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RESEARCH ARTICLE

METHOD NUMERICAL TO ACCOUNT IMPROPERTRIPLE INTEGRALSBY USING COMPOSITE SIMPSON'S RULE AND ROMBERG ACCELERATION

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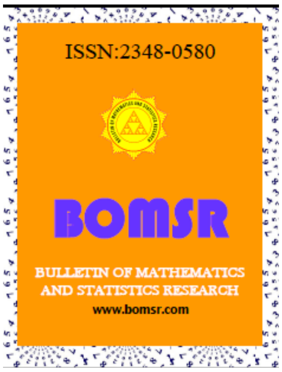

ABSTRACT

In this paper to derive numericalmethod to calculate values of triple integrals its integrands have singular partial derivatives and singular integrals at end of region of integration and to derive the form errors (correction terms) by using Simpson's rule with triple dimensions xand y and z , we will use Romberg accelerationto improve the results we will apply this rule integral particular when *h1=h2=2h3*

Keywords: Triple integral ; Simpson's rule ;Romberg Acceleration

Academic Discipline And Sub-Disciplines Numerical Analysis

1. INTRODUCTION

There are many research studied computations of triple integrals numerical Dheyaa [1]methods of a single integration to make composite to calculate the triple integration by using midpoint rule and Simpson's rule ،and Salman[4] , introduced derive rule to find values of triple integrals, numerical its integrands have singular partial derivatives not on the end of the region of integration by using the midpoint ،In this paper we will use composite Simpson's rule to calculate an approximate values of triple integrals its integrands have singular partial derivatives and singular integrals at end of region of integration and to derive the form errors for it , this method resulted from the applied Romberg accelerating on the values that resulting from the usethe Simpson's rule on the three dimensions indicated by the symbol sim^3 [2] when(n) the number of subinterval on the number of sub intervals on the dimension x and (r) the number of subintervals in the dimension y and (m) the number of subintervals on the dimension z we will applied the base integral particularly when ($n=r$),($m=2n$) and (n ,r, m even numbers) $h_1=h_2=2h_3$.

2.Triple Integrals For Continuous Integrands With Singular Partial Derivatives

We now introduce the style of computing triple integrals numerical when the function $f(x,y,z)$ continuous integration but have singular partial derivatives in one end of region integration.

Theorem

let function $f(x,y,z)$ differentiable and Continuous in every point of region $[a,b]\times[c,d]\times[e,g]$, but at least one of the partial derivatives undefinedat the point (a,c,e) or (b,d,g) is to know the approximate value of integration *I* can be calculated from the following rule

$$I=\int_e^g\int_c^d\int_a^b f(x,y,z)\,dxdydz = sim^3(h_1,h_2,h_3)+E(h_1,h_2,h_3);$$

$$\begin{aligned}
sim^3(h_1,h_2,h_3)&=\frac{h_1h_2h_3}{27}\Big[\ f(x_0,y_0,z_0)+f(x_0,y_0,z_m)+f(x_0,y_r,z_0)+f(x_0,y_r,z_m)+\\
&f(x_n,y_0,z_0)+f(x_n,y_0,z_m)+f(x_n,y_r,z_0)+f(x_n,y_r,z_m)+4\sum_{i=1}^{n/2}\Big[f(x_{2i-1},y_0,z_0)\\
&+f(x_{2i-1},y_0,z_m)+f(x_{2i-1},y_r,z_0)+f(x_{2i-1},y_r,z_m)\Big]+2\sum_{i=1}^{(n/2)-1}\Big[f(x_{2i},y_0,z_0)\\
&+f(x_{2i},y_0,z_m)+f(x_{2i},y_r,z_0)+f(x_{2i},y_r,z_m)\Big]+4\sum_{j=1}^{r/2}\Big[\Big[f(x_0,y_{2j-1},z_0)\\
&+f(x_0,y_{2j-1},z_m)+f(x_n,y_{2j-1},z_0)+f(x_n,y_{2j-1},z_m)+4\sum_{i=1}^{n/2}\Big[f(x_{2i-1},y_{2j-1},z_0)\\
&+f(x_{2i-1},y_{2j-1},z_m)\Big]+2\sum_{i=1}^{(n/2)-1}\Big[f(x_{2i},y_{2j-1},z_0)+f(x_{2i},y_{2j-1},z_m)\Big]\Big]+\\
&2\sum_{j=1}^{(r/2)-1}\Big[\Big[f(x_0,y_{2j},z_0)+f(x_0,y_{2j},z_m)+f(x_n,y_{2j},z_0)+f(x_n,y_{2j},z_m)+\\
&4\sum_{i=1}^{n/2}\Big[f(x_{2i-1},y_{2j},z_0)+f(x_{2i-1},y_{2j},z_m)\Big]+2\sum_{i=1}^{(nl2)-1}\Big[f(x_{2i},y_{2j},z_0)+f(x_{2i},y_{2j},z_m)\Big]\Big]\\
&+4\sum_{k=1}^{m/2}\Big[\ f(x_0,y_0,z_{2k-1})+f(x_0,y_r,z_{2k-1})+f(x_n,y_0,z_{2k-1})+f(x_n,y_r,z_{2k-1})\\
&+4\sum_{i=1}^{n/2}\Big[f(x_{2i-1},y_0,z_{2k-1})+f(x_{2i-1},y_r,z_{2k-1})\Big]+2\sum_{i=1}^{(n/2)-1}\Big[f(x_{2i},y_0,z_{2k-1})+\\
&f(x_{2i},y_r,z_{2k-1})\Big]+4\sum_{j=1}^{r/2}\Big[f(x_0,y_{2j-1},z_{2k-1})+f(x_n,y_{2j-1},z_{2k-1})+4\sum_{i=1}^{n/2}\\
&f(x_{2i-1},y_{2j-1},z_{2k-1})+2\sum_{i=1}^{(n/2)-1}f(x_{2i},y_{2j-1},z_{2k-1})\Big]+2\sum_{j=1}^{(r/2)-1}\Big[f(x_0,y_{2j},z_{2k-1})+\\
&f(x_n,y_{2j},z_{2k-1})+4\sum_{i=1}^{n/2}f(x_{2i-1},y_{2j},z_{2k-1})+2\sum_{i=1}^{(n/2)-1}f(x_{2i},y_{2j},z_{2k-1})\Big]\ \Big]+\\
&2\sum_{k=1}^{(m/2)-1}\Big[\ f(x_0,y_0,z_{2k})+f(x_0,y_r,z_{2k})+f(x_n,y_0,z_{2k})+f(x_n,y_r,z_{2k})
\end{aligned}$$

$$+4\sum_{i=1}^{n/2}\Big[\ f\ (x_{2i-1},y_0,z_{2k})+f\ (x_{2i-1},y_r,z_{2k})\Big]+2\sum_{i=1}^{(n/2)-1}\Big[\ f\ (x_{2i},y_0,z_{2k})+$$

$$f\ (x_{2i},y_r,z_{2k})\Big]+4\sum_{j=1}^{r/2}\Big[\ f\ (x_0,y_{2j-1},z_{2k})+f\ (x_n,y_{2j-1},z_{2k})+4\sum_{i=1}^{n/2}f\ (x_{2i-1},y_{2j-1},z_{2k})$$

$$+2\sum_{i=1}^{(n/2)-1}f\ (x_{2i},y_{2j-1},z_{2k})\Big]+2\sum_{j=1}^{(r/2)-1}\Big[\ f\ (x_0,y_{2j},z_{2k})+f\ (x_n,y_{2j},z_{2k})+$$

$$4\sum_{i=1}^{n/2}f\ (x_{2i-1},y_{2j},z_{2k})+2\sum_{i=1}^{(n/2)-1}f\ (x_{2i},y_{2j},z_{2k})\Big]\ \Big]\ \Big]$$

And

i) $f\ (x,y,z)$ isContinuous and With Singular Partial Derivatives at the point (a,c,e) then

$$E\ (h_1,h_2,h_3)+\Big[\big(a_1h_1^5h_2h_3D_x^4+a_2h_1h_2^5h_3D_y^4+a_3h_1h_2h_3^5D_z^4\big)+\big(a_4h_1^7h_2h_3D_x^6+a_5h_1h_2^7h_3D_y^6$$

$$+a_6h_1h_2h_3^7D_z^6+a_7h_1^6h_2^2h_3D_x^5D_y+\ldots\big)+\big(a_{22}h_1^8h_2h_3D_x^7+a_{23}h_1h_2^8h_3D_y^7+\ldots\big)+\ldots\ \Big]\ f\ (x_1,y_1,z_1)$$

$$+C_1h_1^4+C_2h_2^4+C_3h_3^4+C_4h_1^6+C_5h_2^6+C_6h_3^6+\ldots$$

ii) $f\ (x,y,z)$ is Continuous With Singular Partial Derivatives at the point (b,d,g) then

$$E\ (h_1,h_2,h_3)+\Big[\ \big(c_1h_1^5h_2h_3D_x^4+c_2h_1h_2^5h_3D_y^4+c_3h_1h_2h_3^5D_z^4\big)+\big(c_4h_1^7h_2h_3D_x^6+c_5h_1h_2^7h_3D_y^6$$

$$+c_6h_1h_2h_3^7D_z^6+c_7h_1^6h_2^2h_3D_x^5D_y\ +\big)+\big(c_{22}h_1^8h_2h_3D_x^7\ \ldots+c_{23}h_1h_2^8h_3D_y^7\ +\ldots\big)+$$

$$\ldots\ \Big]\ f\ (x_{n-1},y_{r-1},z_{m-1})+J_1h_1^4+J_2h_2^4+J_3h_3^4+J_4h_1^6+J_5h_2^6+J_6h_3^6+\ldots$$

$,\ x_i=a+ih_1, i=0,1,2,\ldots,n\ \ ,\ y_j=c+jh_2, j=0,1,2,\ldots,r$,suchthat

$a_i, c_i, , C_i J_i, \ldots$ constants for all $i=1,2,\ldots$.

PROOF: i) We can partition integration I to

$$I=\int_e^g\int_c^d\int_a^b f\ (x,y,z)\ dxdydz=\int_{z_0}^{z_m}\int_{y_0}^{y_r}\int_{x_0}^{x_n}f\ (x,y,z)\ dxdydz=\int_{z_0}^{z_2}\int_{y_0}^{y_2}\int_{x_0}^{x_2}f\ (x,y,z)\ dxdydz\ +$$

$$\int_{z_0}^{z_2}\int_{y_0}^{y_2}\int_{x_2}^{x_n}f\ (x,y,z)\ dxdydz+\int_{z_0}^{z_2}\int_{y_2}^{y_r}\int_{x_0}^{x_2}f\ (x,y,z)\ dxdydz+\int_{z_2}^{z_m}\int_{y_0}^{y_2}\int_{x_0}^{x_2}f\ (x,y,z)\ dxdydz$$

$$+\int_{z_0}^{z_2}\int_{y_2}^{y_r}\int_{x_2}^{x_n}f\ (x,y,z)\ dxdydz+\int_{z_2}^{z_m}\int_{y_0}^{y_2}\int_{x_2}^{x_n}f\ (x,y,z)\ dxdydz+\int_{z_2}^{z_m}\int_{y_2}^{y_r}\int_{x_0}^{x_2}f\ (x,y,z)\ dxdydz$$

$$+\int_{z_2}^{z_m}\int_{y_2}^{y_r}\int_{x_2}^{x_n}f\ (x,y,z)\ dxdydz \qquad \ldots(1')$$

All above integrals have continuous integrands in it's region of integrationexceptedthe first integration (I_1) hassingularity in it's region at the point (x_2,y_2,z_2) so it can be the values of these integrals $(I_2,I_3,I_4,I_5,I_6,I_7,I_8)$ account from [2] either to find a first integration spread function using (Taylor's series for a function of several variables) around the point (x_2,y_2,z_2) Sestri [5] so we have

$$f\ (x,y,z)=\Big[\ \ 1+(x-x_2)D_x+(y-y_2)D_y+(z-z_2)D_z+\frac{(x-x_2)^2}{2!}D_x^2+\frac{(y-y_2)^2}{2!}D_y^2+$$

$$\frac{(z-z_2)^2}{2!}D_z^2+(x-x_2)(y-y_2)D_xD_y+(x-x_2)(z-z_2)D_xD_z+(y-y_2)(z-z_2)D_yD_z$$
$$+\ldots\Big]f(x_2,y_2,z_2) \qquad \ldots(2')$$

$$\int_{z_0}^{z_2}\int_{y_0}^{y_2}\int_{x_0}^{x_2}f(x,y,z)\,dxdydz=\Big[\ 8h_1h_2h_3-8h_1^2h_2h_3D_x-8h_1h_2^2h_3D_y-8h_1h_2h_3^2D_z+$$
$$\frac{32}{3!}h_1^3h_2h_3D_x^2+\frac{32}{3!}h_1h_2^3h_3D_y^2+\frac{32}{3!}h_1h_2h_3^3D_z^2+8h_1^2h_2^2h_3D_xD_y+8h_1^2h_2h_3^2D_xD_z+$$
$$8h_1h_2^2h_3^2D_yD_z-\frac{64}{4!}h_1^4h_2h_3D_x^3-\frac{64}{4!}h_1h_2^4h_3D_y^3-\frac{64}{4!}h_1h_2h_3^4D_z^3-\frac{32}{3!}h_1^3h_2^2h_3D_x^2D_y-$$
$$\frac{32}{3!}h_1^2h_2^3h_3D_xD_y^2-\frac{32}{3!}h_1^3h_2h_3^2D_x^2D_z-\frac{32}{3!}h_1^2h_2h_3^3D_xD_z^2-\frac{32}{3!}h_1h_2^3h_3^2D_y^2D_z$$
$$-\frac{32}{3!}h_1h_2^2h_3^3D_yD_z^2-8h_1^2h_2^2h_3^2D_xD_yD_z+\frac{128}{5!}h_1^5h_2h_3D_x^4+\frac{128}{5!}h_1h_2^5h_3D_y^4+$$
$$\frac{128}{5!}h_1h_2h_3^5D_z^4+\frac{64}{4!}h_1^4h_2^2h_3D_x^3D_y+\frac{64}{4!}h_1^2h_2^4h_3D_xD_y^3+\frac{64}{4!}h_1^4h_2h_3^2D_x^3D_z+$$
$$\frac{64}{4!}h_1^2h_2h_3^4D_xD_z^3+\frac{64}{4!}h_1h_2^4h_3^2D_y^3D_z+\frac{64}{4!}h_1h_2^2h_3^4D_yD_z^3+\frac{128}{36}h_1^3h_2^3h_3D_x^2D_y^2+$$
$$\frac{128}{36}h_1^3h_2h_3^3D_x^2D_z^2+\frac{128}{36}h_1h_2^3h_3^3D_y^2D_z^2-\frac{256}{6!}h_1^6h_2h_3D_x^5-\frac{256}{6!}h_1h_2^6h_3D_y^5-$$
$$\frac{256}{6!}h_1h_2h_3^6D_z^5-\frac{128}{5!}h_1^5h_2^2h_3D_x^4D_y-\frac{128}{5!}h_1^2h_2^5h_3D_xD_y^4-\ldots\Big]f(x_2,y_2,z_2)$$
$$\ldots(3')$$

From the forms ($2'$) we have

1) $$f(x_0,y_0,z_0)=\Big[\ 1-2h_1D_x-2h_2D_y-2h_3D_z+2h_1^2D_x^2+2h_2^2D_y^2+2h_3^2D_z^2+4h_1h_2D_xD_y$$
$$+4h_1h_3D_xD_z+4h_2h_3D_yD_z-\frac{8}{3!}h_1^3D_x^3-\frac{8}{3!}h_2^3D_y^3-\frac{8}{3!}h_3^3D_z^3-4h_1^2h_2D_x^2D_y-4h_1h_2^2D_xD_y{}^2$$
$$-4h_1^2h_3D_x^2D_z-4h_1h_3^2D_xD_z^2-4h_2^2h_3D_y^2D_z-4h_2^2h_3D_yD_z^2-8h^3D_xD_yD_z+\frac{16}{4!}h_1^4D_x^4+$$
$$\frac{16}{4!}h_2^4D_y^4+\frac{16}{4!}h_3^4D_z^4+\frac{16}{3!}h_1^3h_2D_x^3D_y+\frac{16}{3!}h_1h_2^3D_xD_y^3+\frac{16}{3!}h_1^3h_3D_x^3D_z+\frac{16}{3!}h_1h_3^3D_xD_z^3$$
$$+\frac{16}{3!}h_2^3h_3D_y^3D_z+\frac{16}{3!}h_2h_3^3D_yD_z^3+4h_1^2h_2^2D_x^2D_y^2+4h_1^2h_3^2D_x^2D_z{}^2+4h_2^2h_3^2D_y^2D_z^2-$$
$$\frac{32}{5!}h_1^5D_x^5-\frac{32}{5!}h_2^5D_y^5-\frac{32}{5!}h_3^5D_z^5-\frac{32}{4!}h^5D_x^4D_y-\frac{32}{4!}h^5D_xD_y^4+\ldots\Big]f(x_2,y_2,z_2)$$

2) $$f(x_2,y_2,z_0)=\Big[\ 1-2h_3D_z+2h_3^2D_z^2-\frac{8}{3!}h_3^3D_z^3+\frac{16}{4!}h_3^4D_z^4-\frac{32}{5!}h_3^5D_z^5+\ldots\Big]f(x_2,y_2,z_2)$$

3) $$f(x_0,y_0,z_2)=\Big[\ 1-2h_1D_x-2h_2D_y+2h_1^2D_x^2+2h_2^2D_y^2+4h_1h_2D_xD_y-\frac{8}{3!}h_1^3D_x^3-\frac{8}{3!}h_2^3D_y^3$$
$$-4h_1^2h_2D_x^2D_y-4h_1h_2^2D_xD_y^2+\frac{16}{4!}h_1^4D_x^4+\frac{16}{4!}h_2^4D_y^4+\frac{16}{3!}h_1^3h_2D_x^3D_y+\frac{16}{3!}h_1h_2^3D_xD_y^3+$$

$$4h_1^2h_2^2D_x^2D_y^2-\frac{32}{5!}h_1^5D_x^5-\frac{32}{5!}h_2^5D_y^5-\frac{32}{4!}h_1^4h_2D_x^4D_y-\frac{32}{4!}h_1h_2^4D_xD_y^4+\dots\Big]f(x_2,y_2,z_2)$$ **4)**

$$f(x_2,y_0,z_2)=\Big[1-2h_2D_y+2h_2^2D_y^2-\frac{8}{3!}h_2^3D_y^3+\frac{16}{4!}h_2^4D_y^4-\frac{32}{5!}h_2^5D_y^5+\dots\Big]f(x_2,y_2,z_2)$$

5) $$f(x_0,y_2,z_2)=\Big[1-2h_1D_x+2h_1^2D_x^2-\frac{8}{3!}h_1^3D_x^3+\frac{16}{4!}h_1^4D_x^4-\frac{32}{5!}h_1^5D_x^5+\dots\Big]f(x_2,y_2,z_2)$$

6) $$f(x_0,y_2,z_0)=\Big[1-2h_1D_x-2h_3D_z+2h_1^2D_x^2+2h_3^2D_z^2+4h_1h_3D_xD_z-\frac{8}{3!}h_1^3D_x^3-\frac{8}{3!}h_3^3D_z^3-4h_1^2h_3D_x^2D_z-4h_1h_3^2D_xD_z^2+\frac{16}{4!}h_1^4D_x^4+\frac{16}{4!}h_3^4D_z^4+\frac{16}{3!}h_1^3h_3D_x^3D_z+\frac{16}{3!}h_1h_3^3D_xD_z^3+4h_1^2h_3^2D_x^2D_z^2-\frac{32}{5!}h_1^5D_x^5-\frac{32}{5!}h_3^5D_z^5+\dots\Big]f(x_2,y_2,z_2)$$

$$f(x_2,y_0,z_0)=\Big[1-2h_2D_y-2h_3D_z+2h_2^2D_y^2+2h_3^2D_z^2+4h_2h_3D_yD_z-\frac{8}{3!}h_2^3D_y^3-\frac{8}{3!}h_3^3D_z^3-4h_2^2h_3D_y^2D_z-4h_2h_3^2D_yD_z^2+\frac{16}{4!}h_2^4D_y^4+\frac{16}{4!}h_3^4D_z^4+\frac{16}{3!}h_2^3h_3D_y^3D_z+\frac{16}{3!}h_2h_3^3D_yD_z^3+4h_2^2h_3^2D_y^2D_z^2-\frac{32}{5!}h_2^5D_y^5-\frac{32}{5!}h_3^5D_z^5+\dots\Big]f(x_2,y_2,z_2)$$ **7)**

8) $$f(x_0+h_1,y_0,z_0)=\Big[1-h_1D_x-2h_2D_y-2h_3D_z+\frac{1}{2}h_1^2D_x^2+2h_2^2D_y^2+2h_3^2D_z^2+2h_1h_2D_xD_y+2h_1h_3D_xD_z+4h_2h_3D_yD_z-\frac{1}{3!}h_1^3D_x^3-\frac{8}{3!}h_2^3D_y^3-\frac{8}{3!}h_3^3D_z^3-h_1^2h_2D_x^2D_y-2h_1h_2^2D_xD_y^2-h_1^2h_3D_x^2D_z-2h_1h_3^2D_xD_z^2-4h_2^2h_3D_y^2D_z-4h_2^2h_3D_yD_z^2-4h_1h_2h_3D_xD_yD_z+\frac{1}{4!}h_1^4D_x^4+\frac{16}{4!}h_2^4D_y^4+\frac{16}{4!}h_3^4D_z^4+\frac{2}{3!}h_1^3h_2D_x^3D_y+\frac{8}{3!}h_1h_2^3D_xD_y^3+\frac{2}{3!}h_1^3h_3D_x^3D_z+\frac{8}{3!}h_1h_3^3D_xD_z^3+\frac{16}{3!}h_2^3h_3D_y^3D_z+\frac{16}{3!}h_2h_3^3D_yD_z^3+h_1^2h_2^2D_x^2D_y^2+h_1^2h_3^2D_x^2D_z^2+4h_2^2h_3^2D_y^2D_z^2-\frac{1}{5!}h_1^5D_x^5-\frac{32}{5!}h_2^5D_y^5-\frac{32}{5!}h_3^5D_z^5-\frac{2}{4!}h_1^4h_2D_x^4D_y-\frac{16}{4!}h_1h_2^4D_xD_y^4+\dots\Big]f(x_2,y_2,z_2)$$

$$f(x_0+h_1,y_0,z_2)=\Big[1-h_1D_x-2h_2D_y+\frac{1}{2}h_1^2D_x^2+2h_2^2D_y^2+2h_1h_2D_xD_y-\frac{1}{3!}h_1^3D_x^3-\frac{8}{3!}h_2^3D_y^3-h_1^2h_2D_x^2D_y-2h_1h_2^2D_xD_y^2+\frac{1}{4!}h_1^4D_x^4+\frac{16}{4!}h_2^4D_y^4+\frac{2}{3!}h_1^3h_2D_x^3D_y+\frac{8}{3!}h_1h_2^3D_xD_y^3+h_1^2h_2^2D_x^2D_y^2-\frac{1}{5!}h_1^5D_x^5-\frac{32}{5!}h_2^5D_y^5-\frac{2}{4!}h_1^4h_2D_x^4D_y-\frac{16}{4!}h_1h_2^4D_xD_y^4+\dots\Big]f(x_2,y_2,z_2)$$ **9)**

10) $$f(x_0+h_1,y_2,z_0)=\Big[1-h_1D_x-2h_3D_z+\frac{1}{2}h_1^2D_x^2+2h_3^2D_z^2+2h_1h_3D_xD_z-\frac{1}{3!}h_1^3D_x^3-\frac{8}{3!}h_3^3D_z^3-h_1^2h_3D_x^2D_z-2h_1h_3^2D_xD_z^2+\frac{1}{4!}h_1^4D_x^4+\frac{16}{4!}h_3^4D_z^4+\frac{2}{3!}h_1^3h_3D_x^3D_z+\frac{8}{3!}h_1h_3^3D_xD_z^3+h_1^2h_3^2D_x^2D_z^2-\frac{1}{5!}h_1^5D_x^5-\frac{32}{5!}h_3^5D_z^5+\dots\Big]f(x_2,y_2,z_2)$$

11) $$f(x_0+h_1,y_2,z_2)=\Big[1-h_1D_x+\frac{1}{2}h_1^2D_x^2-\frac{1}{3!}h_1^3D_x^3+\frac{1}{4!}h_1^4D_x^4-\frac{1}{5!}h_1^5D_x^5+\dots\Big]f(x_2,y_2,z_2)$$

$$f\left(x_0,y_0+h_2,z_0\right)=\Big[\ 1-2h_1D_x-h_2D_y-2h_3D_z+2h_1^2D_x^2+\frac{1}{2}h_2^2D_y^2+2h_3^2D_z^2$$

12) $$+2h_1h_2D_xD_y+4h_1h_3D_xD_z+2h_2h_3D_yD_z-\frac{8}{3!}h_1^3D_x^3-\frac{1}{3!}h_2^3D_y^3-\frac{8}{3!}h_3^3D_z^3-$$
$$2h_1^2h_2D_x^2D_y-h_1h_2^2D_xD_y^2-4h_1^2h_3D_x^2D_z-4h_1h_3^2D_xD_z^2-h_2^2h_3D_y^2D_z-2h_2h_3^2D_yD_z^2-$$
$$4h_1h_2h_3D_xD_yD_z+\frac{16}{4!}h_1^4D_x^4+\frac{1}{4!}h_2^4D_y^4+\frac{16}{4!}h_3^4D_z^4+\frac{8}{3!}h_1^3h_2D_x^3D_y+\frac{2}{3!}h_1h_2^3D_xD_y^3+$$
$$\frac{16}{3!}h_1^3h_3D_x^3D_z+\frac{16}{3!}h_1h_3^3D_xD_z^3+\frac{2}{3!}h_2^3h_3D_y^3D_z+\frac{8}{3!}h_2h_3^3D_yD_z^3+h_1^2h_2^2D_x^2D_y^2+$$
$$4h_1^2h_3^2D_x^2D_z^2+h_2^2h_3^2D_y^2D_z^2-\frac{32}{5!}h_1^5D_x^5-\frac{1}{5!}h_2^5D_y^5-\frac{32}{5!}h_3^5D_z^5-\frac{16}{4!}h_1^4h_2D_x^4D_y-$$
$$4h_1^2h_3^2D_x^2D_z^2+h_2^2h_3^2D_y^2D_z^2-\frac{32}{5!}h_1^5D_x^5-\frac{1}{5!}h_2^5D_y^5-\frac{32}{5!}h_3^5D_z^5-\frac{16}{4!}h_1^4h_2D_x^4D_y-$$
$$\frac{2}{4!}h_1h_2^4D_xD_y^4+\ldots\ \Big]f\left(x_2,y_2,z_2\right)$$

$$f\left(x_0,y_0+h_2,z_2\right)=\Big[\ 1-2h_1D_x-h_2D_y+2h_1^2D_x^2+\frac{1}{2}h_2^2D_y^2+2h_1h_2D_xD_y-\frac{8}{3!}h_1^3D_x^3-$$

13) $$\frac{1}{3!}h_2^3D_y^3-2h_1^2h_2D_x^2D_y-h_1h_2^2D_xD_y^2+\frac{16}{4!}h_1^4D_x^4+\frac{1}{4!}h_2^4D_y^4+\frac{8}{3!}h_1^3h_2D_x^3D_y+\frac{2}{3!}h_1h_2^3D_xD_y^3$$
$$+h_1^2h_2^2D_x^2D_y^2-\frac{32}{5!}h_1^5D_x^5-\frac{1}{5!}h_2^5D_y^5-\frac{2}{4!}h_1^4h_2D_x^4D_y-\frac{16}{4!}h_1h_2^4D_xD_y^4+\ldots\ \Big]f\left(x_2,y_2,z_2\right)$$

14) $$f\left(x_2,y_0+h_2,z_0\right)=\Big[\ 1-h_2D_y-2h_3D_z+\frac{1}{2}h_2^2D_y^2+2h_3^2D_z^2+2h_2h_3D_yD_z-\frac{1}{3!}h_2^3D_y^3-$$
$$\frac{8}{3!}h_3^3D_z^3-h_2^2h_3D_y^2D_z-2h_2h_3^2D_yD_z^2+\frac{1}{4!}h_2^4D_y^4+\frac{16}{4!}h_3^4D_z^4+\frac{2}{3!}h_2^3h_3D_y^3D_z+$$
$$\frac{8}{3!}h_2h_3^3D_yD_z^3+h_2^2h_3^2D_y^2D_z^2-\frac{1}{5!}h_2^5D_y^5-\frac{32}{5!}h_3^5D_z^5-\ldots\ \Big]f\left(x_2,y_2,z_2\right)$$

15) $$f\left(x_2,y_0+h_2,z_2\right)=\Big[\ 1-h_2D_y+\frac{1}{2}h_2^2D_y^2-\frac{1}{3!}h_2^3D_y^3+\frac{1}{4!}h_2^4D_y^4-\frac{1}{5!}h_2^5D_y^5+\ldots\ \Big]f\left(x_2,y_2,z_2\right)$$

16) $$f\left(x_0,y_0,z_0+h_3\right)=\Big[\ 1-2h_1D_x-2h_2D_y-h_3D_z+2h_1^2D_x^2+2h_2^2D_y^2+\frac{1}{2}h_3^2D_z^2$$
$$+4h_1h_2D_xD_y+2h_1h_3D_xD_z+2h_2h_3D_yD_z-\frac{8}{3!}h_1^3D_x^3-\frac{8}{3!}h_2^3D_y^3-\frac{1}{3!}h_3^3D_z^3-$$
$$4h_1^2h_2D_x^2D_y-4h_1h_2^2D_xD_y^2-2h_1^2h_3D_x^2D_z-h_1h_3^2D_xD_z^2-2h_2^2h_3D_y^2D_z-h_2^2h_3D_yD_z^2-$$
$$4h_1h_2h_3D_xD_yD_z+\frac{16}{4!}h_1^4D_x^4+\frac{16}{4!}h_2^4D_y^4+\frac{1}{4!}h_3^4D_z^4+\frac{16}{3!}h_1^3h_2D_x^3D_y+\frac{16}{3!}h_1h_2^3D_xD_y^3$$
$$+\frac{8}{3!}h_1^3h_3D_x^3D_z+\frac{2}{3!}h_1h_3^3D_xD_z^3+\frac{8}{3!}h_2^3h_3D_y^3D_z+\frac{2}{3!}h_2h_3^3D_yD_z^3+4h_1^2h_2^2D_x^2D_y^2$$
$$+h_1^2h_3^2D_x^2D_z^2+h_2^2h_3^2D_y^2D_z^2-\frac{32}{5!}h_1^5D_x^5-\frac{32}{5!}h_2^5D_y^5-\frac{1}{5!}h_3^5D_z^5-\frac{32}{4!}h_1^4h_2D_x^4D_y$$
$$-\frac{32}{4!}h_1h_2^4D_xD_y^4-\ldots\ \Big]f\left(x_2,y_2,z_2\right)$$

17) $f\left(x_0,y_2,z_0+h_3\right)=\Big[\ 1-2h_1D_x-h_3D_z+2h_1^2D_x^2+\frac{1}{2}h_3^2D_z^2+2h_1h_3D_xD_z-\frac{8}{3!}h_1^3D_x^3$

$-\frac{1}{3!}h_3^3D_z^3-2h_1^2h_3D_x^2D_z-h_1h_3^2D_xD_z^2+\frac{16}{4!}h_1^4D_x^4+\frac{1}{4!}h_3^4D_z^4+\frac{8}{3!}h_1^3h_3D_x^3D_z$

$+\frac{2}{3!}h_1h_2^3D_xD_z^3+h_1^2h_3^2D_x^2D_z^2-\frac{32}{5!}h_1^5D_x^5-\frac{1}{5!}h_3^5D_z^5+\ldots\ \Big]\ f\left(x_2,y_2,z_2\right)$

18) $f\left(x_2,y_0,z_0+h_3\right)=\Big[\ 1-2h_2D_y-h_3D_z+2h_2^2D_y^2+\frac{1}{2}h_3^2D_z^2+2h_2h_3D_yD_z-$

$\frac{8}{3!}h_2^3D_y^3-\frac{1}{3!}h_3^3D_z^3-2h_2^2h_3D_y^2D_z-h_2h_3^2D_yD_z^2+\frac{16}{4!}h_2^4D_y^4+\frac{1}{4!}h_3^4D_z^4+\frac{8}{3!}h_2^3h_3D_y^3D_z$

$+\frac{2}{3!}h_2h_3^3D_yD_z^3+h_2^2h_3^2D_y^2D_z^2-\frac{32}{5!}h_2^5D_y^5-\frac{1}{5!}h_3^5D_z^5+\ldots\ \Big]\ f\left(x_2,y_2,z_2\right)$

19) $f\left(x_2,y_2,z_0+h_3\right)=\Big[\ 1-h_3D_z+\frac{1}{2}h_3^2D_z^2-\frac{1}{3!}h_3^3D_z^3+\frac{1}{4!}h_3^4D_z^4-\frac{1}{5!}h_3^5D_z^5+\ldots\ \Big]\ f\left(x_2,y_2,z_2\right)$

20) $f\left(x_0+h_1,y_0+h_2,z_0\right)=\Big[\ 1-h_1D_x-h_2D_y-2h_3D_z+\frac{1}{2}h_1^2D_x^2+\frac{1}{2}h_2^2D_y^2+2h_3^2D_z^2+$

$h_1h_2D_xD_y+2h_1h_3D_xD_z+2h_2h_3D_yD_z-\frac{1}{3!}h_1^3D_x^3-\frac{1}{3!}h_2^3D_y^3-\frac{8}{3!}h_3^3D_z^3-$

$\frac{1}{2}h_1^2h_2D_x^2D_y\ \frac{1}{2}h_1h_2^2D_xD_y^2-h_1^2h_3D_x^2D_z-2h_1h_3^2D_xD_z^2-h_2^2h_3D_y^2D_z-2h_2h_3^2D_yD_z^2$

$-2h_1h_2h_3D_xD_yD_z+\frac{1}{4!}h_1^4D_x^4+\frac{1}{4!}h_2^4D_y^4+\frac{16}{4!}h_3^4D_z^4+\frac{1}{3!}h_1^3h_2D_x^3D_y+\frac{1}{3!}h_1h_2^3D_xD_y^3+$

$\frac{2}{3!}h_1^3h_3D_x^3D_z+\frac{8}{3!}h_1h_3^3D_xD_z^3+\frac{2}{3!}h_2^3h_3D_y^3D_z+\frac{8}{3!}h_2h_3^3D_yD_z^3+\frac{1}{4}h_1^2h_2^2D_x^2D_y^2+$

$h_1^2h_3^2D_x^2D_z^2+h_2^2h_3^2D_y^2D_z^2-\frac{1}{5!}h_1^5D_x^5-\frac{1}{5!}h_2^5D_y^5-\frac{32}{5!}h_3^5D_z^5-\frac{1}{4!}h_1^4h_2D_x^4D_y-$

$\frac{1}{4!}h_1h_2^4D_xD_y^4+\ldots\ \Big]\ f\left(x_2,y_2,z_2\right)$

21) $f\left(x_0+h_1,y_0+h_2,z_2\right)=\Big[\ 1-h_1D_x-h_2D_y+\frac{1}{2}h_1^2D_x^2+\frac{1}{2}h_2^2D_y^2+h_1h_2D_xD_y-$

$\frac{1}{3!}h_1^3D_x^3-\frac{1}{3!}h_2^3D_y^3-\frac{1}{2}h_1^2h_2D_x^2D_y-\frac{1}{2}h_1h_2^2D_xD_y^2+\frac{1}{4!}h_1^4D_x^4+\frac{1}{4!}h_2^4D_y^4+$

$\frac{1}{3!}h_1^3h_2D_x^3D_y+\frac{1}{3!}h_1h_2^3D_xD_y^3+\frac{1}{4}h_1^2h_2^2D_x^2D_y^2-\frac{1}{5!}h_1^5D_x^5-\frac{1}{5!}h_2^5D_y^5-\frac{1}{4!}h_1^4h_2D_x^4D_y$

$-\frac{1}{4!}h_1h_2^4D_xD_y^4+\ldots\ \Big]\ f\left(x_2,y_2,z_2\right)$

22) $f\left(x_0+h_1,y_0,z_0+h_3\right)=\Big[\ 1-h_1D_x-2h_2D_y-h_3D_z+\frac{1}{2}h_1^2D_x^2+2h_2^2D_y^2+\frac{1}{2}h_3^2D_z^2$

$+2h_1h_2D_xD_y+h_1h_3D_xD_z+2h_2h_3D_yD_z-\frac{1}{3!}h_1^3D_x^3-\frac{8}{3!}h_2^3D_y^3-\frac{1}{3!}h_3^3D_z^3-$

$h_1^2h_2D_x^2D_y-2h_1h_2^2D_xD_y^2-\frac{1}{2}h_1^2h_3D_x^2D_z-\frac{1}{2}h_1h_3^2D_xD_z^2-2h_2^2h_3D_y^2D_z-$

$h_2h_3^2D_yD_z^2-2h_1h_2h_3D_xD_yD_z+\frac{1}{4!}h_1^4D_x^4+\frac{16}{4!}h_2^4D_y^4+\frac{1}{4!}h_3^4D_z^4+\frac{2}{3!}h_1^3h_2D_x^3D_y$

$$+\frac{8}{3!}h_1h_2^3D_xD_y^3+\frac{1}{3!}h_1^3h_3D_x^3D_z+\frac{1}{3!}h_1h_3^3D_xD_z^3+\frac{8}{3!}h_2^3h_3D_y^3D_z+\frac{2}{3!}h_2h_3^3D_yD_z^3+$$
$$h_1^2h_2^2D_x^2D_y^2+\frac{1}{4}h_1^2h_3^2D_x^2D_z^2+h_2^2h_3^2D_y^2D_z^2-\frac{1}{5!}h_1^5D_x^5-\frac{32}{5!}h_2^5D_y^5-\frac{1}{5!}h_3^5D_z^5$$
$$-\frac{2}{4!}h_1^4h_2D_x^4D_y-\frac{16}{4!}h_1h_2^4D_xD_y^4+\dots\Big]f\left(x_2,y_2,z_2\right)$$

23) $$f\left(x_0+h_1,y_2,z_0+h_3\right)=\Big[1-h_1D_x-h_3D_z+\frac{1}{2}h_1^2D_x^2+\frac{1}{2}h_3^2D_z^2+h_1h_3D_xD_z-$$
$$\frac{1}{3!}h_1^3D_x^3-\frac{1}{3!}h_3^3D_z^3\frac{1}{2}h_1^2h_3D_x^2D_z-\frac{1}{2}h_1h_3^2D_xD_z^2+\frac{1}{4!}h_1^4D_x^4+\frac{1}{4!}h_3^4D_z^4+\frac{1}{3!}h_1^3h_3D_x^3D_z$$
$$+\frac{1}{3!}h_1h_3^3D_xD_z^3+\frac{1}{4}h_1^2h_3^2D_x^2D_z^2-\frac{1}{5!}h_1^5D_x^5-\frac{1}{5!}h_3^5D_z^5+\dots\Big]f\left(x_2,y_2,z_2\right)$$

$$f\left(x_0,y_0+h_2,z_0+h_3\right)=\Big[1-2h_1D_x-h_2D_y-h_3D_z+2h_1^2D_x^2+\frac{1}{2}h_2^2D_y^2+\frac{1}{2}h_3^2D_z^2$$

24) $$+2h_1h_2D_xD_y+2h_1h_3D_xD_z+h_2h_3D_yD_z-\frac{8}{3!}h_1^3D_x^3-\frac{1}{3!}h_2^3D_y^3-\frac{1}{3!}h_3^3D_z^3-$$
$$2h_1^2h_2D_x^2D_y-h_1h_2^2D_xD_y^2-2h_1^2h_3D_x^2D_z-h_1h_3^2D_xD_z^2-\frac{1}{2}h_2^2h_3D_y^2D_z-$$
$$\frac{1}{2}h_2h_3^2D_yD_z^2-2h_1h_2h_3D_xD_yD_z+\frac{16}{4!}h_1^4D_x^4+\frac{1}{4!}h_2^4D_y^4+\frac{1}{4!}h_3^4D_z^4+\frac{8}{3!}h_1^3h_2D_x^3D_y$$
$$+\frac{2}{3!}h_1h_2^3D_xD_y^3+\frac{8}{3!}h_1^3h_3D_x^3D_z+\frac{2}{3!}h_1h_3^3D_xD_z^3+\frac{1}{3!}h_2^3h_3D_y^3D_z+\frac{1}{3!}h_2h_3^3D_yD_z^3$$
$$+h_1^2h_2^2D_x^2D_y^2+h_1^2h_3^2D_x^2D_z^2+\frac{1}{4}h_2^2h_3^2D_y^2D_z^2-\frac{32}{5!}h_1^5D_x^5-\frac{1}{5!}h_2^5D_y^5-\frac{1}{5!}h_3^5D_z^5$$
$$-\frac{16}{4!}h_1^4h_2D_x^4D_y-\frac{2}{4!}h_1h_2^4D_xD_y^4+\dots\Big]f\left(x_2,y_2,z_2\right)$$

25) $$f\left(x_2,y_0+h_2,z_0+h_3\right)=\Big[1-h_2D_y-h_3D_z+\frac{1}{2}h_2^2D_y^2+\frac{1}{2}h_3^2D_z^2+h_2h_3D_yD_z-$$
$$\frac{1}{3!}h_2^3D_y^3-\frac{1}{3!}h_3^3D_z^3-\frac{1}{2}h_2^2h_3D_y^2D_z-\frac{1}{2}h_2h_3^2D_yD_z^2+\frac{1}{4!}h_2^4D_y^4+\frac{1}{4!}h_3^4D_z^4+$$
$$\frac{1}{3!}h_2^3h_3D_y^3D_z+\frac{1}{3!}h_2h_3^3D_yD_z^3+\frac{1}{4}h_2^2h_3^2D_y^2D_z^2-\frac{1}{5!}h_2^5D_y^5-\frac{1}{5!}h_3^5D_z^5-\dots\Big]f\left(x_2,y_2,z_2\right)$$

26) $$f\left(x_0+h_1,y_0+h_2,z_0+h_3\right)=\Big[1-h_1D_x-h_2D_y-h_3D_z+\frac{1}{2}h_1^2D_x^2+\frac{1}{2}h_2^2D_y^2+\frac{1}{2}h_3^2D_z^2$$
$$+h_1h_2D_xD_y+h_1h_3D_xD_z+h_2h_3D_yD_z-\frac{1}{3!}h_1^3D_x^3-\frac{1}{3!}h_2^3D_y^3-\frac{1}{3!}h_3^3D_z^3-$$
$$\frac{1}{2}h_1^2h_2D_x^2D_y-\frac{1}{2}h_1h_2^2D_xD_y^2-\frac{1}{2}h_1^2h_3D_x^2D_z-\frac{1}{2}h_1h_3^2D_xD_z^2-\frac{1}{2}h_2^2h_3D_y^2D_z-$$
$$\frac{1}{2}h_2h_3^2D_yD_z^2-h_1h_2h_3D_xD_yD_z+\frac{1}{4!}h_1^4D_x^4+\frac{1}{4!}h_2^4D_y^4+\frac{1}{4!}h_3^4D_z^4+\frac{1}{3!}h_1^3h_2D_x^3D_y+$$
$$\frac{1}{3!}h_1h_2^3D_xD_y^3+\frac{1}{3!}h_1^3h_3D_x^3D_z+\frac{1}{3!}h_1h_3^3D_xD_z^3+\frac{1}{3!}h_2^3h_3D_y^3D_z+\frac{1}{3!}h_2h_3^3D_yD_z^3+$$
$$\frac{1}{4}h_1^2h_2^2D_x^2D_y^2+\frac{1}{4}h_1^2h_3^2D_x^2D_z^2+\frac{1}{4}h_2^2h_3^2D_y^2D_z^2-\frac{1}{5!}h_1^5D_x^5-\frac{1}{5!}h_2^5D_y^5-\frac{1}{5!}h_3^5D_z^5-$$

$$\frac{1}{4!}h_1^4h_2D_x^4D_y-\frac{1}{4!}h_1h_2^4D_xD_y^4+\ldots\ \Big]\ f\ (x_2,y_2,z_2)$$

from ($3'$) , (1) - (26) and $f(x_2,y_2,z_2)=Ef(x_1,y_1,z_1)$ where $E=e^{(h_1D_x+h_2D_y+h_3D_z)}$ we get:-

$$\therefore I_1=\int_{z_0}^{z_2}\int_{y_0}^{y_2}\int_{x_0}^{x_2}f\ (x,y,z)\ dxdydz=\frac{h_1h_2h_3}{27}\Big[\ f\ (x_0,y_0,z_0)+f\ (x_0,y_0,z_2)+f\ (x_0,y_2,z_0)+$$
$$f\ (x_0,y_2,z_2)+f\ (x_2,y_0,z_0)+f\ (x_2,y_0,z_2)+f\ (x_2,y_2,z_0)+f\ (x_2,y_2,z_2)+4\ \Big[$$
$$f\ (x_0+h_1,y_0,z_0)+f\ (x_0+h_1,y_0,z_2)+f\ (x_0+h_1,y_2,z_0)+f\ (x_0+h_1,y_2,z_2)$$
$$+f\ (x_0,y_0+h_2,z_0)+f\ (x_0,y_0+h_2,z_2)+f\ (x_2,y_0+h_2,z_0)+f\ (x_2,y_0+h_2,z_2)$$
$$+f\ (x_0,y_0,z_0+h_3)+f\ (x_0,y_2,z_0+h_3)+f\ (x_2,y_0,z_0+h_3)+f\ (x_2,y_2,z_0+h_3)$$
$$+4\Big[\ f\ (x_0+h_1,y_0+h_2,z_0)+f\ (x_0+h_1,y_0+h_2,z_2)+f\ (x_0,y_0+h_2,z_0+h_3)$$
$$+f\ (x_2,y_0+h_2,z_0+h_3)+f\ (x_0+h_1,y_0,z_0+h_3)+f\ (x_0+h_1,y_2,z_0+h_3)$$
$$4\ f\ (x_0+h_1,y_0+h_2,z_0+h_3)\ \Big]\Big]\ \Big]+\Big[\left(a_1h_1^5h_2h_3D_x^4+a_2h_1h_2^5h_3D_y^4+a_3h_1h_2h_3^5D_z^4\right)+$$
$$\left(a_4h_1^7h_2h_3D_x^6+a_5h_1h_2^7h_3D_y^6+a_6h_1h_2h_3^7D_z^6+a_7h_1^6h_2^2h_3D_x^5D_y+\ldots\right)+$$
$$+\left(a_{22}h_1^8h_2h_3D_x^7+a_{23}h_1h_2^8h_3D_y^7+\ldots\right)+\ldots\ \Big]\ f\ (x_{1,}y_1,z_1) \qquad \ldots(4')$$

$$I_2=\int_{z_0}^{z_2}\int_{y_0}^{y_2}\int_{x_2}^{x_n}f\ (x,y,z)\ dxdydz=\int_{z_0}^{z_2}\int_{y_0}^{y_2}\sum_{l=1}^{(n/2)-1}\int_{x_{2l}}^{x_{2l+2}}f\ (x,y,z)\ dxdydz=\frac{h_1h_2h_3}{27}\sum_{l=1}^{(n/2)-1}\Big[$$
$$f\ (x_{2l},y_0,z_0)+f\ (x_{2l},y_0,z_2)+f\ (x_{2l},y_2,z_0)+f\ (x_{2l},y_2,z_0)+f\ (x_{2l+2},y_0,z_2)$$
$$+f\ (x_{2l+2},y_0,z_2)+f\ (x_{2l+2},y_2,z_0)+f\ (x_{2l+2},y_2,z_2)+4\Big[\ f\ (x_{2l}+h_1,y_0,z_0)$$
$$+f\ (x_{2l}+h_1,y_0,z_2)+f\ (x_{2l}+h_1,y_2,z_0)+f\ (x_{2l}+h_1,y_2,z_2)+f\ (x_{2l},y_0+h_2,z_0)$$
$$+f\ (x_{2l},y_0+h_2,z_2)+f\ (x_{2l+2},y_0+h_2,z_0)+f\ (x_{2l+2},y_0+h_2,z_2)+f\ (x_{2l},y_0,z_0+h_3)$$
$$+f\ (x_{2l+2},y_0,z_0+h_3)+f\ (x_{2l},y_2,z_0+h_3)+f\ (x_{2l+2},y_2,z_0+h_3)+4\ \Big[\ f\ (x_{2l}+h_1,y_0+h_2,z_0)$$
$$+f\ (x_{2l}+h_1,y_0+h_2,z_2)+f\ (x_{2l},y_0+h_2,z_0+h_3)+f\ (x_{2l+2},y_0+h_2,z_0+h_3)+$$
$$f\ (x_{2l}+h_1,y_0,z_0+h_3)+f\ (x_{2l}+h_1,y_2,z_0+h_3)+4f\ (x_{2l}+h_1,y_0+h_2,z_0+h_3)\ \Big]\ \Big]\Big]$$
$$+e_1h_1^4+e_2h_2^4+e_3h_3^4+e_4h_1^6+e_5h_2^6+e_6h_3^6+\ldots\ldots(5')$$

$$I_3=\int_{z_0}^{z_2}\int_{y_2}^{y_r}\int_{x_0}^{x_2}f\ (x,y,z)\ dxdydz=\int_{z_0}^{z_2}\sum_{s=1}^{(r/2)-1}\int_{y_{2s}}^{y_{2s+2}}\int_{x_0}^{x_2}f\ (x,y,z)\ dxdydz=\frac{h_1h_2h_3}{27}\sum_{s=1}^{(r/2)-1}\Big[$$
$$f\ (x_0,y_{2s},z_0)+f\ (x_0,y_{2s},z_2)+f\ (x_0,y_{2s+2},z_0)+f\ (x_0,y_{2s+2},z_2)+f\ (x_2,y_{2s},z_0)$$
$$+f\ (x_2,y_{2s},z_2)+f\ (x_2,y_{2s+2},z_0)+f\ (x_2,y_{2s+2},z_2)+4\Big[\ f\ (x_0+h_1,y_{2s},z_0)+$$
$$f\ (x_0+h_1,y_{2s},z_2)+f\ (x_0+h_1,y_{2s+2},z_0)+f\ (x_0+h_1,y_{2s+2},z_2)+f\ (x_0,y_{2s}+h_2,z_0)+$$
$$f\ (x_0,y_{2s}+h_2,z_2)+f\ (x_2,y_{2s}+h_2,z_0)+f\ (x_2,y_{2s}+h_2,z_2)+f\ (x_0,y_{2s},z_0+h_3)$$
$$+f\ (x_2,y_{2s},z_0+h_3)+f\ (x_0,y_{2s+2},z_0+h_3)+f\ (x_2,y_{2s+2},z_0+h_3)+4\Big[$$
$$f\ (x_0+h_1,y_{2s}+h_2,z_0)+f\ (x_0+h_1,y_{2s}+h_2,z_2)+f\ (x_0,y_{2s}+h_2,z_0+h_3)+$$

$$f\left(x_2,y_{2s}+h_2,z_0+h_3\right)+f\left(x_0+h_1,y_{2s},z_0+h_3\right)+f\left(x_0+h_1,y_{2s+2},z_0+h_3\right)+$$
$$4f\left(x_0+h_1,y_{2s}+h_2,z_0+h_3\right)\ \Big]\Big]\Big]+g_1h_1^4+g_2h_2^4+g_3h_3^4+g_4h_1^6+g_5h_2^6+g_6h_3^6+\ldots$$

…(6′)

$$I_4=\int_{z_2}^{z_m}\int_{y_0}^{y_2}\int_{x_0}^{x_2}f\left(x,y,z\right)dxdydz=\sum_{t=1}^{(m/2)-1}\int_{z_{2t}}^{z_{2t+2}}\int_{y_0}^{y_2}\int_{x_0}^{x_2}f\left(x,y,z\right)dxdydz=\frac{h_1h_2h_3}{27}\sum_{t=1}^{(m/2)-1}\Big[$$
$$f\left(x_0,y_0,z_{2t}\right)+f\left(x_0,y_0,z_{2t+2}\right)+f\left(x_0,y_2,z_{2t}\right)+f\left(x_0,y_2,z_{2t+2}\right)+f\left(x_2,y_0,z_{2t}\right)+$$
$$f\left(x_2,y_0,z_{2t+2}\right)+f\left(x_2,y_2,z_{2t}\right)+f\left(x_2,y_2,z_{2t+2}\right)+4\Big[\ f\left(x_0+h_1,y_0,z_{2t}\right)+$$
$$f\left(x_0+h_1,y_0,z_{2t+2}\right)+f\left(x_0+h_1,y_2,z_{2t}\right)+f\left(x_0+h_1,y_2,z_{2t+2}\right)+f\left(x_0,y_0+h_2,z_{2t}\right)+$$
$$f\left(x_0,y_0+h_2,z_{2t+2}\right)+f\left(x_2,y_0+h_2,z_{2t}\right)+f\left(x_2,y_0+h_2,z_{2t+2}\right)+f\left(x_0,y_0,z_{2t}+h_3\right)+$$
$$f\left(x_0,y_2,z_{2t}+h_3\right)+f\left(x_2,y_0,z_{2t}+h_3\right)+f\left(x_2,y_2,z_{2t}+h_3\right)+4\ \Big[\ f\left(x_0+h_1,y_0+h_2,z_{2t}\right)$$
$$+f\left(x_0+h_1,y_0+h_2,z_{2t+2}\right)+f\left(x_0,y_0+h_2,z_{2t}+h_3\right)+f\left(x_2,y_0+h_2,z_{2t}+h_3\right)$$
$$+f\left(x_0+h_1,y_0,z_{2t}+h_3\right)+f\left(x_0+h_1,y_2,z_{2t}+h_3\right)+4f\left(x_0+h_1,y_0+h_2,z_{2t}+h_3\right)\ \Big]\Big]\Big]$$
$$+j_1h_1^4+j_2h_2^4+j_3h_3^4+j_4h_1^6+j_5h_2^6+j_6h_3^6+\ldots$$

…(7′)

$$I_5=\int_{z_0}^{z_2}\int_{y_2}^{y_r}\int_{x_2}^{x_n}f\left(x,y,z\right)dxdydz=\int_{z_0}^{z_2}\sum_{s=1}^{(r/2)-1}\int_{y_{2s}}^{y_{2s+2}}\sum_{l=1}^{(n/2)-1}\int_{x_{2l}}^{x_{2l+2}}f\left(x,y,z\right)dxdydz=$$
$$\frac{h_1h_2h_3}{27}\sum_{s=1}^{(r/2)-1}\sum_{l=1}^{(n/2)-1}\Big[\ f\left(x_{2l},y_{2s},z_0\right)+f\left(x_{2l},y_{2s},z_2\right)+f\left(x_{2l},y_{2s+2},z_0\right)+$$
$$f\left(x_{2l},y_{2s+2},z_2\right)+f\left(x_{2l+2},y_{2s},z_0\right)+f\left(x_{2l+2},y_{2s},z_2\right)+f\left(x_{2l+2},y_{2s+2},z_0\right)+$$
$$f\left(x_{2l+2},y_{2s+2},z_2\right)+4\Big[\ f\left(x_{2l}+h_1,y_{2s},z_0\right)+f\left(x_{2l}+h_1,y_{2s},z_2\right)+$$
$$f\left(x_{2l}+h_1,y_{2s+2},z_0\right)+f\left(x_{2l}+h_1,y_{2s+2},z_2\right)+f\left(x_{2l},y_{2s}+h_2,z_0\right)+f\left(x_{2l},y_{2s}+h_2,z_2\right)$$
$$+f\left(x_{2l+2},y_{2s}+h_2,z_0\right)+f\left(x_{2l+2},y_{2s}+h_2,z_2\right)+f\left(x_{2l},y_{2s},z_0+h_3\right)+$$
$$f\left(x_{2l+2},y_{2s},z_0+h_3\right)+f\left(x_{2l},y_{2s+2},z_0+h_3\right)+f\left(x_{2l+2},y_{2s+2},z_0+h_3\right)+4\Big[$$
$$f\left(x_{2l}+h_1,y_{2s}+h_2,z_0\right)+f\left(x_{2l}+h_1,y_{2s}+h_2,z_2\right)+f\left(x_{2l},y_{2s}+h_2,z_0+h_3\right)+$$
$$f\left(x_{2l+2},y_{2s}+h_2,z_0+h_3\right)+f\left(x_{2l}+h_1,y_{2s},z_0+h_3\right)+f\left(x_{2l}+h_1,y_{2s+2},z_0+h_3\right)+$$
$$+\ 4\ f\left(x_{2l}+h_1,y_{2s}+h_2,z_0+h_3\right)\ \Big]\Big]\Big]+p_1h_1^4+p_2h_2^4+p_3h_3^4+p_4h_1^6+p_5h_2^6+p_6h_3^6+\ldots$$

…(8′)

$$I_6=\int_{z_2}^{z_m}\int_{y_0}^{y_2}\int_{x_2}^{x_n}f\left(x,y,z\right)dxdydz=\sum_{t=1}^{(m/2)-1}\int_{z_{2t}}^{z_{2t+2}}\int_{y_0}^{y_2}\sum_{l=1}^{(n/2)-1}\int_{x_{2l}}^{x_{2l+2}}f\left(x,y,z\right)dxdydz=\frac{h_1h_2h_3}{27}$$
$$\sum_{t=1}^{(m/2)-1}\sum_{l=1}^{(n/2)-1}\Big[\ f\left(x_{2l},y_0,z_{2t}\right)+f\left(x_{2l},y_0,z_{2t+2}\right)+f\left(x_{2l},y_2,z_{2t}\right)+f\left(x_{2l},y_2,z_{2t+2}\right)$$
$$+f\left(x_{2l+2},y_0,z_{2t}\right)+f\left(x_{2l+2},y_0,z_{2t+2}\right)+f\left(x_{2l+2},y_2,z_{2t}\right)+f\left(x_{2l+2},y_2,z_{2t+2}\right)+$$
$$4\Big[\ f\left(x_{2l}+h_1,y_0,z_{2t}\right)+f\left(x_{2l}+h_1,y_0,z_{2t+2}\right)+f\left(x_{2l}+h_1,y_2,z_{2t}\right)+f\left(x_{2l}+h_1,y_2,z_{2t+2}\right)$$
$$+f\left(x_{2l},y_0+h_2,z_{2t}\right)+f\left(x_{2l},y_0+h_2,z_{2t+2}\right)+f\left(x_{2l+2},y_0+h_2,z_{2t}\right)+$$
$$f\left(x_{2l+2},y_0+h_2,z_{2t+2}\right)+f\left(x_{2l},y_0,z_{2t}+h_3\right)+f\left(x_{2l},y_2,z_{2t}+h_3\right)+$$
$$f\left(x_{2l+2},y_0,z_{2t}+h_3\right)+f\left(x_{2l+2},y_2,z_{2t}+h_3\right)+4\Big[\ f\left(x_{2l}+h_1,y_0,z_{2t}+h_3\right)$$

$$+f\left(x_{2l}+h_1,y_2,z_{2t}+h_3\right)+f\left(x_{2l}+h_1,y_0+h_2,z_{2t}\right)+f\left(x_{2l}+h_1,y_0+h_2,z_{2t+2}\right)+$$
$$f\left(x_{2l},y_0+h_2,z_{2t}+h_3\right)+f\left(x_{2l+2},y_0+h_2,z_{2t}+h_3\right)+4f\left(x_{2l}+h_1,y_0+h_2,z_{2t}+h_3\right)\Big]\Big]\Big]$$
$$+q_1h_1^4+q_2h_2^4+q_3h_3^4+q_4h_1^6+q_5h_2^6+q_6h_3^6+\ldots \qquad \ldots(9')$$

$$I_7=\int_{z_2}^{z_m}\int_{y_2}^{y_r}\int_{x_0}^{x_2}f\left(x,y,z\right)dxdydz=\sum_{t=1}^{(m/2)-1}\int_{z_{2t}}^{z_{2t+2}}\sum_{s=1}^{(r/2)-1}\int_{y_{2s}}^{y_{2s+2}}\int_{x_0}^{x_2}f\left(x,y,z\right)dxdydz=\frac{h_1h_2h_3}{27}\sum_{t=1}^{(m/2)-1}\sum_{s=1}^{(r/2)-1}$$
$$\Big[\ f\left(x_0,y_{2s},z_{2t}\right)+f\left(x_0,y_{2s},z_{2t+2}\right)+f\left(x_0,y_{2s+2},z_{2t}\right)+f\left(x_0,y_{2s+2},z_{2t+2}\right)+$$
$$f\left(x_2,y_{2s},z_{2t}\right)+f\left(x_2,y_{2s},z_{2t+2}\right)+f\left(x_2,y_{2s+2},z_{2t}\right)+f\left(x_2,y_{2s+2},z_{2t+2}\right)+$$
$$4\Big[\ f\left(x_0+h_1,y_{2s},z_{2t}\right)+f\left(x_0+h_1,y_{2s},z_{2t+2}\right)+f\left(x_0+h_1,y_{2s+2},z_{2t}\right)+$$
$$f\left(x_0+h_1,y_{2s+2},z_{2t+2}\right)+f\left(x_0,y_{2s}+h_2,z_{2t}\right)+f\left(x_0,y_{2s}+h_2,z_{2t+2}\right)+f\left(x_2,y_{2s}+h_2,z_{2t}\right)$$
$$+f\left(x_2,y_{2s}+h_2,z_{2t+2}\right)+f\left(x_0,y_{2s},z_{2t}+h_3\right)+f\left(x_2,y_{2s},z_{2t}+h_3\right)+f\left(x_0,y_{2s+2},z_{2t}+h_3\right)$$
$$+f\left(x_2,y_{2s+2},z_{2t}+h_3\right)+4\Big[\ f\left(x_0+h_1,y_{2s}+h_2,z_{2t}\right)+f\left(x_0+h_1,y_{2s}+h_2,z_{2t+2}\right)+$$
$$f\left(x_0,y_{2s}+h_2,z_{2t}+h_3\right)+f\left(x_2,y_{2s}+h_2,z_{2t}+h_3\right)+f\left(x_0+h_1,y_{2s},z_{2t}+h_3\right)+$$
$$f\left(x_0+h_1,y_{2s+2},z_{2t}+h_3\right)+4f\left(x_0+h_1,y_{2s}+h_2,z_{2t}+h_3\right)\Big]\Big]\Big]+u_1h_1^4+u_2h_2^4+$$
$$u_3h_3^4+u_4h_1^6+u_5h_2^6+u_6h_3^6+\ldots \qquad \ldots(10')$$

$$I_8=\int_{z_2}^{z_m}\int_{y_2}^{y_r}\int_{x_2}^{x_n}f\left(x,y,z\right)dxdydz=\sum_{t=1}^{(m/2)-1}\int_{z_{2t}}^{z_{2t+2}}\sum_{s=1}^{(r/2)-1}\int_{y_{2s}}^{y_{2s+2}}\sum_{l=1}^{(n/2)-1}\int_{x_{2l}}^{x_{2l+2}}f\left(x,y,z\right)dxdydz=\frac{h_1h_2h_3}{27}$$
$$\sum_{t=1}^{(m/2)-1}\sum_{s=1}^{(r/2)-1}\sum_{l=1}^{(n/2)-1}\Big[\ f\left(x_{2l},y_{2s},z_{2t}\right)+f\left(x_{2l},y_{2s},z_{2t+2}\right)+f\left(x_{2l},y_{2s+2},z_{2t}\right)+$$
$$f\left(x_{2l},y_{2s+2},z_{2t+2}\right)+f\left(x_{2l+2},y_{2s},z_{2t}\right)+f\left(x_{2l+2},y_{2s},z_{2t+2}\right)+f\left(x_{2l+2},y_{2s+2},z_{2t}\right)$$
$$+f\left(x_{2l+2},y_{2s+2},z_{2t+2}\right)+4\Big[\ f\left(x_{2l}+h_1,y_{2s},z_{2t}\right)+f\left(x_{2l}+h_1,y_{2s},z_{2t+2}\right)+$$
$$f\left(x_{2l}+h_1,y_{2s+2},z_{2t}\right)+f\left(x_{2l}+h_1,y_{2s+2},z_{2t+2}\right)+f\left(x_{2l},y_{2s}+h_2,z_{2t}\right)+$$
$$f\left(x_{2l},y_{2s}+h_2,z_{2t+2}\right)+f\left(x_{2l+2},y_{2s}+h_2,z_{2t}\right)+f\left(x_{2l+2},y_{2s}+h_2,z_{2t+2}\right)+$$
$$f\left(x_{2l},y_{2s},z_{2t}+h_3\right)+f\left(x_{2l},y_{2s+2},z_{2t}+h_3\right)+f\left(x_{2l+2},y_{2s+2},z_{2t}+h_3\right)+$$
$$f\left(x_{2l},y_{2s+2},z_{2t}+h_3\right)+4\ \Big[\ f\left(x_{2l}+h_1,y_{2s}+h_2,z_{2t}\right)+f\left(x_{2l}+h_1,y_{2s}+h_2,z_{2t+2}\right)+$$
$$f\left(x_{2l},y_{2s}+h_2,z_{2t}+h_3\right)+f\left(x_{2l+2},y_{2s}+h_2,z_{2t}+h_3\right)+f\left(x_{2l}+h_1,y_{2s},z_{2t}+h_3\right)+$$
$$f\left(x_{2l}+h_1,y_{2s+2},z_{2t}+h_3\right)+4f\left(x_{2l}+h_1,y_{2s}+h_2,z_{2t}+h_3\right)\Big]\Big]\Big]+v_1h_1^4+v_2h_2^4+v_3h_3^4+$$
$$v_4h_1^6+v_5h_2^6+v_6h_3^6+\ldots \qquad \ldots(11')$$

where $e_i,g_i,j_i,p_i,q_i,u_i,v_i,\ i=1,2,\ldots$, Collection of formulas from ($4'$)-($11'$)

$$I=\int_e^g\int_b^d\int_a^b f\left(x,y,z\right)dxdydz=sim^3\left(h_1,h_2,h_3\right)+\Big[\left(a_1h_1^5h_2h_3D_x^4+a_2h_1h_2^5h_3D_y^4+a_3h_1h_2h_3^5D_z^4\right)+$$
$$\left(a_4h_1^7h_2h_3D_x^6+a_5h_1h_2^7h_3D_y^6+a_6h_1h_2h_3^7D_z^6+a_7h_1^6h_2^2h_3D_x^5D_y+\ldots\right)+\left(a_{22}h_1^8h_2h_3D_x^7+\right.$$
$$\left.a_{23}h_1h_2^8h_3D_y^7+\ldots\right)+\ldots\ \Big]\ f\left(x_1,y_1,z_1\right)+C_1h_1^4+C_2h_2^4+C_3h_3^4+C_4h_1^6+C_5h_2^6+C_6h_3^6+\ldots$$
$$\ldots(12')$$

where $a_i,C_i,\ i=1,2,\ldots$

□

ii) We can partition integration *I* to

$$I=\int_e^g\int_c^d\int_a^b f\,(x,y,z)\,dxdydz=\int_{z_0}^{z_m}\int_{y_0}^{y_r}\int_{x_0}^{x_n} f\,(x,y,z)\,dxdydz=\int_{z_0}^{z_{m-2}}\int_{y_0}^{y_{r-2}}\int_{x_0}^{x_{n-2}} f\,(x,y,z)\,dxdydz+$$

$$\int_{z_{m-2}}^{z_m}\int_{y_{r-2}}^{y_r}\int_{x_0}^{x_{n-2}} f\,(x,y,z)\,dxdydz+\int_{z_{m-2}}^{z_m}\int_{y_0}^{y_{r-2}}\int_{x_{n-2}}^{x_n} f\,(x,y,z)\,dxdydz+\int_{z_0}^{z_{m-2}}\int_{y_{r-2}}^{y_r}\int_{x_{n-2}}^{x_n} f\,(x,y,z)\,dxdydz$$

$$+\int_{z_{m-2}}^{z_m}\int_{y_0}^{y_{r-2}}\int_{x_0}^{x_{n-2}} f\,(x,y,z)\,dxdydz+\int_{z_0}^{z_{m-2}}\int_{y_{r-2}}^{y_r}\int_{x_0}^{x_{n-2}} f\,(x,y,z)\,dxdydz+\int_{z_0}^{z_{m-2}}\int_{y_0}^{y_{r-2}}\int_{x_{n-2}}^{x_n} f\,(x,y,z)\,dxdydz$$

$$+\int_{z_{m-2}}^{z_m}\int_{y_{r-2}}^{y_r}\int_{x_{n-2}}^{x_n} f\,(x,y,z)\,dxdydz \qquad ...(13')$$

All above integrals have continuous integrands in it's region of integration excepted the first integration (I_8) has singularity in its region at the point (x_n,y_r,z_m) so it can be the values of these integrals $(I_1,I_2,I_3,I_4,I_5,I_6,I_7)$ account from [2] either to find a first integration spread function using (Taylor's series for a function of several variables) around the point $(x_{n-2},y_{r-2},z_{m-2})$ Sestri [5] so we use the same method of proof (i).

NOT:-

1. From the theorem if $h_1=h_2=2h_3$ we get

$$I=\int_e^g\int_b^d\int_a^b f\,(x,y,z)dxdydz=sim^3(h_1,h_1,0.5h_1)+E\,(h_1,h_1,0.5h_1)$$

*$f\,(x,y,z)$ is Continuous and With Singular Partial Derivatives at the point (a,c,e) then

$$E_{sim^3}(h_1,h_1,0.5h_1)=\Big[h_1^7\left(b_1D_x^4+b_2D_y^4+b_3D_z^4\right)+h_1^9\left(b_4D_x^6+b_5D_y^6+b_6D_z^6+b_7D_x^5D_y+...\right)+h_1^{10}\left(b_{22}D_x^7+b_{23}D_y^7+...\right)+...\Big]\,f\,(x_1,y_1,z_1)+G_1h_1^4+G_2h_1^6+\cdots \qquad ...(14')$$

**$f\,(x,y,z)$ is Continuous With Singular Partial Derivatives at the point (b,d,g) then

$$E_{sim^3}(h_1,h_1,0.5h_1)=\Big[\;h_1^7\left(d_1D_x^4+d_2D_y^4+d_3D_z^4\right)+h_1^9\left(d_4D_x^6+d_5D_y^6+d_6D_z^6+d_7D_x^5D_y+...\right)+h_1^{10}\left(d_{22}D_x^7+d_{23}D_y^7+...\right)+...\Big]\,f\,(x_{n-1},y_{r-1},z_{m-1})+K_1h_1^4+K_2h_1^6+... \qquad ...(15')$$

Such that b_i,d_i,J_i,K_i are constants ,for all $i=1,2,...$.

2. If the integrand of integral has singularity at one end of region of integration we will use the suggestion ofDavias and Rabinowitz [3] to evaluate the integral .

3.Examples.

$I=\int_{0.5}^1\int_{0.5}^1\int_{0.5}^1 zy\sqrt{1-xyz}\,dxdydz$ The integration Integrandcontinuous the integration period But a single partial derivatives in point $(x,y,z)=(1,1,1)$, And the type of radical singularity .and his analytic value (0.0514008709909)Rounded to (13) decimal placestheformula of the correction terms for the integrationby using formula $(15')$ is

$E_{sim^3}=a_1h_1^{3.5}+A_1h_1^4+a_2h_1^{4.5}+a_3h_1^{5.5}+A_2h_1^6+a_4h_1^{6.5}+...\;i=1,2,3,...\;,\quad A_i,a_i$. We conclude from the tableThe value of integration using the simpson's Rule with Romberg accelerating that value equal to the analytical valuewhen (subintervals $n = 64, r = 64, m = 128$)(2^{19} subintervals)

n=r	m	Simpson value	K=3.5	K=4	K=4.5	K=5.5	K=6
2	4	0.0513372307920					
4	8	0.0513942765097	0.0513998075675				
8	16	0.0514002076099	0.0514007826795	0.0514008476870			
16	32	0.0514008061995	0.0514008642377	0.0514008696749	0.0514008706916		
32	64	0.0514008648205	0.0514008705043	0.0514008709221	0.0514008709797	0.0514008709863	
64	128	0.0514008704149	0.0514008709573	0.0514008709875	0.0514008709906	0.0514008709908	0.0514008709909

Table (1)

And The integration Integrand $I = \int_0^1\int_0^1\int_0^1 x^5 \log(x+y+z)\,dxdydz$ has singleIn point $(x,y,z) = (0,0,0)$ And the type of singularity here logarithm, and his analytic value (0.09840938817995)Rounded to (14) decimal placesandcorrection terms by usingformula $(14')$ $E_{sim^3} = A_1h_1^4 + A_2h_1^6 + a_1h_1^8 \ln h_1 + A_3h_1^8 + \ldots\ i = 1,2,3,\ldots$, A_i, a_i since $a_1h_1^8 \ln h_1 + A_3h_1^8$ appear in the correction terms ,so will use shanke suggestion [6] andas suggested by Davias and Rabinowitz[3] for evaluation the value ,We conclude from the table(2)The value of integration using the simpson's rule with Romberg accelerating that value equal to the analytical valuewhen (subintervals $n = 64, r = 64, m = 128$)(2^{19} subintervals)

n=r	m	Simpson value	K=4	K=6	K=8	K=8	K=10
2	4	0.11940653750843					
4	8	0.09983154814037	0.09852654884917				
8	16	0.09850023996758	0.09841148608940	0.09840965969639			
16	32	0.09841509905467	0.09840942299381	0.09840939024626	0.09840938918960		
32	64	0.09840974563139	0.09840938873650	0.09840938819274	0.09840938818468	0.09840938818074	
64	128	0.09840941052888	0.09840938818871	0.09840938818002	0.09840938817997	0.09840938817995	0.09840938817995

Table (2)

The integration Integrand $I = \int_0^1\int_0^1\int_0^1 \frac{2xyz}{\sqrt{x^2+y^2+z^2}}\,dxdydz$ has single point in $(x,y,z) = (0,0,0)$

And the type of Radical and relative at the same time, And his analytic value (0.2157192692857)Rounded to thirteen decimal places ,and use suggested by Davias and Rabinowitz [3] andcorrection terms by Using formula $(14')$ $E_{sim^3} = A_1h_1^4 + a_1h_1^5 + A_2h_1^6 + A_3h_1^8 + \ldots$ where $i = 1,2,3,\ldots$, A_i, a_1 Constants ,We conclude from the table(3) The value of integration using the simpson's rule with Romberg accelerating that value equal to the analytical valuewhen (subintervals $n = 64, r = 64, m = 128$)(2^{19} subintervals).

n=r	m	Simpson value	K=4	K=5	K=6	K=8	K=10
2	4	0.2176470369505					
4	8	0.2158859756477	0.2157685715608				
8	16	0.2157301928727	0.2157198073543	0.2157182343154			
16	32	0.2157199567951	0.2157192743899	0.2157192571975	0.2157192734337		
32	64	0.2157193122553	0.2157192692859	0.2157192691213	0.2157192693106	0.2157192692944	
64	128	0.2157192719690	0.2157192692833	0.2157192692832	0.2157192692858	0.2157192692857	0.2157192692857

Table (3)

4. The discussion

It is clear according to tables of results apove that when we calculate the approximate value for the improper triple integralswhen Continuous Integrands With Singular Partial Derivatives and Singular Integrandsby the composite simpson's rule with Romberg accelerating method we can get a best results with respect to convergence to value of integration with a few number from subintervals Thus, we can depend on it'smethod in a calculation This type of the triple integrals.

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