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RESEARCH ARTICLE

A CLASS OF COMBINED ESTIMATORS FOR ESTIMATING POPULATION MEAN USING AUXILIARY VARIABLE WITH DOUBLE SAMPLING IN PRESENCE OF NON-RESPONSE

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ABSTRACT

In this article, we propose a class of combined estimators for estimating the population mean of the study variable under double sampling in presence of non-response. The variances of the proposed class of estimators are obtained to first degree of approximation. Numerically, the proposed estimators are compared with some competitor estimators. It is identified through numerical study that the proposed estimators are more efficient as compared to all other competitor estimators.

Keywords: Auxiliary and study Variable, Double Sampling bias and MSE, Non-response in sampling

1. INTRODUCTION

In sample survey for estimating the population parameter under different situations or different schemes are most important part and different author may leads to different estimators. Auxiliary information and off-course it is related to the study variable gives us an efficient estimate of population parameters like as population mean, total and variance, under some crucial conditions. This information may be used for drawing a random sample using SRSWOR / SRSWR, to stratification, systematic or probability proportional to size sampling strategy or for estimating the population parameter or at both purposes. Auxiliary information gives us a variety of techniques by means of ratio, product, regression and other methods. Dalenius (1934) states that "As demonstrated by the developments in the last half-century, supplementary information may be exploited for all aspects of the sample design, the definition of sampling units, the selection design and the estimation method." In real life situation almost all surveys suffer from non-response. For this scheme Hansen and Hurwitz (1946) suggesting the first attempt by mail questionnaire and the second attempt by a personal interview with assumed that a sub-sample of initial non-respondents is recontacted.Other authors such as Chand (1975), Cochran (1977), Rao (1986, 1987, 1990), Khare and Srivastava (1993, 1995, 1996, 1997), Sodipo and Obisesan (2007), Khare and Kumar (2009, 2010), Khare and Rehman (2014), Okafor and Lee (2000) and Tabasum and Khan (2004, 2006), Singh and Kumar (2008a,b), Hazra (2015) and many more authors have studied the problem of non-response under double (two-phase) sampling.

2. SAMPLING SCHEME

The double sampling in presence of non-response sampling scheme is that; let a population have N units and divided N_r of responding units and N_{nr} of non-responding units. Now we draw a sample of size n_1(<N) from population of size N by using simple random sampling without replacement (SRSWOR) scheme. Further we select a smaller second phase sample of size n_2 is selected from n_1 by SRSWOR scheme and observe that Non-response occurs on the second phase sample of size n_2 in which n_r units respond and n_{nr} units do not. From the n_{nr} non-respondents by SRSWOR a sample of $r = n_{nr}/k$; $k > 1$ units is selected where k is the inverse sampling rate at the second phase sample of size n_2. All the r units respond at this time round.

3. NOTATION AND TERMINOLOGY

In this article we used different notations and terminology, they are described as follows: $(\bar{y}_r, \bar{y}_{nr})$ be the sample mean based on n_r and n_{nr} units of study variable and $(\bar{x}_1, \bar{x}_2, \bar{x}_r, \bar{x}_{nr})$ be the sample mean of auxiliary variable based on n_1, n_2, n_r, n_{nr} units respectively. $\bar{Y}$ and $\bar{X}$ be the population mean of study variable and auxiliary variable based on population size $N=N_r+N_{nr}$. Also $\bar{Y}_{nr}$, $\bar{X}_{nr}$ be the population mean of study variable and auxiliary variable based on population size N_{nr} (non-response part).

$$S_{y(w)}^2 = \sum_{i=1}^{N} (y_i - \bar{Y})^2 \Big/ (N-1),\ S_{x(w)}^2 = \sum_{i=1}^{N} (x_i - \bar{X})^2 \Big/ (N-1),\ S_{y(nr)}^2 = \sum_{i=1}^{N_{nr}} (y_i - \bar{Y}_{nr})^2 \Big/ (N_{nr}-1),$$

$$S_{x(nr)}^2 = \sum_{i=1}^{N_{nr}} (x_i - \bar{X}_{nr})^2 \Big/ (N_{nr}-1),\ C_{y(w)}^2 = S_{y(w)}^2/\bar{Y}^2,\quad C_{y(nr)}^2 = S_{y(nr)}^2/\bar{Y}^2,\quad C_{x(w)}^2 = S_{x(w)}^2/\bar{X}^2,$$

$$C_{x(nr)}^2 = S_{x(nr)}^2/\bar{X}^2,\ S_{yx(W)} = \frac{\sum_{i=1}^{N}(y_i - \bar{Y})(x_i - \bar{X})}{N-1},\ S_{yx(nr)} = \frac{\sum_{i=1}^{N_{nr}}(y_i - \bar{Y})(x_i - \bar{X})}{N_{nr}-1},\ \rho_{yx(w)} = \frac{S_{yx(w)}}{S_{y(w)}S_{x(w)}},$$

$$\rho_{yx(nr)} = \frac{S_{yx(nr)}}{S_{y(nr)}S_{x(nr)}},\ \beta_{yx(w)} = \frac{S_{yx(w)}}{S_{x(w)}^2},\ \beta_{yx(nr)} = \frac{S_{yx(nr)}}{S_{x(nr)}^2},\ K_{yx(w)} = \frac{\rho_{yx(w)}C_{y(w)}}{C_{x(w)}},\ K_{yx(nr)} = \frac{\rho_{yx(nr)}C_{y(nr)}}{C_{x(nr)}},$$

$$C_{yx(w)} = \frac{S_{yx(W)}}{\bar{X}\bar{Y}},\quad C_{yx(nr)} = \frac{S_{yx(nr)}}{\bar{X}\bar{Y}}$$

$\rho_{yx(r)}$ and $\rho_{yx(nr)}$ are respectively the correlation coefficient of response and non-response group between study variable y and auxiliary variable x. $w_r = n_r/n_2$, $w_{nr} = n_{nr}/n_2$, $f = n_2/N$, $W_{nr}=N_{nr}/N$, $\lambda = (1-f)/n_2$, $\lambda' = (1-f_1)/n_1$, $\boldsymbol{\lambda}^* = \frac{W_{nr}(k-1)}{n_2}$, $f_1 = n_1/N$.

4. PROPOSED ESTIMATOR

An unbiased estimator for the population mean $\bar{Y}$ of y, proposed by Hansen and Hurwitz (1946), is defined by

$\bar{\boldsymbol{y}}^* = \boldsymbol{w_r}\bar{\boldsymbol{y}}_{\boldsymbol{r}} + \boldsymbol{w_{nr}}\bar{\boldsymbol{y}}_{\boldsymbol{nr}}$

Let x denote an auxiliary variable with population mean $\bar{X}$. Then the Hansen and Hurwitz (1946) estimator

$\bar{\boldsymbol{x}}^* = \boldsymbol{w_r}\bar{\boldsymbol{x}}_{\boldsymbol{r}} + \boldsymbol{w_{nr}}\bar{\boldsymbol{x}}_{\boldsymbol{nr}}$

In the given sampling scheme we have propose a class of combined estimator for estimating population mean using auxiliary variables with double sampling in the presence of non-response based on Hansen and Hurwitz estimator is given below

$$\boldsymbol{T_P} = \bar{\boldsymbol{y}}^* \left\{ \left(\frac{\bar{\boldsymbol{x}}_1}{\bar{\boldsymbol{x}}^*}\right)^{\boldsymbol{\alpha}} + \left(\frac{\bar{\boldsymbol{x}}_1}{\bar{\boldsymbol{x}}_2}\right)^{\boldsymbol{\beta}} - \mathbf{1} \right\}$$

where α and β are constant.

Define

$$\bar{y}^* = \bar{Y}(1 + \Delta_0), \bar{x}^* = \bar{X}(1 + \Delta_{00}), \bar{x}_1 = \bar{X}(1 + \Delta_1), \bar{x}_2 = \bar{X}(1 + \Delta_2)$$

such that, $E(\Delta_0) = E(\Delta_{00}) = E(\Delta_1) = E(\Delta_2) = 0$ and

$$E(\Delta_0{}^2) = \lambda C^2_{y(w)} + \lambda^* C^2_{y(nr)}, E(\Delta_{00}{}^2) = \lambda C^2_{x(w)} + \lambda^* C^2_{x(nr)}, E(\Delta_1{}^2) = \lambda' C^2_{x(w)}, E(\Delta_2{}^2) = \lambda C^2_{x(w)},$$

$$E(\Delta_0\Delta_1) = \lambda' \rho_{yx(w)} C_{y(w)} C_{x(w)}, E(\Delta_0\Delta_{00}) = \lambda \rho_{yx(w)} C_{y(w)} C_{x(w)} + \lambda^* \rho_{yx(nr)} C_{y(nr)} C_{x(nr)},$$

$$E(\Delta_0\Delta_2) = \lambda \rho_{yx(w)} C_{y(w)} C_{x(w)}, E(\Delta_{00}\Delta_1) = \lambda' C^2_{x(w)}, E(\Delta_{00}\Delta_2) = \lambda C^2_{x(w)}, E(\Delta_2\Delta_1) = \lambda' C^2_{x(w)}$$

The T_p can be written in terms of Δ's in following manner

$$T_P = \bar{Y}(1 + \Delta_0)\left\{\left(\frac{1+\Delta_1}{1+\Delta_{00}}\right)^{\alpha} + \left(\frac{1+\Delta_1}{1+\Delta_2}\right)^{\beta} - 1\right\}$$

$$T_P = \bar{Y}(1 + \Delta_0)\left\{(1 + \Delta_1)^{\alpha}(1 + \Delta_{00})^{-\alpha} + (1 + \Delta_1)^{\beta}(1 + \Delta_2)^{-\beta} - 1\right\}$$

Here we assume that $|\Delta_{00}| < 1$, $|\Delta_2| < 1$ and approximate upto second degree of Δ's to the right hand side of the above form, we get

$$T_P = \bar{Y}(1 + \Delta_0)\left\{\begin{matrix}\left(1 + \alpha\Delta_1 + \frac{\alpha(\alpha-1)}{2}\Delta_1{}^2\right)\left(1 - \alpha\Delta_{00} - \frac{\alpha(\alpha+1)}{2}\Delta_{00}{}^2\right) + \left(1 + \beta\Delta_1 + \frac{\beta(\beta-1)}{2}\Delta_1{}^2\right) \\ \left(1 - \beta\Delta_2 - \frac{\beta(\beta+1)}{2}\Delta_2{}^2\right) - 1\end{matrix}\right\}$$

$$T_P = \bar{Y}(1 + \Delta_0)\left\{\begin{matrix}\left(1 + \alpha\Delta_1 - \alpha\Delta_{00} - \alpha^2\Delta_1\Delta_{00} + \frac{\alpha(\alpha-1)}{2}\Delta_1{}^2 - \frac{\alpha(\alpha+1)}{2}\Delta_{00}{}^2\right) + \\ \left(1 + \beta\Delta_1 - \beta\Delta_2 - \beta^2\Delta_1\Delta_2 + \frac{\beta(\beta-1)}{2}\Delta_1{}^2 - \frac{\beta(\beta+1)}{2}\Delta_2{}^2\right) - 1\end{matrix}\right\}$$

$$T_P = \bar{Y}\left\{\begin{matrix}\left(1 + \alpha\Delta_1 - \alpha\Delta_{00} - \alpha^2\Delta_1\Delta_{00} + \frac{\alpha(\alpha-1)}{2}\Delta_1{}^2 - \frac{\alpha(\alpha+1)}{2}\Delta_{00}{}^2 + \Delta_0 + \alpha\Delta_1\Delta_0 - \alpha\Delta_{00}\Delta_0\right) + \\ \left(1 + \beta\Delta_1 - \beta\Delta_2 - \beta^2\Delta_1\Delta_2 + \frac{\beta(\beta-1)}{2}\Delta_1{}^2 - \frac{\beta(\beta+1)}{2}\Delta_2{}^2 + \Delta_0 + \beta\Delta_1\Delta_0 - \beta\Delta_2\Delta_0\right) - 1 - \Delta_0\end{matrix}\right\}$$

$$T_P = \bar{Y}\left\{\begin{matrix}1 + \Delta_0 + \alpha\Delta_1 + \beta\Delta_1 - \alpha\Delta_{00} - \beta\Delta_2 + \alpha\Delta_1\Delta_0 - \alpha\Delta_{00}\Delta_0 + \beta\Delta_1\Delta_0 - \beta\Delta_2\Delta_0 \\ -\alpha^2\Delta_1\Delta_{00} - \beta^2\Delta_1\Delta_2 + \frac{\beta(\beta-1)}{2}\Delta_1{}^2 - \frac{\beta(\beta+1)}{2}\Delta_2{}^2 + \frac{\alpha(\alpha-1)}{2}\Delta_1{}^2 - \frac{\alpha(\alpha+1)}{2}\Delta_{00}{}^2\end{matrix}\right\}$$

So

$$T_P - \bar{Y} = \bar{Y}\left\{\begin{matrix}\Delta_0 + \alpha\Delta_1 + \beta\Delta_1 - \alpha\Delta_{00} - \beta\Delta_2 + \alpha\Delta_1\Delta_0 - \alpha\Delta_{00}\Delta_0 + \beta\Delta_1\Delta_0 - \beta\Delta_2\Delta_0 \\ -\alpha^2\Delta_1\Delta_{00} - \beta^2\Delta_1\Delta_2 + \frac{\beta(\beta-1)}{2}\Delta_1{}^2 - \frac{\beta(\beta+1)}{2}\Delta_2{}^2 + \frac{\alpha(\alpha-1)}{2}\Delta_1{}^2 - \frac{\alpha(\alpha+1)}{2}\Delta_{00}{}^2\end{matrix}\right\}$$

Taking expectation both sides we get the bias of the proposed estimator and it is

$$\mathbf{Bias}(T_P) = -\bar{Y}\left\{\begin{matrix}\rho_{yx(w)} C_{y(w)} C_{x(w)}(\lambda - \lambda')(\alpha + \beta) + \lambda' C^2_{x(w)}(\alpha^2 + \beta^2) - \\ \lambda' C^2_{x(w)}\left(\frac{\alpha(\alpha-1) + \beta(\beta-1)}{2}\right) + \lambda C^2_{x(w)}\left(\frac{\alpha(\alpha+1) + \beta(\beta+1)}{2}\right) + \frac{\alpha(\alpha+1)}{2}\lambda^* C^2_{x(nr)}\end{matrix}\right\}$$

Rewrite the form as follow

$$T_P - \bar{Y} = \bar{Y}\{\Delta_0 + \alpha\Delta_1 + \beta\Delta_1 - \alpha\Delta_{00} - \beta\Delta_2\}$$

Squaring both sides of and neglecting terms of Δ's involving power greater than two, we have

$$(T_P - \bar{Y})^2 = \bar{Y}^2\{\Delta_0 + \alpha\Delta_1 + \beta\Delta_1 - \alpha\Delta_{00} - \beta\Delta_2\}^2$$

$$(T_P - \bar{Y})^2 = \bar{Y}^2\left\{\begin{matrix}\Delta_0{}^2 + \alpha^2\Delta_1{}^2 + \beta^2\Delta_1{}^2 + \alpha^2\Delta_{00}{}^2 + \beta^2\Delta_2{}^2 + 2\alpha\Delta_1\Delta_0 + 2\beta\Delta_1\Delta_0 - 2\alpha\Delta_{00}\Delta_0 - \\ 2\beta\Delta_2\Delta_0 + 2\alpha\beta\Delta_1{}^2 - 2\alpha^2\Delta_1\Delta_{00} - 2\alpha\beta\Delta_1\Delta_2 - 2\alpha\beta\Delta_1\Delta_{00} - 2\beta^2\Delta_1\Delta_2 + 2\alpha\beta\Delta_2\Delta_{00}\end{matrix}\right\}$$

$$(T_P - \bar{Y})^2 = \bar{Y}^2\left\{\begin{matrix}\Delta_0{}^2 + \alpha^2(\Delta_1{}^2 + \Delta_{00}{}^2 - 2\Delta_1\Delta_{00}) + \beta^2(\Delta_1{}^2 + \Delta_2{}^2 - 2\Delta_1\Delta_2) + 2\alpha(\Delta_1\Delta_0 - \Delta_{00}\Delta_0) + \\ 2\beta(\Delta_1\Delta_0 - \Delta_2\Delta_0) + 2\alpha\beta(\Delta_1{}^2 - \Delta_1\Delta_2 - \Delta_1\Delta_{00} + \Delta_2\Delta_{00})\end{matrix}\right\}$$

Taking expectation on both sides, we get the MSE of the estimator T_P to the first degree of approximation, we get

$$MSE(T_P) = \bar{Y}^2 \begin{bmatrix} (\lambda C_{y(w)}^2 + \lambda^* C_{y(nr)}^2) + \alpha^2 (C_{x(w)}^2 (\lambda - \lambda') + \lambda^* C_{x(nr)}^2) + \beta^2 C_{x(w)}^2 (\lambda - \lambda') \\ -2\alpha(\rho_{yx(w)} C_{y(w)} C_{x(w)} (\lambda - \lambda') + \lambda^* \rho_{yx(nr)} C_{y(nr)} C_{x(nr)}) - 2\beta \rho_{yx(w)} C_{y(w)} C_{x(w)} (\lambda - \lambda') + \\ 2\alpha\beta C_{x(w)}^2 (\lambda - \lambda') \end{bmatrix}$$

The MSE is minimized with respect to α and β i.e., we get two normal equations which are given bellow

$$\alpha(C_{x(w)}^2(\lambda - \lambda') + \lambda^* C_{x(nr)}^2) - (\rho_{yx(w)} C_{y(w)} C_{x(w)} (\lambda - \lambda') + \lambda^* \rho_{yx(nr)} C_{y(nr)} C_{x(nr)}) + \beta C_{x(w)}^2 (\lambda - \lambda') = 0$$

and $$\beta C_{x(w)}^2 (\lambda - \lambda') - \rho_{yx(w)} C_{y(w)} C_{x(w)} (\lambda - \lambda') + \alpha C_{x(w)}^2 (\lambda - \lambda') = 0$$

Hence the optimal value of α and β are

$$\alpha = \frac{\rho_{yx(nr)} C_{y(nr)}}{C_{x(nr)}}$$

and β is solved by the equation $\beta C_{x(w)} - \rho_{yx(w)} C_{y(w)} + \alpha C_{x(w)} = 0$

i.e,

$$\beta = \left(\frac{\rho_{yx(w)} C_{y(w)}}{C_{x(w)}} - \frac{\rho_{yx(nr)} C_{y(nr)}}{C_{x(nr)}} \right)$$

Table-1: Some competitor estimators with its optimal variance.

Proposal Name and Estimator	Variances
Hansen and Hurwitz(1946) $\bar{y}^* = w_r \bar{y}_r + w_{nr} \bar{y}_{nr}$	$\bar{Y}^2 (\lambda C_{y(w)}^2 + \lambda^* C_{y(nr)}^2)$
Khare and Srivastava (1993), Tabasum and Khan's (2004) $T_1 = \bar{y}^* \frac{\bar{x}_1}{\bar{x}^*}$	$\bar{Y}^2 [(\lambda - \lambda')\{C_{y(w)}^2 + (1 - 2K_{yx(w)}) C_{x(w)}^2\} + \lambda^* \{C_{y(nr)}^2 + (1 - 2K_{yx(nr)}) C_{x(nr)}^2\} + +\lambda' C_{y(w)}^2]$
Khare and Srivastava (1993), Tabasum and Khan's (2006) $T_2 = \bar{y}^* \frac{\bar{x}_1}{\bar{x}_2}$	$\bar{Y}^2 [(\lambda - \lambda')\{C_{y(w)}^2 + (1 - 2K_{yx(w)}) C_{x(w)}^2\} + \lambda^* C_{y(nr)}^2 + \lambda' C_{y(w)}^2)]$
Singh and Kumar's (2008a) $T_3 = \bar{y}^* \left(\frac{\bar{x}_1}{\bar{x}^*}\right) \left(\frac{\bar{x}_1}{\bar{x}_2}\right)$	$\bar{Y}^2 [(\lambda - \lambda')\{C_{y(w)}^2 + 4(1 - K_{yx(w)}) C_{x(w)}^2\} + \lambda^* \{C_{y(nr)}^2 + (1 - 2K_{yx(nr)}) C_{x(nr)}^2\} + \lambda' C_{y(w)}^2)]$
Singh and Kumar's (2008a) $T_4 = \bar{y}^* \left(\frac{\bar{x}^*}{\bar{x}_1}\right) \left(\frac{\bar{x}_2}{\bar{x}_1}\right)$	$\bar{Y}^2 [(\lambda - \lambda')\{C_{y(w)}^2 + 4(1 - K_{yx(w)}) C_{x(w)}^2\} + \lambda^* \{C_{y(nr)}^2 + (1 + 2K_{yx(nr)}) C_{x(nr)}^2\} + \lambda' C_{y(w)}^2)]$
Singh and Ruiz Espejo (2007) $T_5 = \bar{y}^* \left\{ b \frac{\bar{x}_1}{\bar{x}^*} + (1 - b) \frac{\bar{x}^*}{\bar{x}_1} \right\}$	$\left[\lambda' C_{y(w)}^2 + (\lambda - \lambda') \left\{ S_{y(w)}^2 + \frac{D^*}{D} \left(\frac{D^*}{D} - 2\beta_{yx(w)} \right) S_{x(w)}^2 \right\} + \lambda^* \left\{ S_{y(nr)}^2 + \frac{D^*}{D} \left(\frac{D^*}{D} - 2\beta_{yx(nr)} \right) S_{x(nr)}^2 \right\} \right]$ *Where* $D = \{\lambda' S_{x(w)}^2 + \lambda^* S_{x(nr)}^2\}$, $D^* = \{(\lambda - \lambda') K_{yx(w)} S_{x(w)}^2 + \lambda^* K_{yx(nr)} S_{x(nr)}^2\}$
Khare and Srivastava (1995) $T_6 = \bar{y}^* + b^{**}(\bar{x}_1 - \bar{x}_2)$	$Var(\bar{y}^*) - \bar{Y}^2 \left(\frac{1}{n} - \frac{1}{n'} \right) \rho_{yx(w)}^2 C_{y(w)}^2$

Khare and Rehman (2014) $T_7 = \bar{y}^* \left(\frac{\bar{x}_1}{\bar{x}^*}\right)^a + b^*(\bar{x}_1 - \bar{x}^*)$	$Var(\bar{y}^*) + \left(\frac{1}{n} - \frac{1}{n'}\right)\{\bar{Y}^2\alpha^2 C^2_{x(w)} + \bar{X}^2 B^2 C^2_{x(w)} - 2\bar{Y}^2\alpha C_{yx(w)} -$ ***2XYBCyx(w)+2XYBαCx(w)2+λ∗Y2α2Cx(nr)2+X2B2Cx(nr)2−2Y2αCyx(nr)−2XYBCyx(nr)+2XYBαCx(nr)2,*** Where $a^{opt} = \left[\begin{matrix}\left(\frac{1}{n} - \frac{1}{n'}\right)\left(\bar{Y}C_{yx(w)} - \bar{X}BC^2_{x(w)}\right) \\ +\lambda^*\left(\bar{Y}C_{yx(nr)} - \bar{X}BC^2_{x(nr)}\right)\end{matrix}\right] \Big/ \left[\begin{matrix}\left(\frac{1}{n} - \frac{1}{n'}\right)\bar{X}C^2_{x(w)} \\ +\lambda^*\bar{Y}C^2_{x(nr)}\end{matrix}\right]$ and $B = \frac{\rho_{yx(w)}S_{y(w)}}{S_{x(w)}}$
Hazra(2015) $T_8 = \bar{y}^*\left\{\alpha\frac{\bar{x}_1}{\bar{x}^*} + (1-\alpha)\frac{\bar{x}_1}{\bar{x}_2}\right\}$	$\bar{Y}^2\left[\left(\lambda C^2_{y(w)} + \lambda^* C^2_{y(nr)} - (\lambda' - \lambda)C^2_{x(w)} + 2(\lambda' - \lambda)\rho_{yx(w)}C_{y(w)}C_{x(w)}\right) - \lambda^*\rho^2_{yx(nr)}C^2_{y(nr)}\right]$

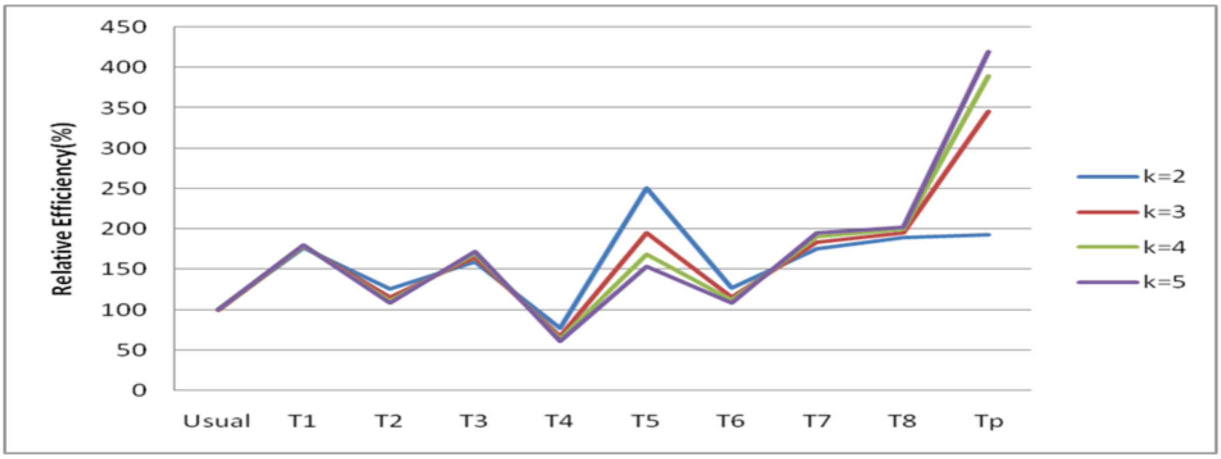
5. PRACTICAL SITUATIONS

To proving the results discuss earlier we considered the data earlier consider by Hazra (2015). The description of the data is given below: Here we consider the population of 96 village of rural area under District -Hooghly, West Bengal from the District Census Handbook 1981. The 25% villages (i.e. 24 villages) whose area is greater than 160 hectares have been considered as non-response group of the population. The number of agricultural labors in the village is taken as study information (y) while the number of cultivators in the village are taken as auxiliary information x.

Table-2: Relative efficiency of the estimators (in %) and MSE with respect to $\bar{y}^*$ for fixed values of n_2, n_1 and different values of k (N =96, n_1= 70 and n_2 =40).

MSE					Relative efficiency(%)				
Estimator	k=2	k=3	k=4	k=5	Estimator	k=2	k=3	k=4	k=5
$\bar{y}^*$	1002.0	1518.4	2034.7	2551.0	$\bar{y}^*$	100	100	100	100
T_1	565.6	849.6	1133.7	1417.7	T_1	177	179	179	180
T_2	797.9	1314.2	1830.5	2346.9	T_2	126	116	111	109
T_3	629.9	914.0	1198.0	1482.0	T_3	159	166	170	172
T_4	1305.0	2264.1	3223.2	4182.3	T_4	77	67	63	61
T_5	399.6	777.2	1208.3	1651.6	T_5	251	195	168	154
T_6	788.8	1305.1	1821.4	2337.8	T_6	127	116	112	109
T_7	568.3	824.9	1069.4	1308.7	T_7	176	184	190	195
T_8	527.3	772.9	1018.6	1264.3	T_8	190	196	200	202
T_p	518.2	438.7	521.8	606.8	T_p	193	346	390	420

Figure-1: Line diagram to showing the relative efficiency (%) with respect to other competitor estimator.



Hence we observed that the following situation was held by the different estimators

The relative efficiency (%) increase when k is increase for the estimatorsT_2, T_4, T_5, T_6.

The relative efficiency (%) decrease when k is increase for the estimatorsT_1, T_3, T_7, T_8,T_p.

So we conclude that the proposed estimator should be better with other competitor estimators for the estimating population mean using auxiliary variable with double sampling in presence of non-response.

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