}$ is not intuitionistic fuzzy d-open set in X. Hence f is not an intuitionistic fuzzy d-irresolute mapping.

Example 4.16: Let $X = \{a, b\}$. $A = \{\langle x, (0.5, 0.1), (0.4, 0.6) \rangle\}$ where $\tau = \{0 \sim, 1 \sim, A\}$ be IFTS. Define an intuitionistic fuzzy mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = a, f(b) = b. Then f is an intuitionistic fuzzy d-irresolute mapping. Now $B = \{\langle y, (0.7, 0.4), (0.2, 0.5) \rangle\}$ is an IFdOS in X and

 $f^{-1}(B) = \{ \langle x, (0.7, 0.4), (0.2, 0.5) \rangle \}$ is not intuitionistic fuzzy d-closed set in X. Hence f is not an intuitionistic fuzzy contra d-irresolute mapping.

Theorem 4.17: Let $f: X \rightarrow Y$ be a mapping from IFTS X into an IFTS Y. Then the following conditions are equivalent:

- 1. f is intuitionistic fuzzy contra d-irresolute.
- 2. The inverse image of each intuitionistic fuzzy d-closed set in Y is an intuitionistic fuzzy d-open set in X.

Theorem 4.18: Every intuitionistic fuzzy contra d-irresolute map is an intuitionistic fuzzy apd-irresolute mapping.

Proof: Let $f: X \rightarrow Y$ be an intuitionistic fuzzy contra d-irresolute mapping and B be an intuitionistic

fuzzy d-open set in Y. By our assumption $f^{-1}(B)$ is an intuitionistic fuzzy d-closed set in X. By Theorem 4.7, f is an intuitionistic fuzzy apd-irresolute mapping.

The converse of the above theorem need not be true shown in the following example.

Example 4.19: Let $X = \{u, v\}$. $W = \{\langle x, (0.7, 0.3), (0.3, 0.6) \rangle\}$ where $\tau = \{0 \sim 1 \sim W\}$ be an IFTS.

Define an intuitionistic fuzzy mapping $f: (X, \tau) \rightarrow (Y, \sigma)$

by $f(v_A(\mathbf{u})) = |\mu_A(\mathbf{u}), f(v_A(\mathbf{v})) = |\mu_A(v)|$. Then f is an intuitionistic fuzzy apd-irresolute map. Now

 $E = \{ \langle x, (0.9, 0.5), (0.1, 0.5) \rangle \}$ is an IFdOS in X but $f^{-1}(E) = \{ \langle x, (0.9, 0.5), (0.1, 0.5) \rangle \}$ is not

intuitionistic fuzzy d-closed set in X. Hence f is not an intuitionistic fuzzy contra d-irresolute mapping. **Definition 4.20:** A mapping $f: X \rightarrow Y$ is said to be an intuitionistic fuzzy perfectly contra d-irresolute mapping if the inverse image of every intuitionistic fuzzy d-open set in Y is an intuitionistic fuzzy d-clopen set in X.

Theorem 4.21: Every intuitionistic fuzzy perfectly contra d-irresolute mapping is an intuitionistic fuzzy contra d- irresolute mapping.

Proof: Let $f: X \to Y$ be intuitionistic fuzzy perfectly contra d-irresolute mapping and let B be an intuitionistic fuzzy d-open set in Y. Then by our assumption $f^{-1}(B)$ is an intuitionistic fuzzy d-clopen set in X. Thus $f^{-1}(B)$ is an intuitionistic fuzzy d-closed set in X. Hence f is an intuitionistic fuzzy contra d-irresolute mapping.

The converse of the above theorem need not be true shown in the following example.

Example 4.22: Let $X = \{u, v\}$. $W = \{\langle x, (0.7, 0.3), (0.2, 0.5) \rangle\}$ where $\tau = \{0 \sim 1 \sim W\}$ be an IFTS.

Define an intuitionistic fuzzy mapping $f:(X,\tau) \rightarrow (Y,\sigma)$ by

 $f(v_A(\mathbf{u})) = |\mu_A(\mathbf{u}), f(v_A(\mathbf{v})) = |\mu_A(\mathbf{v})$. Then f is an intuitionistic fuzzy contra d-irresolute map. Now W is an IFdOS in X and $f^{-1}(M) = \{\langle x, (0.2, 0.5), (0.7, 0.3) \rangle\}$ is not intuitionistic fuzzy d-open set in X. Hence f is not an intuitionistic fuzzy perfectly contra d-irresolute mapping.

Theorem 4.23: Every intuitionistic fuzzy perfectly contra d-irresolute mapping is an intuitionistic fuzzy d- irresolute mapping.

Proof: Obvious.

The converse of the above theorem need not be true seen from the following example.

Example 4.24: Let $X = \{a, b\}$. $W = \{\langle x, (0.5, 0.1), (0.4, 0.6) \rangle\}$ where $\tau = \{0 \sim, 1 \sim, W\}$ be an IFTS. Define an intuitionistic fuzzy mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = a, f(b) = b. Then f is an intuitionistic fuzzy d-irresolute map. Now $E = \{\langle x, (0.7, 0.4), (0.2, 0.5) \rangle\}$ is an IFdOS in X and $f^{-1}(B) = \{\langle x, (0.7, 0.4), (0.2, 0.5) \rangle\}$ is not intuitionistic fuzzy d-closed set in X. Hence f is not an intuitionistic fuzzy mapping d irresolute mapping

intuitionistic fuzzy perfectly contra d-irresolute mapping.

Theorem 4.25: Let $f: X \to Y$ be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then the following conditions are equivalent

- 1. f is an intuitionistic fuzzy perfectly contra d-irresolute mapping.
- 2. f is an intuitionistic fuzzy contra d-irresolute mapping and intuitionistic fuzzy d-irresolute mapping

Proof:

 $1 \Rightarrow 2$ Let f be an intuitionistic fuzzy perfectly d-irresolute mapping. Let B be an IFdOS in Y. By our assumption is an intuitionistic fuzzy clopen set in X. Thus $f^{-1}(B)$ is both IFdOS and IFdCS in X. Hence f is an intuitionistic fuzzy contra d-irresolute and intuitionistic fuzzy d-irresolute map.

 $2 \Rightarrow 1$ Let f be both intuitionistic fuzzy contra d-irresolute and intuitionistic fuzzy d-irresolute and let B be an intuitionistic fuzzy d-open set in Y. By our assumption is both IFdOS and IFdCS in X. (i.e) $f^{-1}(B)$ is an intuitionistic fuzzy d-clopen set in X. Hence f is intuionistic fuzzy perfectly d-contra irresolute mapping.

Theorem 4.26: If a map $f: X \to Y$ be intuitionistic fuzzy irresolute and intuitionistic fuzzy apdclosed then $f^{-1}(A)$ is an IFdGCS whenever A is an IFdGCS.

Proof: Let A be an IFdGCS in Y. Suppose that $f^{-1}(A) \subseteq B$, where B is an intuitionistic fuzzy d-open set in X. Taking complements we obtain $\overline{B} \subseteq f^{-1}(\overline{A})$ or $f(\overline{B}) \subseteq (\overline{A})$. Since f is intuitionistic fuzzy apd-closed set $f(\overline{B}) \subseteq d$ in $t(\overline{A}) = \overline{dcl(A)}$. It follows that $\overline{B} \subseteq f^{-1}(\overline{dcl(A)}) = \overline{f^{-1}(dcl(A))}$ and hence $f^{-1}(dcl(A)) \subseteq B$. Since f is intuitionistic fuzzy d-irresolute $f^{-1}(dcl(A))$ is an intuitionistic fuzzy d-closed set. Thus we have $d \operatorname{cl}(f^{-1}(A)) \subseteq d \operatorname{cl}(f^{-1}(dcl(A))) = f^{-1}(dcl(A)) \subseteq B$. This implies $f^{-1}(A)$ is an IFdGCS in X.

A similar argument shows that the inverse image of IFdGOSs is IFdGOSs.

Theorem 4.27: If a map $f: X \rightarrow Y$ is an intuitionistic fuzzy apd-irresolute and intuitionistic fuzzy dclosed, then for every IFdGCS B of X, f(B) is an IFdGCS in Y. **Proof:** Let B be an IFdGCS in X and $f(B) \subseteq G$, where G is an intuitionistic fuzzy d-open set in Y. Then $B \subseteq f^{-1}(G)$ holds. Since f is an intuitionistic fuzzy apd-irresolute, $dcl(B) \subseteq f^{-1}(G)$ and hence $f(dcl(B)) \subseteq G$. Since f is intuitionistic fuzzy d-closed f(dcl(B)) is an intuitionistic uzzy d-closed set in Y. Therefore we have $dcl(f(B)) \subseteq dcl(f(d \operatorname{cl}(B)) = f(\operatorname{dcl}(B)) \subseteq G$. Hence f(B) is an IFdGCS in Y.

Theorem 4.28: Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be two mappings. Then

- 1. $g \circ f$ is an intuitionistic fuzzy apd-irresolute, if f is an intuitionistic fuzzy apd-irresolute and g is an intuitionistic fuzzy d-irresolute mapping.
- 2. $g \circ f$ is an intuitionistic fuzzy apd-closed map if f is intuitionistic fuzzy d-closed and g is an intuitionistic fuzzy apd-closed map.
- 3. $g \circ f$ is an intuitionistic fuzzy apd-closed map if f is an intuitionistic fuzzy apd-closed and g is an intuitionistic fuzzy d-open map and g^{-1} preserves IFdGOS.

Proof:

- 1. Assume that A is an IFdGCS of X and B be an IFdOS in Z for which $A \subseteq (g \circ f)^{-1}(B)$. Since g is an intuitionistic fuzzy d-irresolute map $g^{-1}(B)$ is an intuitionistic fuzzy d-open set in Y. Since f is an intuitionistic fuzzy apd-irresolute map $scl(A) \subseteq f^{-1}[g^{-1}(B)] = (g \circ f)^{-1}(B)$. This proves that $g \circ f$ is intuitionistic fuzzy apd-irresolute mapping.
- 2. Suppose that B is an intuitionistic fuzzy d-closed set in X and A is an IFdGOS in Z for which $(g \circ f)^{-1}(B) \subseteq A$. Then f(B) is an intuitionistic fuzzy d-closed set in Y as f is an intuitionistic fuzzy d-closed map. Since g is intuitionistic fuzzy apd-closed $g(f(B))=(g \circ f)(B) \subseteq d \operatorname{int}(A)$. This implies that $g \circ f$ is an intuitionistic fuzzy apd-closed map.
- 3. Assume that B is an IFdCS in X and A is an IFdGOS in Z for which $(g \circ f)^{-1}(B) \subseteq A$. Hence $f(B) \subseteq g^{-1}(A)$. Since A is IFdGOS in Z and g^{-1} preserves IFdGOS $g^{-1}(A)$ is IFdGOS in Y, which implies $f(B) \subseteq \sin t(g^{-1}(A))$. Thus $(g \circ f)(B) = g(f(B)) \subseteq g(dint(g^{-1}(A))) \subseteq d \operatorname{int}(gg^{-1}(A)) \subseteq \operatorname{dint}(A)$. This implies that $g \circ f$ is intuitionistic fuzzy apd-

Theorem 4.29: Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be two mappings. Then

- 1. $g \circ f$ is an intuitionistic fuzzy perfectly contra d-irresolute, if f and g are intuitionistic fuzzy erfectly contra d-irresolute mapping.
- 2. $g \circ f$ is an intuitionistic fuzzy contra d-irresolute if f and g are intuitionistic fuzzy perfectly contra d-irresolute and g is an intuitionistic fuzzy contra d-irresolute.
- 3. $g \circ f$ is an intuitionistic fuzzy d-irresolute, if f is intuitionistic fuzzy perfectly contra d-irresolute and g ids intuitionistic fuzzy contra d-irresolute.
- 4. $g \circ f$ is an intuitionistic fuzzy d-irresolute, if f is intuitionistic fuzzy perfectly contra d-irresolute and g is intuitionistic fuzzy contra d-irresolute.
- 5. $g \circ f$ is an intuitionistic fuzzy contra d-irresolute, if f is intuitionistic fuzzy perfectly contra d-irresolute and g is intuitionistic fuzzy d-irresolute.
- 6. $g \circ f$ is an intuitionistic fuzzy d-irresolute, if f is intuitionistic fuzzy contra d-irresolute and g is intuitionistic fuzzy contra d-irresolute.

closed map.

7. $g \circ f$ is an intuitionistic fuzzy contra d-irresolute, if f is intuitionistic fuzzy contra d-irresolute and g is intuitionistic fuzzy d-irresolute.

Proof: Follows from definition.

REFERENCES

- [1] I. Arockiarani and H. Jude Immaculate, *A Note on intuitionistic fuzzy d-continuous mappings*, Proceeding of National Seminar on Recent Development in Topology, (2015)236-244.
- [2] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and System, 20 (1986), 87-96.
- [3] M. Caldas, *Weak and strong forms of Irresolute maps*, International Journal of Mathematics and Mathematical Sciences, Vol.23, No.4 (2000) 253-259.
- [4] M. Caldas and S. Jafari, Weak and Strong Forms of β-Irresoluteness, The Arabian Journal for Science and Engineering, Volume 31, Number 1A (2006), 31-39.
- [5] C.L. Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl, 24(1968), 182-190.
- [6] D. Coker, *An introduction to intuitionistic fuzzy topological spaces*, Fuzzy sets and systems, 88(1997), 81-89.
- [7] Ekici, E., E. Kerre, *On fuzzy contra-continuities*, Advances in Fuzzy Mathematics, 1(2006), 35–44.
- [8] Gurcay, H., D. Coker, On fuzzy continuity in intuitionistic fuzzy topological spaces, J. Fuzzy. Math, 5 (1997), 365–378. [5]
- [9] I. M. Hanafy and El-Arish, *Completely continuous functions in intuitionistic fuzzy topological spaces*, Czechoslovak Mathematical journal, (2003), 793-803.
- [10] Joung Kon Jeon, Young Bae Jun and Jin Han Park, *Intuitionistic fuzzy alpha continuity and intuitionistic fuzzy pre continuity*, International journal of Mathematics and Mathematical Sciences, (2005), 3091-3101.
- [11] Krsteska, B., E. Ekici, *Intuitionistic fuzzy contra strong precontinuity*, Faculty of Sciences and Mathematics, University of Nis, Siberia, (2007), 273–284.
- [12] P. Rajarajeswari and L. Senthil Kumar, *Intuitionistic Fuzzy Completely Regular Weakly Generalized Continuous Mappings*, International Journal of Applied Information Systems 3(2012), 34-38.
- [13] R.K. Saraf, Seema Mishra and M. Caldas, *Weak and strong forms of Irresolute Fuzzy maps*, Proc. Pakistan Acad. Sci, 45(2), (2008), 75-80.
- [14] Seok Jong Lee and Eun Pyo Lee, *The category of intuitionistic fuzzy topological spaces*, Bull. Korean Math. Soc, (2000), 63-76.
- [15] T. Shyla Isac Mary and P. Thangavelu, *On Regular Pre-Semi closed Sets In Topological Spaces*, KBM Journal of Mathematical Sciences & Computer Applications, 1(2010), 9-17.
- [16] S.S. Thakur and Rekha Chaturvedi, *Regular Generalized closed sets in intuitionistic fuzzy topological spaces*, Universitatea Din Bacau Studii Si Cercertari Stiintifice, 6(2006), 257-272.
- [17] Thakur, S. S. and Singh, S., *On fuzzy semi-pre open sets and fuzzy semi-precontinuity*, Fuzzy sets and systems, (1998), 383-391.
- [18] Young Bae Jun, Jung ok Kang and Seok Zun Kang, *Intuitionistic fuzzy irresolute and continuous mappings*, Far East J. Math. Sci [FJMS] 17(2) (2005), 201-216.
- [19] L. A. Zadeh. *Fuzzy sets*, Information and control, 9(1965), 338-353.