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RESEARCH ARTICLE

A FINANCIAL MATHEMATICAL MODEL TO CALCULATE A SAVING SCHEME THROUGH ARITHMETIC AND GEOMETRIC GRADIENT

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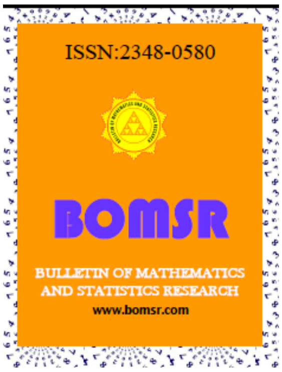


ABSTRACT

The aim of this paper is to show through a financial mathematical model, how to develop a permanent savings scheme. To do this, we use the theorems of financial mathematics relating to annuities that run with gradients: arithmetic and geometric. The result allows us to observe how we carry out the savings scheme and, most importantly, the increase reflected in each of the deposits is applied in a way of arithmetic or geometric gradient. Thus, it allows to have more savings over time, i.e. increase the capital (principal + interest) generating with it, higher dividends.

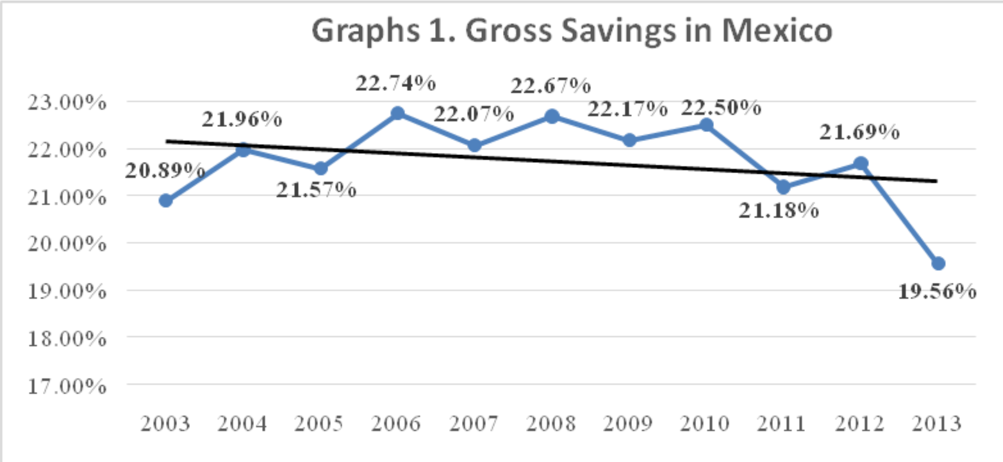
2000 AMS Subject classification: 62P, 62P05, 97M, 97M30.

KEYWORDS: gradient, arithmetic, geometric, saving, investment, interest rate

1. BACKGROUND

In the past fifty years in Latin America, Mexico had reported apositive trend in matters of savings. However, in recent years the indicators show that the trend today is decreasing.According to data from the World Bank (2014, 2015), saving trends in Mexico have kept in decline since 2013 as we can see in Graph 1.

According to Schmidt-Hebbel, Webb and Corsetti [6], households are saving a substantial partinthe developed countries and developing countries. They concluded that, in developing countries, savings in households are regulated mainly by the income and wealth, and current fiscal policies. However, Ogaki, Ostry and Reinhart [5] suggest that the stage when the country is in development, is crucial to make a difference in the behavior of investors, and also the rates of return depend on the classification of income that the country has.Besides, as Baxter and Crucini[1] have noted "National saving and investment rates are highly positively correlated in virtually all countries. It is puzzling, as it apparently implies a low degree of international capital mobility".



Source: The World Bank (2014)

Mexico, for instance, is listed as a country of the medium-high socioeconomic level (World Bank, 2015). A Financial Group study revealed that only 7.2% of the population saved in the year 2013 with the purpose to confront an economiccontingency, or for retirement (Forbes, 2013).

With this consideration, now the problem arises in the development of the hypothetical case

1.1. Statement of problem

It is thought that the main problem of Mexican youth, is the lack of adequate financial education, according to Moreno, García-Santillán and Managua[4]. Also, little knowledge of investment instruments, which serve to grow (increase) of their income in the short and long term.Added to this, it is also important to consider as main factors: socioeconomic status and current fiscal policies.

Regarding information published in Forbes Mexico (2013) it is necessary to generate knowledge and confidence in the use of financial products among active consumers corresponding to 40% of the unbanked population of the country.

With these arguments, we may observe that the lack of savings may be associatedwith the lack of knowledge in this field. Also, the percentage of the population that does not use financial services is very high. For that reason in this document, it is shown, from a financial mathematical model,how may be developing a savings scheme in two scenarios: with arithmetic gradients or geometric gradients.

1.1.1. Development of the case

To develop the model, firstly the arithmetic formula post-payment gradient is used with a fixed interest rate monthly, and subsequently, the model with a post-payment geometric gradient is developed similarly, with a fixed monthly rate.We remember that ordinary interest is calculated on the basis of a 360-day year or a 30-day month; exact interest is calculated on a 365-day year. The interest formulas for both ordinary and exact interest are the same, with atime slightly differing when given as number of days.

Scenario a) Gradient Arithmetic Post payment

The data to calculate both cases are:

Overdue annuity (post-payment)		
Ag =	\$1,000.00	Arithmetic gradient
Gg=	5.5%	Geometric gradient
n =	24	Both cases
i_r=	1.50% monthly	Both cases
Rp1 =	\$10,000.00	Both cases

Where:

Mag= Amount accumulated with arithmetic gradient

n= time

Ag= arithmetic gradient

i_r= interest rate

$Rp_{1=}$ the first deposit (periodical deposit)

The formula that we use:

$$Mag = \left[R_{p1} + \frac{Ag}{i_r} \right] \left[\frac{\left(1+i_r\right)^n - 1}{i_r} \right] - \left(\frac{n * Ag}{i_r} \right) \quad (1)$$

Therefore, we have:

$$Mag = \left[\$10,000.00 + \frac{\$1,000.00}{0.015} \right] \left[\frac{\left(1+0.015\right)^{24} - 1}{0.015} \right] - \left(\frac{24 * \$1,000.00}{0.015} \right) \quad (1.1)$$

$$Mag = \left[\$10,000.00 + \$66,666.67 \right] \left[\frac{\left(1.015\right)^{24} - 1}{0.015} \right] - \left(\frac{\$24,000.00}{0.015} \right) \quad (1.2)$$

$$Mag = \left[\$76,666.67 \right] \left[\frac{1.4295028 - 1}{0.015} \right] - \left(\$1'600,000.00 \right) \quad (1.3)$$

$$Mag = \left[\$76,666.67 \right] \left[\frac{0.4295028}{0.015} \right] - \$1'600,000.00 \quad (1.4)$$

$$Mag = \left[\$76,666.67 \right] \left[28.63352 \right] - \$1'600,000.00 \quad (1.5)$$

$$Mag = \$2'195,236.63 - \$1'600,000.00 \quad (1.6)$$

$$Mag = \$595,236.63 \quad (1.7)$$

Now we will check it with an amortization table:

Table 1. Saving found (annuityoverdue)

Deposit number	Annuity	Interest	Balance
1	$10,000.00	0.00	$10,000.00
2	$11,000.00	$150.00	$21,150.00
3	$12,000.00	$317.25	$33,467.25
4	$13,000.00	$502.01	$46,969.26
5	$14,000.00	$704.54	$61,673.80
6	$15,000.00	$925.11	$77,598.90
7	$16,000.00	$1,163.98	$94,762.89
8	$17,000.00	$1,421.44	$113,184.33
9	$18,000.00	$1,697.76	$132,882.10
10	$19,000.00	$1,993.23	$153,875.33
11	$20,000.00	$2,308.13	$176,183.46

12	$21,000.00	$2,642.75	$199,826.21
13	$22,000.00	$2,997.39	$224,823.60
14	$23,000.00	$3,372.35	$251,195.96
15	$24,000.00	$3,767.94	$278,963.90
16	$25,000.00	$4,184.46	$308,148.35
17	$26,000.00	$4,622.23	$338,770.58
18	$27,000.00	$5,081.56	$370,852.14
19	$28,000.00	$5,562.78	$404,414.92
20	$29,000.00	$6,066.22	$439,481.14
21	$30,000.00	$6,592.22	$476,073.36
22	$31,000.00	$7,141.10	$514,214.46
23	$32,000.00	$7,713.22	$553,927.68
24	$33,000.00	$8,308.92	$595,236.59

Source: own

Note: The differences are due to rounding.

Scenario b) Gradient Geometric Post-payment

The formula that we use:

$$MGg = R_{p1}\left[\frac{(1+i_r)^n-(1+Gg)^n}{i_r-Gg}\right] \tag{2}$$

Therefore, we have:

$$MGg = \$10,000.00\left[\frac{(1+0.015)^{24}-(1+0.055)^{24}}{0.015-0.055}\right] \tag{2.1}$$

$$MGg = \$10,000.00\left[\frac{(1.015)^{24}-(1.055)^{24}}{0.015-0.055}\right] MGg = \$10,000.00\left[\frac{1.4295028-3.6145899}{0.015-0.055}\right] \tag{2.2}$$

$$MGg = \$10,000.00\left[\frac{-2.1850871}{-0.04}\right] MGg = \$10,000.00(54.6271775) \tag{2.3}$$

$$MGg = \$546,271.78 \tag{2.4}$$

Now we will check it with an amortization table:

Table 2. Saving found (annuity overdue)

Deposit number	Annuity	Interest	Balance
1	$10,000.00		$10,000.00
2	$10,550.00	$150.00	$20,700.00
3	$11,130.25	$310.50	$32,140.75
4	$11,742.41	$482.11	$44,365.28
5	$12,388.25	$665.48	$57,419.00
6	$13,069.60	$861.29	$71,349.89
7	$13,788.43	$1,070.25	$86,208.56
8	$14,546.79	$1,293.13	$102,048.48
9	$15,346.87	$1,530.73	$118,926.07
10	$16,190.94	$1,783.89	$136,900.91
11	$17,081.44	$2,053.51	$156,035.87

12	$18,020.92	$2,340.54	$176,397.33
13	$19,012.07	$2,645.96	$198,055.36
14	$20,057.74	$2,970.83	$221,083.93
15	$21,160.91	$3,316.26	$245,561.11
16	$22,324.76	$3,683.42	$271,569.29
17	$23,552.63	$4,073.54	$299,195.45
18	$24,848.02	$4,487.93	$328,531.41
19	$26,214.66	$4,927.97	$359,674.04
20	$27,656.47	$5,395.11	$392,725.62
21	$29,177.57	$5,890.88	$427,794.08
22	$30,782.34	$6,416.91	$464,993.33
23	$32,475.37	$6,974.90	$504,443.60
24	$34,261.52	$7,566.65	$546,271.77

Source: own

Note: The differences are due to rounding.

DISCUSSION

The development of both models gradient leads to the following assumptions: firstly in both cases the initial payment or first instalment is similar ($10,000.00 dls.), as well as the interest rate that will generate dividends or yields (1.5% monthly) and shall be valid, during the time in which this fund savings shall be constituted (24 months).

In the case of arithmetic gradient a gradual increase of $ 1,000.00 from the second installment and so on until the end, was established. In the case of geometric gradient a gradual percentage increase of 5.5% from the second fee or deposit, it was established as well.

At the beginning, we may think that the arithmetic gradient model is the best choice versus geometric gradient, because as we can see in Table 3, in each of the months increased annuity is higher in the arithmetic gradient versus geometric gradient.Apparently, it is, or at least until the annuity number 23 where we can see that in the first case (*Ga)*the annuity is $32,000.00 while on the other scenario (*Gg*) reached the amount of $32,475.37 and hereinafter, in this format (*Gg*) the amount of the annuity versus the arithmetic gradient, is exceeded.

In both cases, we can observe a practical usefulness for the investor. Probably some savers prefer to set an amount corresponding to the increase made in each one of annuities, while other investors prefer to establish it as a percentage.

In both cases, it is recommended that, in addition to having practical usefulness both formats, these must be aligned to the theoretical assumption that says: greater deposit with higher capitalization, higher returns are obtained. It is based on the effect of compound interest which states García Santillán [2, 3]

Table 3. Comparative investment found

Arithmetic Gradient (annuity overdue)				Geometric Gradient (annuity overdue)			
Deposit number	Annuity	Interest	Balance	Deposit number	Annuity	Interest	Balance
1	$10,000.00	0	$10,000.00	1	$10,000.00		$10,000.00
2	$11,000.00	$150.00	$21,150.00	2	$10,550.00	$150.00	$20,700.00
3	$12,000.00	$317.25	$33,467.25	3	$11,130.25	$310.50	$32,140.75
4	$13,000.00	$502.01	$46,969.26	4	$11,742.41	$482.11	$44,365.28
5	$14,000.00	$704.54	$61,673.80	5	$12,388.25	$665.48	$57,419.00

6	$15,000.00	$925.11	$77,598.90	6	$13,069.60	$861.29	$71,349.89
7	$16,000.00	$1,163.98	$94,762.89	7	$13,788.43	$1,070.25	$86,208.56
8	$17,000.00	$1,421.44	$113,184.33	8	$14,546.79	$1,293.13	$102,048.48
9	$18,000.00	$1,697.76	$132,882.10	9	$15,346.87	$1,530.73	$118,926.07
10	$19,000.00	$1,993.23	$153,875.33	10	$16,190.94	$1,783.89	$136,900.91
11	$20,000.00	$2,308.13	$176,183.46	11	$17,081.44	$2,053.51	$156,035.87
12	$21,000.00	$2,642.75	$199,826.21	12	$18,020.92	$2,340.54	$176,397.33
13	$22,000.00	$2,997.39	$224,823.60	13	$19,012.07	$2,645.96	$198,055.36
14	$23,000.00	$3,372.35	$251,195.96	14	$20,057.74	$2,970.83	$221,083.93
15	$24,000.00	$3,767.94	$278,963.90	15	$21,160.91	$3,316.26	$245,561.11
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20	$29,000.00	$6,066.22	$439,481.14	20	$27,656.47	$5,395.11	$392,725.62
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23	$32,000.00	$7,713.22	$553,927.68	23	$32,475.37	$6,974.90	$504,443.60
24	$33,000.00	$8,308.92	$595,236.59	24	$34,261.52	$7,566.65	$546,271.77

Source: own

CONCLUSIONS

The development of this exercise contributes to provide the type of demonstration and explanation required to move gradually society from a culture of saving to the possibility of investing. In Mexico, 61% of the adult population have a savings account, but only 2% have an investment account. To generate the education and confidence needed to increase savings culture in Mexico's people is stepping forward to the possibility that those savings will also become a smart investment.

Gradients are financial transactions in which it is agreed to cover the obligation on a series of increasing or decreasing periodic payments with certain conditions. The result of this exercise allowsto appreciate that the difference between a scenario with arithmetic gradient and one with geometric gradient is minimal. However, in the first scenario the monthly payment obtained is greater than the second one.

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