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RESEARCH ARTICLE

STOCHASTIC MODEL TO FIND THE EFFECT OF GLUCOSE-DEPENDENT INSULINOTROPIC HORMONE IN CHOLECYSTECTOMIZED PATIENTS USING BOUNDARY CONDITION IN HAMILTON JACOBI-BELLMANN EQUATION

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ABSTRACT

The purpose of the Study was to evaluate the effect gut hormones with focus on Glucose-dependent Insulinotropic Hormone (GIP) in ten cholecystectomized subjects and ten healthy humans. The Subjects were given a standardized fat-rich liquid meal (2,200kJ). Basal and postprandial plasma concentrations of GIP were measured. The concentration of GIP was similar in the two groups. In this paper, the problem is investigated by considering the boundary condition of Hamilton Jacobi-Bellmann equation.

Key Words: GIP, Incretin hormone, Cholecystectomized Patients, HJB Equation & Ornstein-Uhlenbeck Process.

1. INTRODUCTION

The Glucose-dependent insulinotropic polypeptides are the main incretin hormones and act in concert to generate the so-called incretin effect [4]. The incretin effect has been defined augmentations of insulin secretion after oral glucose compared with insulin sectetion after isoglycemic intravenous glucose. The incretin effect accounts for up to 70% of the insulin secteter after ingestion of glucose, depending on the amount of glucose. The flow of bile acids into the intestine, subsequent to gall bladder emptying, has been suggested to play an important role in the regulation of postprandial glucose homeostasis as well as overall human metabolism [6] & [10]. The unexplained high prevalence of gluco-metabolic disturbances in cholecystectomized patients [3]. 10 cholecystectomized subjects and 10 healthy control subjects were included in the study. The

subjects were without diabetes and had normal glucose tolerance according to 75-g oral glucose tolerance test. The plasma concentration of GIP was measured by radioimmunoassay [5]. The subjects were given a standardized fat-rich liquid meal (2,200kJ). The plasma concentration of total GIP is shown in figure (1). The basal plasma concentrations were observed in the two groups. It was found that there were no differences in basal values or postprandial responses in cholecystectomized subjects compared with control subjects. Cholecystectomy did not have a major impact on gastro intestinal hormones.

In this paper the problem is investigated by using the boundary condition of Halmilton-Jacobi-Bellman equation [11]. The continuous-time portfolio optimization problem in Kim and Omberg [12]. The sufficient conditions to verify that a solution derived from the Hamilton-Jacobi-Bellman equation are in fact an optimal solution to the portfolio selection problem. Many studies have been done on continuous-time portfolio optimization problem with the Merton's seminal work [1], [7] & [9]. In particular, there has been increasing interest in finding an optiomal portfolio strategy when investment opportunities are stochastic, because many empirical works conclude that investment opportunities are time-varying. There are two main approaches in solving continuous-time portfolio optimization problem. One is the stochastic control approach and the other is the martingale approach. In the stochastic control approach, an optimal solution is conjectured by guessing a solution to the HJB equation. It is necessary to verify that the conjectured solution is in fact solution to the original problem. Korn and kraft [8] pointed out, the verification is often skipped since it is mathematically demanding for kim and omberg examined the finiteness of conjectured value function very carefully, but they could not provide verification conditions. The sufficient condition to verify that the conjectured solution is in fact the solution to the original problem.

2. Notations:

B	Brownian motion
Y	State process
σ	Stochastic process
Z	Value function
δ	Relative risk aversion coefficient

3. Stochastic Model:

Let (ψ, D, P) be a complete probability space on which we define a two-dimensional standard Brownian motion $B = (B^1, B^2)^T$ and we also fix a time interval $[0,T]$. Let $D(t)$ be the augmentation of the filtration $D^B(t) := \omega(B(a); 0 < a < t)$, $0 < t < T$.

Let Y be an Ornstein-Uhlenbeck process:

$$dY(t) = \alpha(\bar{Y} - Y(t))dt + \omega_Y\left(\beta dB^1(t) + \sqrt{1-\beta^2}\,dB^2(t)\right) \qquad (1)$$

$$Y(0) = y_0 \in \mathbb{R}.$$

$\beta \in [-1,1], \alpha > 0, \omega_Y > 0, and \bar{Y} \in \mathbb{R}.$ We call Y a state process, because it determines an investment opportunity set in our portfolio problem. There is one riskless asset and one risky asset. Suppose the price A_0 of the riskless asset satisfies $dA_0(t) = qA_0(t)dt, A_0(0) = 1,$

Where $q \geq 0$ is constant. The risky asset price A satisfies the stochastic differential equation

$$dA(t) = A(t)\gamma(Y(t))dt + A(t)\omega dB^1(t), A(0) = a > 0, \qquad (2)$$

Where $\gamma: \mathbb{R} \to \mathbb{R}$ satisfies $\gamma(y)$-q)/ω=y *for* $y \in \mathbb{R}$ Then (2) can be written by

$$dA(t) = A(t)(q + \omega Y(t))dt + A(t)\omega dB^1(t).$$

We consider the division between the riskless asset and the risky assets. Let $\mathbb{N}^2(t_0,t_1)$ be a set of $\mathbb{Z}(t)$-progressively measurable processes $\sigma:\psi x[t_0,t_1]\rightarrow \mathbb{R}$ such that

$$P\left(\int_{t_0}^{t_1}\sigma(t)^2\,dt<\infty\right)=1 \tag{3}$$

We call an element of $\mathbb{N}^2(t_0,t_1)$ a portfolio strategy. We regard $\sigma_i(t)$ as a fraction of the wealth invested in the risky asset at time t. The wealth process U^σ corresponding to $\sigma\in\mathbb{N}^2(0,T)$ is given by

$$U^\sigma(0)=u_0>0 \text{ and}$$

$$\begin{aligned}dU(t)&=U(t)\left[\sigma(t)(\gamma(Y(t))-q)+q\right]dt+U(t)\sigma(t)\omega dB^1(t)\\&=U(t)\left[\sigma(t)\omega Y(t)+q\right]dt+U(t)\sigma(t)\omega dB^1(t).\end{aligned} \tag{4}$$

There is incompleteness in the sense that there are some random processes that are not replicated by the self-financing portfolio strategy σ. The investor maximizes the expected utility of his wealth at terminal date T. We assume that the investor has a power utility function with a relative risk aversion coefficient δ:

$$\max_{\sigma\in Q_\delta(0,T)} E\left[\frac{U^\sigma(T)^{1-\delta}}{1-\delta}\right]. \tag{5}$$

Here Q_δ denotes the set of admissible portfolio strategies defined as follows. A Stochastic process σ is said to be an admissible portfolio strategy on $[t_0,t_1]$ if

(a) $\sigma\in\mathbb{N}^2(t_0,t_1)$, when $0<\delta<1$

(b) For some function $\tilde{\sigma}:[0,T]\times\mathbb{R}\rightarrow\mathbb{R}$ satisfying the linear growth condition,
$\sigma(t)=\tilde{\sigma}(t,Y(t))\,on\left[t_0,t_1\right],\,when\,\delta>1.$

The set of all admissible strategies on $[t_0,t_1]$ is denoted by $Q_\delta[t_0,t_1]$. The choice of our set of portfolio strategies seems to be restrictive.

Because of incompleteness there is no unique equivalent martingale measure, and we cannot apply the so-called martingale approach directly. It is thus common to apply the dynamic programming approach using Hamilton-Jacobi-Bellman equation. Let

$$K(t,u,y;\sigma)=E^{t,u,y}\left[\frac{U^\sigma(T)^{1-\delta}}{1-\delta}\right],$$

Here and in the sequel, we use the notation $E^{t,u,y}[.]=E[.|U(t)=u,Y(t)=y]$.

Let $S=[0,T)\times(0,\infty)\times\mathbb{R}$. We then define $\zeta:S\rightarrow\mathbb{R}$ by

$\zeta(t,u,y)=\sup_{\sigma\in Q_\delta(t,T)} K(t,u,y;\sigma)$.

The function ζ is called a value function. The Hamilton-Jacobi-Bellman equation related to the problem (5) is $\sup_{\sigma\in\square} C^\sigma H(t,u,y)=0$ (6)

With the boundary condition $H(T,u,y)=\frac{u^{1-\delta}}{1-\delta}$, (7)

Where $C^\sigma H(t,u,y)=H_t+u(\sigma\omega y+q)H_u+\alpha(\bar{Y}-y)H_y$

$$+\frac{1}{2}u^2\sigma^2\omega^2H_{uu}+\frac{1}{2}\omega^2{}_YH_{yy}+\omega_Yu\sigma\omega\beta H_{uy}.$$

It is well-known from Kim and Omberg and others that the function H is separable and has the following form: $H(t,u,y)=\frac{u^{1-\delta}}{1-\delta}g(t,y),$ (8)

Where $g(t,y)=\exp\left\{p(t)+s(t)y+\frac{1}{2}r(t)y^2\right\}$

With the boundary conditions $p(T)=s(T)=r(T)=0.$

It follows from the first order condition for (6) that the candidate optimal portfolio strategy is given by $\sigma^*(t)=\frac{1}{\delta}\frac{Y(t)}{\omega}+\frac{1}{\delta}\frac{\beta\omega_Y}{\omega}(s(t)+r(t)Y(t)).$ (9)

Substituting this conjectured solution into the Hamilton-Jacobi-Bellman equation, we obtain the differential equation for $p(.), s(.), \text{ and } r(.)$ as follows:

$$\dot{r}(t)=-\omega^2{}_Y\left(\frac{1-\delta}{\delta}\beta^2+1\right)r(t)^2-2\left(\frac{1-\delta}{\delta}\omega_Y\beta-\alpha\right)r(t)-\frac{1-\delta}{\delta} \quad (10)$$

$$\dot{s}(t)=-\omega^2{}_Y\left(\frac{1-\delta}{\delta}\beta^2+1\right)s(t)r(t)-\left(\frac{1-\delta}{\delta}\omega_Y\beta-\alpha\right)s(t)-\alpha\bar{Y}r(t) \quad (11)$$

$$\dot{p}(t)=-\frac{1}{2}\omega^2{}_Y\left(\frac{1-\delta}{\delta}\beta^2+1\right)s(t)^2-\frac{1}{2}\omega^2{}_Y r(t)-\alpha\bar{Y}s(t)-(1-\delta)q \quad (12)$$

4. Example

Ten cholecystectomized subjects and 10 healthy control subjects were included in the study. The subjects were without diabetes and had normal glucose tolerance according to 75-g oral glucose tolerance test. The plasma concentration of GIP was measured by radioimmunoassay. The subjects were given a standardized fat-rich liquid meal (2,200kJ). The plasma concentration of total GIP is shown in figure (1). The basal plasma concentrations were observed in the two groups. It was found that there was no difference in basal values or postprandial responses in cholecystectomized subjects compared with control subjects. Cholecystectomy did not have a major impact on gastro intestinal hormones [2].

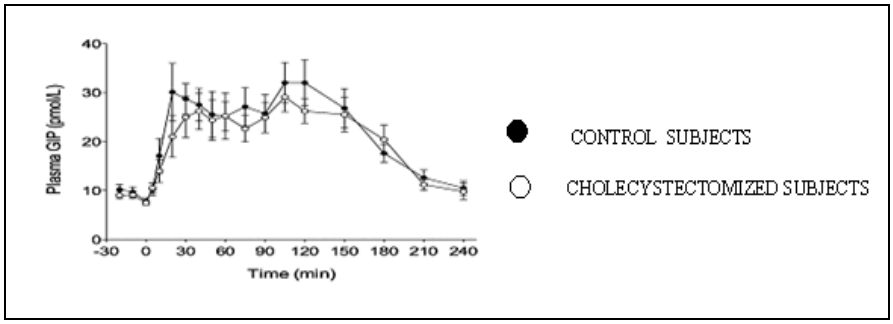

Figure: (1)

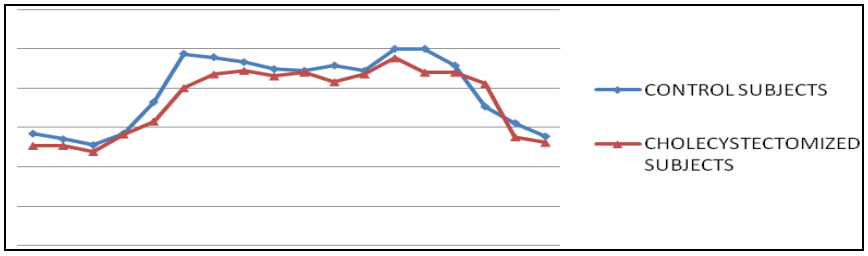

Figure: (2)

5. Conclusion

Evaluations of the effect gut hormones with focus on Glucose-dependent Insulinotropic Hormone (GIP) in ten cholecystectomized subjects and ten healthy humans. The concentration of GIP was similar in the two groups. In this paper, the problem is investigated by considering the boundary condition of Hamilton Jacobi-Bellmann equation. The result coincides with the mathematical and medical report.

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