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ON SOME WEAKER CLASS OF $\zeta-$CONTINUOUS MAPPINGS IN INTUITIONISTIC FUZZY TOPOLOGICAL SPACES

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ABSTRACT

In this paper we initiate the concepts of intuitionistic fuzzy almost $\zeta-$continuous mappings and intuitionistic fuzzy slightly $\zeta-$continuous mappings in intuitionistic fuzzy topological space. We also apply these notions of $\zeta-$continuous mappings to analyse the covering properties and separation axioms in intuitionistic fuzzy spaces.

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1. INTRODUCTION

Ever since the introduction of fuzzy sets by Zadeh [22], the fuzzy concept has invaded almost all branches of mathematics. Atanassov [2] generalised this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [5] introduced intuitionistic fuzzy topological spaces. Using the notion of intuitionistic fuzzy sets Joen [11] defined the concepts of intuitionistic fuzzy α-continuity. Many researchers Ilija Kovacevic [9], T.Noiri [20] have extended these notions to analyse different types of continuity. In this paper different classes of $\zeta-$ continuous functions are defined. Separation axioms and covering properties are also analysed using these $\zeta-$continuous mappings.

2. PRELIMINARIES

Definition 2.1:[2] An intuitionistic fuzzy set (IFS, in short) A in X is an object having the form $A=\{x,\mu_A(x),\upsilon_A(x)/x\in X\}$ where the functions $\mu_A: X\to I$ and $\upsilon_A: X\to I$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\upsilon_A(x)$) of each element $x \in X$ to the set A on a nonempty set X and $0 \le \mu_A(x) + \upsilon_A(x) \le 1$ for each $x \in X$. Obviously every fuzzy set A on a nonempty set X is an IFS's A and B be in the form $A = \{x, \mu_A(x), 1 - \mu_A(x) / x \in X\}$

Definition 2.2:[2] Let X be a nonempty set and the IFS's A and B be in the form $A = \{x, \mu_A(x), \upsilon_A(x) / x \in X\}$, $B = \{x, \mu_B(x), \upsilon_B(x) / x \in X\}$ and let $A = \{A_j : j \in J\}$ be an arbitrary family of IFS's in X. Then we define

(i) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\upsilon_A(x) \ge \upsilon_B(x)$ for all $x \in X$.

(ii) A=B if and only if $A \subseteq B$ and $B \subseteq A$.

(iii) $\overline{A} = \{x, \upsilon_A(x), \mu_A(x) / x \in X\}$.

(iv) $A \cap B = \{x, \mu_A(x) \cap \mu_B(x), \upsilon_A(x) \cup \upsilon_B(x) / x \in X\}$.

(v) $A \cup B = \{x, \mu_A(x) \cup \mu_B(x), \upsilon_A(x) \cap \upsilon_B(x) / x \in X\}$

(vi) $1_\sim = \{\langle x, 1, 0 \rangle x \in X\}$ and $0_\sim = \{\langle x, 0, 1 \rangle x \in X\}$.

Definition 2.3:[5]An intuitionistic fuzzy topology (IFT, in short) on a nonempty set X is a family τ of an intuitionistic fuzzy set (IFS, in short) in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$.

(ii) $A_1 \cap A_2 \in \tau$ for any $A_1, A_2 \in \tau$.

(iii) $\bigcup A_j \in \tau$ for any $A_j : j \in J \subseteq \tau$.

In this paper we denote intuitionistic fuzzy topological space (IFTS, in short) by $(X, \tau), (Y, \kappa)$ or X,Y. Each IFS which belongs to τ is called *an intuitionistic fuzzy open set* (*IFOS*, in short) in X. The complement $\overline{A}$ of an IFOS A in X is called an *intuitionistic fuzzy closed set* (*IFCS*, in short). An IFS X is called *intuitionistic fuzzy clopen* [19] (*IF clopen*)(*IFCO*, for short) iff it is both intutionistic fuzzy open and intuitionistic fuzzy closed.

Definition 2.4:[5] Let (X, τ) be an IFTS and $A = \{x, \mu_A(x), \upsilon_A(x)\}$ be an IFS in X. Then the fuzzy interior and closure of A are denoted by

(i) $cl(A) = \bigcap$ {K: K is an IFCS in X and $A \subseteq K$ }.

(ii) $\text{int}(A) = \bigcup$ {G: G is an IFOS in X and $G \subseteq A$}.

Note that, for any IFS A in (X, τ), we have $cl(\overline{A}) = \overline{\text{int}(A)}$ and $\text{int}(\overline{A}) = \overline{cl(A)}$.

Definition 2.5:[7] Let A be an IFS in an IFTS (X, τ), then A is

(i) An *intuitionistic fuzzy regular open set* (IFROS) if $A = \text{int}(cl(A))$.

(ii) An *intuitionistic fuzzy semi open set* (IFSOS) if $A \subseteq cl(\text{int}(A))$.

(iii) An *intuitionistic fuzzy preopen set* (IFPOS) if $A \subseteq \text{int}(cl(A))$.

(iv) An *intuitionistic fuzzy d open set* (IFdOS) if $A \subseteq scl(b\,\text{int}(A)) \cup cl(\text{int}(A))$.

(v) An *intuitionistic fuzzy α-open set* (IFα OS) if $A \subseteq \text{int}(cl(\text{int}(A)))$.

(vi) An *intuitionistic fuzzy β-open set* (IFβ OS) if $A \subseteq cl(\text{int}(cl((A))))$.

(vii) An *intuitionistic fuzzy γ-open set* (IFγ OS) if $A \subseteq cl(\text{int}(A)) \cup \text{int}(cl(A))$.

The complement of the above said sets are *intuitionistic fuzzy regular closed* set, *intuitionistic fuzzy semiclosed* set, *intuitionistic fuzzy preclosed* set, *intuitionistic fuzzy d closed* set, *intuitionistic fuzzy α-closed* set, *intuitionistic fuzzy β-closed* set, *intuitionistic fuzzy γ-closed* set, (*IFRCS, IFSCS, IFPCS, IFdCS, IFα CS, IFβ CS, IFγ CS* respectively).

Definition 2.6:[12] An IFS $p(\alpha,\beta)=\langle x,C_{\alpha},C_{1-\beta}\rangle$ where $\alpha\in(0,1],\beta\in[0,1)$ and $\alpha+\beta\leq 1$ is called an *intuitionistic fuzzy point* (IFP) in X.

Note that an IFP $p(\alpha,\beta)$ is said to belong to an IFS $A=\langle X,\mu_A,\upsilon_A\rangle$ of X denoted by $p(\alpha,\beta)\in A$ if $\alpha\leq\mu_A$ and $\beta\geq\upsilon_A$.

Definition 2.7:[12] Let $p(\alpha,\beta)$ be an IFP of an IFTS (X,τ). An IFS A of X is called an *intuitionistic fuzzy neighbourhood* (IFN) of $p(\alpha,\beta)$ if there exists an IFOS B in X such that $p(\alpha,\beta)\in B\subseteq A$.

Definition 2.8: [16] An IFTS X is called CO-T_1 if and only if for each pair of distinct IFP $x_{(\alpha,\beta)},y_{(\gamma,\delta)}$) in X there exists IFclopen sets U and V in X $x_{(\alpha,\beta)}\in U$, $x_{(\alpha,\beta)}\notin V$, $y_{(\gamma,\delta)}\notin U$, $y_{(\gamma,\delta)}\in V$.

Definition 2.9:[6]An IFTS X will be called regular if for each IFP $p(\alpha,\beta)$ and each IFCS such $p\cap C=0_{\sim}$ there exists intuitionistic fuzzy open sets U and V such that $p\subseteq U,C\subseteq V$ and $U\cap V=0$.

Definition 2.10:[6] An IFTS X will be called normal if for each IFCSs U and V such that $U\cap V=0$ there exists IFOSs U_1 and V_1 such that $U\subseteq U_1$, $V\subseteq V_1$ and $U_1\cap V_1=0_{\sim}$.

Definition 2.11:[14 **]**An IFS A is said to be an intuitionistic fuzzy dense (IFD for short) in another IFS B in an IFTS (X, τ), if cl(A) = B.

Definition 2.12:[15] An IFTS X is called *hyperconnected* if every IF open set in X is dense.

Definition 2.13:[5] Let X and Y be two IFTSs. Let $A=\{\langle X,\mu_A(x),\upsilon_A(x)\rangle:x\in X\}$ and $B=\{\langle Y,\mu_B(y),\upsilon_B(y)\rangle y\in Y\}$ be IFSs of X and Y respectively. Then is an IFS $A\times B$ of $X\times Y$ defined by $A\times B(x,y)=\langle (X,Y),\min(\mu_A(x),\mu_B(y)),\max(\upsilon_A(x),\upsilon_B(y))\rangle$.

Definition 2.14:[17] Let A be an IFTS (X,τ). Then A is called an *intuitionistic fuzzy* ζ *open set*($IF\zeta OS$, in short) in X if $A\subseteq bcl(\operatorname{int}(A))$.

Definition 2.15:[17] Let A be an IFTS (X,τ). Then A is called an *intuitionistic fuzzy* ζ *closed set* (IFζ CS, in short) in X if $b\operatorname{int}(cl(A))\subseteq A$.

Definition 2.16:[18] An IFTS (X,τ) is said to be *intuitionistic fuzzy* ζ *$T_{1/2}$ space* ($IF\zeta T_{1/2}$, in short) if every $IF\zeta OS$ in X is an IFOS in X.

Definition 2.17:[5] Let X and Y be two non-empty sets and $f:X\to Y$ be a function.

If $B=\{<y,\mu_B(y),\upsilon_B(y)>/y\in Y\}$ is an IFS in Y, then the pre-image of B under f is denoted and defined by $f^{-1}(B)=\{<x,f^{-1}(\mu_B(x)),f^{-1}(\upsilon_B(x))>/x\in X\}$. Since $\mu_B(x),\upsilon_B(x)$ are fuzzy sets, we explain that $f^{-1}(\mu_B(x))=\mu_B(x)(f(x)),f^{-1}(\upsilon_B(x))=\upsilon_B(x)(f(x))$.

Definition 2.18: Let $f:X\to Y$ from an IFTS X into an IFTS Y. Then f is said to be an

(i) *Intuitionistic fuzzy* ζ *continuous* ($IF\zeta-$cont, in short)[16] if $f^{-1}(B)\in IF\zeta OS(X)$ for every $B\in\kappa$.

(ii) *Intuitionistic fuzzy continuous* [4] if $f^{-1}(B)\in IFO(X)$ for every $B\in\kappa$.

(iii) *Intuitionistic fuzzy semi-continuous* [7] if $f^{-1}(B)\in IFSO(X)$ for every $B\in\kappa$.

(iv) *Intuitionistic fuzzy precontinuous* [7] if $f^{-1}(B)\in IFPO(X)$ for every $B\in\kappa$.

(v) *Intuitionistic fuzzy d continuous* [7] if $f^{-1}(B)\in IFdO(X)$ for every $B\in\kappa$.

(vi) *Intuitionistic fuzzy α-continuous* [7] if $f^{-1}(B)\in IF\alpha O(X)$ for every $B\in\kappa$.

(vii) *Intuitionistic fuzzy β-continuous* [7] if $f^{-1}(B) \in IF\beta O(X)$ for every $B \in \kappa$.

(viii) *Intuitionistic fuzzy γ-continuous* [7] if $f^{-1}(B) \in IF\gamma O(X)$ for every $B \in \kappa$.

(ix) *Intuitionistic fuzzy clopen-continuous*[19] if for each IFP $p(\alpha,\beta)$ of X and each open set V containing , $f(p(\alpha,\beta))$ there exists a clopen set U containing $p(\alpha,\beta)$, such that $f(u) \subseteq V$.

(x) *intuitionistic fuzzy totally continuous*[13] if and only if $f^{-1}(B)$ is an IF clopen sets in X, for every $B \in \kappa$.

Definition 2.19:[17] Let f be a mapping from IFTS (X,τ) into an IFTS (Y,κ). Then f is said to be *intuitionistic fuzzy* $\zeta -$*irresolute* ($IF\zeta -$*irresolute*, in short) if $f^{-1}(B) \in IF\zeta O(X)$ for every $IF\zeta OS$ B in Y.

Definition 2.20:[7] Let $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$. The product $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ is defined by $f_1 \times f_2(x_1, x_2) = (f_1(x_1), f_2(x_2))$ for every $(X_1, X_2) \in X_1 \times X_2$.

3. INTUISTIONISTIC FUZZY ALMOST $\zeta -$CONTINUOUS MAPPINGS

Definition 3.1 A mapping $f : X \to Y$ from an IFTS X into an IFTS Y is called an *intuitionistic fuzzy almost* $\zeta -$*continuous* ($IFa\zeta -$continuous, in short) mapping if $f^{-1}(B)$ is an IFζ CS in X, for every IFRCS B in Y.

Example 3.2:

Let $X = \{a,b\}, Y = \{u,v\}$ $G_1 = \langle x,(0.2,0.2,0.1),(0.4,0.4,0.6)\rangle$,

$G_2 = \langle x,(0.3,0.2,0.5),(0.2,0.2,0.4)\rangle$

$H = \langle y,(0.3,0.4,0.3),(0.4,0.5,0.4)\rangle$

Then Then $\tau = \{0_{\sim}, 1_{\sim}, G_1, G_2\}$ and $\kappa = \{0_{\sim}, 1_{\sim}, H\}$ are IFT on X and Y respectively.

Define a mapping $f : (X,\tau) \to (Y,\kappa)$ by $f(a) = u$ and $f(b) = v$.

Then f is an IF*a* ζ continuous mapping.

Theorem 3.3: Every IF continuous mapping is $IFa\zeta -$continuous but not conversely.

Proof: Let $f : (X,\tau) \to (Y,\kappa)$ be an IF continuous mapping and B be an IFRCS in Y. Since every IFRCS is an IFCS, B is an IFCS in Y. Then f $^{-1}$(B) is an IFCS in X, by hypothesis. Since every IFCS is an *IFζ CS*, f $^{-1}$(B) is an IFζ CS in X. Hence *f* is an $IFa\zeta -$continuous mapping.

Example 3.4: Let $X = \{a,b\}$, $Y = \{u,v\}$

$G_1 = \langle x,(0.3,0.4),(0.7,0.6)\rangle$, $G_2 = \langle y,(0.7,0.8),(0.3,0.2)\rangle$

Then $\tau = \{0_{\sim}, 1_{\sim}, G_1\}$ and $\kappa = \{0_{\sim}, 1_{\sim}, G_2\}$ are IFT on X and Y respectively.

Define a mapping $f : (X,\tau) \to (Y,\kappa)$ by $f(a) = u$ and $f(b) = v$.

Then f is an $IFa\zeta -$continuous mapping but not an IF continuous mapping.

Theorem 3.5: Every IFζ continuous mapping is an $IFa\zeta -$continuous but not conversely.

Proof: Let $f : (X,\tau) \to (Y,\kappa)$ be an IF ζ continuous mapping and B be an IFRCS in Y. Then f $^{-1}$(B) is IFζ CS in X. Hence f is an $IFa\zeta -$continuous mapping.

Example 3.6: Let $X = \{a,b\}$, $Y = \{u,v\}$

$G_1 = \langle x,(0.7,0.8),(0.3,0.2)\rangle$, $G_2 = \langle y,(0.6,0.7),(0.4,0.3)\rangle$

Then $\tau = \{0_{\sim}, 1_{\sim}, G_1\}$ and $\kappa = \{0_{\sim}, 1_{\sim}, G_2\}$ are IFT on X and Y respectively.

Define a mapping $f : (X, \tau) \to (Y, \kappa)$ by $f(a) = u$ and $f(b) = v$.

Then f is an $IFa\zeta-$continuous mapping but not an IF$\zeta-$continuous mapping.

Theorem 3.7: Every $IFa\zeta-$continuous mapping is an IFS continuous, but not conversely.

Proof: Let $f : (X, \tau) \to (Y, \kappa)$ be an $IFa\zeta-$continuous mapping and B be an IFRCS in Y. By hypothesis f $^{-1}$(B) is an IFζ CS in X. Since every IFζ CS is an IFSCS, f $^{-1}$(B) is an IFSCS in X. Hence f is IFS continuous.

Example 3.8 : Let $X = \{a, b\}$, $Y = \{u, v\}$

$G_1 = \langle x, (0.5, 0.4), (0.5, 0.6) \rangle$, $G_2 = \langle y, (0.2, 0.3), (0.8, 0.7) \rangle$

Then $\tau = \{0_{\sim}, 1_{\sim}, G_1\}$ and $\kappa = \{0_{\sim}, 1_{\sim}, G_2\}$ are IFT on X and Y respectively.

Define a mapping $f : (X, \tau) \to (Y, \kappa)$ by $f(a) = u$ and $f(b) = v$.

Then f is an IFS continuous mapping but not an $IFa\zeta-$continuous mapping.

Theorem 3.9: Every $IFa\zeta-$continuous mapping is an IFd continuous but not conversely.

Proof: Let $f : (X, \tau) \to (Y, \kappa)$ be an $IFa\zeta-$continuous mapping and B be an IFRCS in Y. Then f $^{-1}$(B) is an IFζ CS in X. Since every IFζ CS is an IFdCS, f $^{-1}$(B) is an IFdCS in X. Hence f is IFd continuous.

Example 3.10 : Let $X = \{a, b\}$, $Y = \{u, v\}$

$G_1 = \langle x, (0.1, 0.1), (0.6, 0.5) \rangle$, $G_2 = \langle y, (0.2, 0.2), (0.3, 0.5) \rangle$

Then $\tau = \{0_{\sim}, 1_{\sim}, G_1\}$ and $\kappa = \{0_{\sim}, 1_{\sim}, G_2\}$ are IFT on X and Y respectively.

Define a mapping $f : (X, \tau) \to (Y, \kappa)$ by $f(a) = u$ and $f(b) = v$.

Then f is an IFd continuous mapping but not an $IFa\zeta-$continuous mapping.

Theorem 3.11: Every $IFa\zeta-$continuous mapping is an IF$\zeta-$continuous but not conversely.

Proof: : Let $f : (X, \tau) \to (Y, \kappa)$ be an $IFa\zeta-$continuous mapping and B be an IFRCS in Y. Then f $^{-1}$(B) is an $IF\zeta CS$ in X. Hence f is IF$\zeta-$continuous.

Example 3.12 : Let $X = \{a, b, c\}$, $Y = \{u, v, w\}$

$G_1 = \langle x, (0.3, 0.1, 0.4), (0.3, 0.3, 0.4) \rangle$, $G_2 = \langle y, (0.2, 0.1, 0.3), (0.4, 0.4, 0.4) \rangle$

Then $\tau = \{0_{\sim}, 1_{\sim}, G_1\}$ and $\kappa = \{0_{\sim}, 1_{\sim}, G_2\}$ are IFT on X and Y respectively.

Define a mapping $f : (X, \tau) \to (Y, \kappa)$ by $f(a) = u$ and $f(b) = v$.

Then f is an IFζ -continuous mapping but not an $IFa\zeta-$continuous mapping.

Theorem 3.13: If $f : X \to Y$ is an IFcζ continuous, then f is an $IFa\zeta-$continuous mapping, but not conversely.

Proof: Let B be an IFRCS in Y. Since every IFRCS is an IFζ CS, B is an IFζ CS in Y. Since f is an IFcζ continuous, $f^{-1}(B)$ is an IFRCS in X. Thus $f^{-1}(B)$ is an IFζ CS in X. Hence f is an IFaζ continuous mapping.

Example 3.14: Let $X = \{a, b\}$, $Y = \{u, v\}$

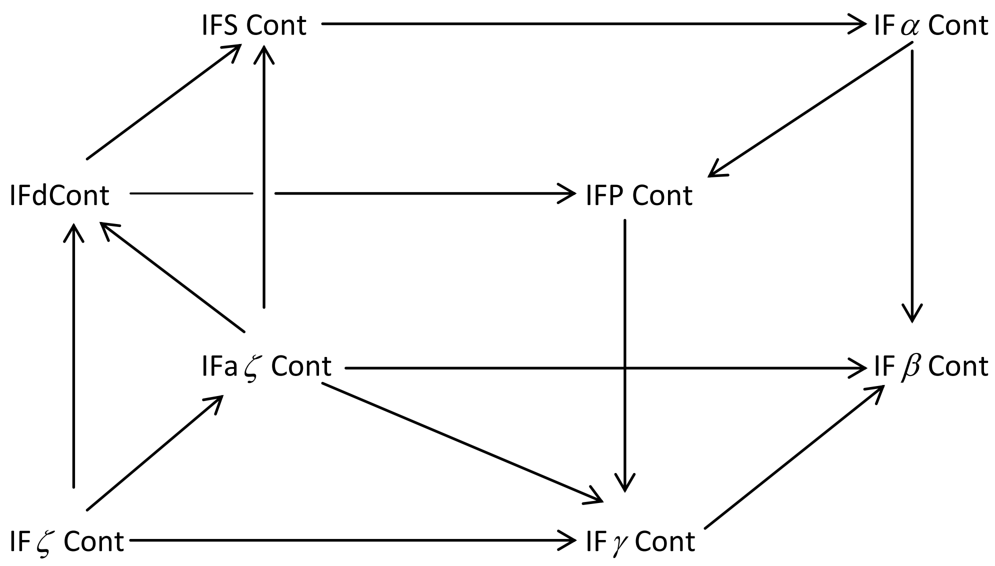
$G_1 = \langle x, (0.5, 0.4), (0.5, 0.6) \rangle$, $G_2 = \langle y, (0.2, 0.3), (0.8, 0.7) \rangle$

Then $\tau = \{0_{\sim}, 1_{\sim}, G_1\}$ and $\kappa = \{0_{\sim}, 1_{\sim}, G_2\}$ are IFT on X and Y respectively.

Define a mapping $f : (X, \tau) \to (Y, \kappa)$ by $f(a) = u$ and $f(b) = v$.

Then f is an $IFa\zeta -$continuous mapping but not an IFcζ continuous mapping.

From the above theorems and examples we have the following implications.



Theorem 3.15: Let $f:(X,\tau)\to(Y,\kappa)$ be a mapping where $f^{-1}(B)$ is an IFRCS in X for every IFCS B in Y. Then f is an $IFa\zeta -$*continuous* mapping but not conversely.

Proof: Let B be an IFRCS in Y. Since every IFRCS is an IFCS, B is an IFCS in Y. Then $f^{-1}(B)$ is an IFRCS in X. As every IFRCS is an IFζ CS, $f^{-1}(B)$ is an IFζ CS in X. Hence f is an $IFa\zeta -$*continuous* mapping .

Example 3.16: : Let $X=\{a,b\}$, $Y=\{u,v\}$

$G_1=\langle x,(0.5,0.6),(0.5,0.4)\rangle$, $G_2=\langle y,(0.5,0.3),(0.5,0.7)\rangle$

Then $\tau=\{0_{\sim},1_{\sim},G_1\}$ and $\kappa=\{0_{\sim},1_{\sim},G_2\}$ are IFT on X and Y respectively.

Define a mapping $f:(X,\tau)\to(Y,\kappa)$ by $f(a)=u$ and $f(b)=v$.

Then f is an $IFa\zeta -$*continuous* mapping but not a mapping as defined in theorem above.

Theorem 3.17: Let $f:X\to Y$ be a mapping. Then the following are equivalent:

(i) f is an $\mathrm{IFa}\zeta -$*continuous mapping.*

(ii) $f^{-1}(B)$ is an IFζ OS in X for every IFROS B in Y.

Proof: $(i)\Rightarrow(ii)$ Let B be an IFROS in Y. Then $\overline{B}$ is an IFRCS in Y. By hypothesis, $f^{-1}(\overline{B})$ is an IFζ CS in X. That is $\overline{f^{-1}(B)}$ is an IFζ CS in X. Therefore $f^{-1}(B)$ is an $IF\zeta OS$ in X.

$(ii)\Rightarrow(i)$ Let B be an IFRCS in Y. Then $\overline{B}$ is an IFROS in Y. By hypothesis, $f^{-1}(\overline{B})$ is an IFζ OS in X. That is $\overline{f^{-1}(B)}$ is an $IF\zeta OS$ in X. Therefore $f^{-1}(B)$ is an IFζ CS in X. Then f is an $IFa\zeta -$*continuous* mapping.

Theorem 3.18: Let $f:X\to Y$ be a mapping, if $f^{-1}(\zeta\,\mathrm{int}(B))\subseteq\zeta\,\mathrm{int}(f^{-1}(B))$ for every IFS B in Y, then f is an $IFa\zeta -$*continuous* mapping.

Proof: Let B be an IFROS in Y. By hypothesis $f^{-1}(\zeta\operatorname{int}(B)) \subseteq \zeta\operatorname{int}(f^{-1}(B))$. Since B is an IFROS, it is an $IF\zeta OS$ in Y. Therefore $\zeta\operatorname{int}(B) = B$. Hence
$f^{-1}(B) = f^{-1}(\zeta\operatorname{int}(B)) \subseteq \zeta\operatorname{int}(f^{-1}(B)) \subseteq f^{-1}(B)$. Therefore $f^{-1}(B) = \zeta\operatorname{int}(f^{-1}(B))$.
This implies $f^{-1}(B)$ is an $IF\zeta OS$ in X and thus f is an $IFa\zeta -$ *continuous* mapping.

Theorem 3.19: Let $f : X \to Y$ be a mapping, if $\zeta cl(f^{-1}(B)) \subseteq f^{-1}(\zeta cl(B))$ for every IFS B in Y, then f is an $IFa\zeta -$ *continuous* mapping.

Proof: Let B be an IFRCS in Y. By hypothesis $\zeta cl(f^{-1}(B)) \subseteq f^{-1}(\zeta cl(B))$. Since B is an IFRCS, it is an IFζ CS in Y. Therefore $\zeta cl(B) = B$. Hence $f^{-1}(B) = f^{-1}(\zeta cl(B)) \supseteq \zeta cl(f^{-1}(B)) \supseteq f^{-1}(B)$. Therefore $f^{-1}(B) = \zeta cl(f^{-1}(B))$. This implies $f^{-1}(B)$ is an IFζ CS in X and thus f is an $IFa\zeta -$ *continuous* mapping.

Remark 3.20: The converse of the above Theorem 3.21 is true if B is an IFRCS in Y and X is an IFζ $T_{1/2}$ space.

Proof: Let f is an $IFa\zeta -$ *continuous* mapping. Let B be an IFRCS in Y. Then $f^{-1}(B)$ is an IFζ CS in X. Since X is an IFζ $T_{1/2}$ space, $f^{-1}(B)$ is an IFOS in X. This implies $\zeta cl(f^{-1}(B)) = f^{-1}(B)$. Now $f^{-1}(\zeta cl(B)) \supseteq f^{-1}(B) = \zeta cl(f^{-1}(B))$. Therefore $f^{-1}(\zeta cl(B)) \supseteq \zeta cl(f^{-1}(B))$.

Theorem 3.21: Let $f : X \to Y$ be a mapping and $g : X \to X \times Y$ be the graph of the mapping. If g is an $IFa\zeta -$ *continuous* mapping, then f is so.

Proof: Let B be an IFROS in Y. Then $f^{-1}(B) = f^{-1}(1_{\sim} \cap f^{-1}(B)) = g^{-1}(1_{\sim} \times B)$. Since $1_{\sim} \times B$ is an IFROS in $X \times Y$ and as g is an $IFa\zeta -$ *continuous* mapping, is an IFζ OS in X. Hence $f^{-1}(B)$ is an $IF\zeta OS$ in X and so f is an $IFa\zeta -$ *continuous* mapping.

Theorem 3.22: Let $f : X \to Y$ and $g : Y \to Z$ be any two mappings. Then the following properties hold :

(i) If f is an IF continuous mapping and g is an $IFa\zeta -$ *continuous* mapping , then gof is an $IFa\zeta -$ *continuous* mapping.

(ii) If f is an IFζ continuous mapping and g is an $IFa\zeta -$ *continuous* mapping, then gof is an $IFa\zeta -$ *continuous* mapping.

Proof: (i) Let B be an IFROS in Z. Since g is an $IFa\zeta -$ *continuous* mapping, $g^{-1}(B)$ is an IFOS in Y. Since f is an IF continuous mapping, $f^{-1}(g^{-1}(B))$ is an IFOS in X. This implies $f^{-1}(g^{-1}(B))$ is an $IF\zeta OS$ in X, since IFOS is an $IF\zeta OS$. But $f^{-1}(g^{-1}(B)) = (gof)^{-1}(B)$. This implies gof $IFa\zeta -$ *continuous* mapping.

(ii) Let B be an IFROS in Z. Since g is an $IFa\zeta -$ *continuous* mapping, $g^{-1}(B)$ is an IFOS in Y. Since f is an IFζ continuous mapping, $f^{-1}(g^{-1}(B))$ is an IFOS in X. Since $f^{-1}(g^{-1}(B)) = (gof)^{-1}(B)$, gof $IFa\zeta -$ *continuous* mapping.

4. INTUISTIONISTIC FUZZY SLIGHTLY $\zeta -$ CONTINUOUS MAPPINGS

Definiton 4.1: A mapping $f : X \to Y$ from an IFTS X into an IFTS Y is called an *intuitionistic fuzzy slightly* $\zeta -$ continuous if for each intuitionistic fuzzy point $p(\alpha,\beta) \in X$ and each intuitionistic fuzzy clopen set B in Y containing $f(p(\alpha,\beta))$, there exists a fuzzy intuitionistic fuzzy ζ open set A in X such that $f(A)\subseteq B$.

Theorem 4.2: For a function $f: X \to Y$, the following statements are equivalent:

(i) f is *intuitionistic fuzzy slightly* $\zeta-$*continuous*;

(ii) for every intuitionistic fuzzy clopen set B in Y, $f^{-1}(B)$ is intuitionistic fuzzy ζ -open;

(iii) for every intuitionistic fuzzy clopen set B in Y, $f^{-1}(B)$ is intuitionistic fuzzy ζ closed;

(iv) for every intuitionistic fuzzy clopen set B in Y, $f^{-1}(B)$ is intuitionistic fuzzy ζ -clopen.

Proof: $(i)\Rightarrow(ii)$ Let B be IF clopen set in Y and let $p_{(\alpha,\beta)} \in f^{-1}(B)$. Since $f(p_{(\alpha,\beta)})\in B$, by (i) there exists a IF ζ OS $A_{p(\alpha,\beta)}$ in X containing $p(\alpha,\beta)$ such that $A_{p(\alpha,\beta)} \subseteq f^{-1}(B)$. We obtain that $f^{-1}(B) = \bigcup_{p(\alpha,\beta)\in f^{-1}(B)} A_{p(\alpha,\beta)}$. Thus $f^{-1}(B)$ is IF ζ -open.

$(ii)\Rightarrow(iii)$ Let B be IF clopen set in Y. Then $\overline{B}$ is IF clopen. By (ii), $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$ is IF ζ -open. Thus $f^{-1}(B)$ is intuitionistic fuzzy ζ -closed.

$(iii)\Rightarrow(iv)$ Let B be IF clopen set in Y. Then by (iii) $f^{-1}(B)$ is intuitionistic fuzzy ζ -closed. Also $\overline{B}$ is IF clopen and (iii) implies $f^{-1}(\overline{B}) = \overline{f^{-1}(B)}$ is IF ζ -closed. Hence $f^{-1}(B)$ is intuitionistic fuzzy ζ - open. Thus $f^{-1}(B)$ is intuitionistic fuzzy ζ -clopen.

$(iv)\Rightarrow(i)$ Let B be IF clopen set in Y containing $f(p_{(\alpha,\beta)})$. By (iv), $f^{-1}(B)$ is intuitionistic fuzzy ζ - open. Let us take $A = f^{-1}(B)$. Thus $f(A)\subseteq B$. Hence f is *intuitionistic fuzzy slightly* $\zeta-$*continuous.*

Lemma 4.3: Let $g: X \to X\times Y$ be the graph of the mapping $f: X\to Y$. If A and B are IFS's of X and Y respectively, then $g^{-1}(1_{\sim}\times B) = (1_{\sim} \cap f^{-1}(B))$

Lemma 4.4: Let X and Y be intuitionistic fuzzy topological spaces, then (X,τ) is product related to (Y,κ) if for any IFS C in X, D in Y whenever $C \not\subset \overline{A}$, $D \not\subset \overline{B}$ implies $\overline{A}\times 1_{\sim} \bigcup 1_{\sim}\times \overline{B} \supseteq C\times D$ there exists $A_1 \in \tau$, $B_1 \in \kappa$ such that $\overline{A_1} \supseteq C$ and $\overline{B_1} \supseteq D$ and $\overline{A_1}\times 1_{\sim} \bigcup 1_{\sim}\times \overline{B_1} = \overline{A}\times 1_{\sim} \bigcup 1_{\sim}\times \overline{B}$

Theorem 4.5: Let $f: X\to Y$ be a function and assume that X is a product related to Y. If the graph $g: X\to X\times Y$ of f is *IF slightly* $\zeta-$*continuous* then so is f .

Proof: Let B be IF clopen set in Y. Then by lemma 3.3, $f^{-1}(B) = (1_{\sim} \cap f^{-1}(B) = g^{-1}(1_{\sim}\times B)$. Now $1_{\sim}\times B$ is a IF clopen set in $X\times Y$. Since g is *IF slightly* $\zeta-$*continuous* then $g^{-1}(1_{\sim}\times B)$ is IF ζ -open in X. Hence is $f^{-1}(B)$ IF ζ -open in X. Thus f is *IF slightly* $\zeta-$*continuous.*

Theorem 4.6: A mapping $f: X\to Y$ from and IFTS X into an IFTS Y is *IF slightly* $\zeta-$*continuous* if and only if for each IFP $p_{(\alpha,\beta)}$ in X and IF clopen set B in Y such that $f(p(\alpha,\beta))\in B$, $cl(f^{-1}(B))$ is IFN of IFP $p_{(\alpha,\beta)}$ in X.

Proof: Let f be an *IF slightly* $\zeta-$*continuous* mapping, $p(\alpha,\beta)$ be an IFP in X and B be any IF clopen set in Y such that $f(p_{(\alpha,\beta)})\in B$. Then

$p(\alpha,\beta)\in f^{-1}(B) \subseteq bcl(\text{int}(f^{-1}(B))) \subseteq cl(f^{-1}(B))$. Hence $cl(f^{-1}(B))$ is an IFN of $p_{(\alpha,\beta)}$ in X.

Conversely, let B be any IF clopen set in Y and $p_{(\alpha,\beta)}$ be IFP in X such that $f(p(\alpha,\beta)) \in B$. Then $p_{(\alpha,\beta)} \in f^{-1}(B)$. According to assumption $cl(f^{-1}(B))$ is IFN of IFP $p_{(\alpha,\beta)}$ in X.

So $p_{(\alpha,\beta)} \in bcl(\mathrm{int}(f^{-1}(B))) \subseteq cl(bcl(\mathrm{int}(f^{-1}(B))))$. So $f^{-1}(B) \subseteq bcl(\mathrm{int}(f^{-1}(B)))$. Hence $f^{-1}(B)$ is IF ζ OS in X. Therefore f is *IF slightly* $\zeta-$*continuous*.

Proposition 4.7: Every *intuitionistic fuzzy* $\zeta-$*continuous* function is *IF slightly* $\zeta-$*continuous*. But the converse need not be true, as shown by the following example.

Example 4.8: Let $X=\{a,b\}$, $Y=\{u,v\}$

$G_1=\langle x,(0.7,0.8),(0.3,0.2)\rangle$, $G_2=\langle y,(0.6,0.7),(0.4,0.3)\rangle$

Then $\tau=\{0_\sim,1_\sim,G_1\}$ and $\kappa=\{0_\sim,1_\sim,G_2\}$ are IFT on X and Y respectively.

Define a mapping $f:(X,\tau)\to(Y,\kappa)$ by $f(a)=u$ and $f(b)=v$.

Then f is an *IF slightly* $\zeta-$*continuous* but not an IF $\zeta-$continuous mapping.

Proposition 4.9: Every $IF\zeta-$*irresolute* function is *IF slightly* $\zeta-$*continuous*. But the converse need not be true, as shown by the following example.

Example 4.10: Let $X=\{a,b\}$, $Y=\{u,v\}$

$G_1=\langle x,(0.3,0.4),(0.6,0.5)\rangle$, $G_2=\langle y,(0.4,0.5),(0.5,0.5)\rangle$

Then $\tau=\{0_\sim,1_\sim,G_1\}$ and $\kappa=\{0_\sim,1_\sim,G_2\}$ are IFT on X and Y respectively.

Define a mapping $f:(X,\tau)\to(Y,\kappa)$ by $f(a)=u$ and $f(b)=v$.

Then f is an *IF slightly* $\zeta-$*continuous* but not an IF $\zeta-$irresolute.

Theorem 4.11: Suppose that Y has a base consisting of IF clopen sets. If $f:X\to Y$ is IF slightly ζ continuous, then f is *IF* $\zeta-$*continuous*.

Proof: Let $p_{(\alpha,\beta)} \in X$ and let C be IFOS in Y containing $f(p_{(\alpha,\beta)})$. Since Y has a base consisting of IF clopen sets, there exists an IF clopen set B containing $f(p_{(\alpha,\beta)})$ such that $B\subseteq C$. Since f is *IF slightly* $\zeta-$*continuous*, then there exists an IF ζ OS A in X containing $p_{(\alpha,\beta)}$ such that $f(A)\subseteq B\subseteq C$. Thus f is $IF\zeta-$*continuous*.

Theorem 4.12: If a function $f:X\to\prod Y_i$ is an IF slightly fuzzy ζ continuous, then $P_i of:X\to Y_i$ is *IF slightly* $\zeta-$*continuous*, where P_i is the projection of $\prod Y_i$ onto Y_i.

Proof: Let B_i be any IF clopen sets of Y_i. Since P_i is IF continuous and IF open mapping, and $P_i:\prod Y_i\to Y_i$, $P_i^{-1}(B_i)$ is IF clopen sets in $\prod Y_i$. Now $(P_i of)^{-1}(B_i)= f^{-1}(P_i^{-1}(B_i))$. As IF slightly ζ continuous and $P_i^{-1}(B_i)$ is IF clopen sets, $f^{-1}(P_i^{-1}(B_i))$ is IF ζ OS in X. Hence $P_i of$ is *IF slightly* $\zeta-$*continuous*.

Theorem 4.13: The following hold for functions $f:X\to Y$ and $g:Y\to Z$

(i) If f is *IF slightly* $\zeta-$*continuous* and g is *IF totally continuous* then gof is IF ζ continuous.

(ii) If f is IF ζ -irresolute and g is *IF slightly* $\zeta-$*continuous* then gof is IF slightly ζ continuous.

Proof: (i) Let B be an IFOS in Z. Since g if IF totally continuous, $g^{-1}(B)$ is an IF clopen set in Y. Now $(gof)^{-1}(B)= f^{-1}(g^{-1}(B))$. Since f is *IF slightly* ζ *continuous*, $f^{-1}(g^{-1}(B))$ IF ζ OS in X. Hence gof is IF ζ continuous.

(ii)Let B be IF clopen set in Z. Since and g is *IF slightly* $\zeta-$*continuous*, $g^{-1}(B)$ is an $IF\zeta OS$ in Y. Now $(gof)^{-1}(B)=f^{-1}(g^{-1}(B))$ Since f is *IF* ζ *-irresolute*, $f^{-1}(g^{-1}(B))$ IF ζ OS in X which implies gof is *IF slightly* $\zeta-$*continuous*.

5. INTUISTIONISTIC FUZZY ζ SEPARATION AXIOMS

Definition 5.1: An IFTS (X,τ) is called $\zeta-T_1$ if and only if for each pair of distinct intuitionistic fuzzy points $x_{(\alpha,\beta)},y_{(\gamma,\delta)}$ in X there exists $IF\zeta OS$ such that $x_{(\alpha,\beta)}\in U$, $y_{(\gamma,\delta)}\notin U$ and $x_{(\alpha,\beta)}\notin V$, $y_{(\gamma,\delta)}\in V$.

Theorem 5.2: If $f:X\to Y$ is *IF slightly* $\zeta-$*continuous* injection and Y is $CO-T_1$, then X is IF$\zeta-T_1$.

Proof: Suppose that Y is IF$CO-T_1$. For any distinct intuitionistic fuzzy points $x_{(\alpha,\beta)},y_{(\gamma,\delta)}$ in X there exists IF clopen sets U,V in Y such that $f(x_{(\alpha,\beta)})\in U, f(y_{(\gamma,\delta)})\notin U, f(x_{(\alpha,\beta)})\in V, f(y_{(\gamma,\delta)})\notin V$. Since f is *IF slightly* $\zeta-$*continuous*, $f^{-1}(U)$ and $f^{-1}(V)$ are IFζ –open sets in X such that $x_{(\alpha,\beta)}\in f^{-1}(U), y_{(\gamma,\delta)}\notin f^{-1}(U), x_{(\alpha,\beta)}\notin f^{-1}(V), y_{(\gamma,\delta)}\in f^{-1}(V)$. This shows that X is IF$\zeta-T_1$.

Definition 5.3: An IFTS (X,τ) is called $\zeta-T_2$ or $\zeta-Hausdorff$ if for all pair of distinct intuitionistic fuzzy points $x_{(\alpha,\beta)},y_{(\gamma,\delta)}$ in X there exists IFζ –open sets $U,V\in Y$ such that $x_{(\alpha,\beta)}\in U$, $y_{(\gamma,\delta)}\in V$ and $U\cap V=0$.

Theorem 5.4: If $f:X\to Y$ is *IF slightly* $\zeta-$*continuous* , injection and Y is $CO-T_2$, then X is IF$\zeta-T_2$.

Proof: Suppose that Y is IF$CO-T_2$. For any distinct intuitionistic fuzzy points $x_{(\alpha,\beta)},y_{(\gamma,\delta)}$ in X there exists IF clopen sets U,V in Y such that $f(x_{(\alpha,\beta)})\in U$ and $f(y_{(\gamma,\delta)})\in V$. Since f is *IF slightly* $\zeta-$*continuous*, $f^{-1}(U)$ and $f^{-1}(V)$ are IFζ –open sets in X such that $x_{(\alpha,\beta)}\in f^{-1}(U), y_{(\gamma,\delta)}\in f^{-1}(V)$. Also we have $f^{-1}(A)\cap f^{-1}(V)=0$. Hence then X is IF$\zeta-T_2$.

Definition 5.5: An IFTS (X,τ) is called IF strongly$\zeta-regular$ if for each IFζ –closed set C and IFP $x_{(\alpha,\beta)}\notin C$, there exists intuitionistic fuzzy open sets U and V such that $C\subseteq U, x_{(\alpha,\beta)}\in V$ and $U\cap V=0$.

Theorem 5.6: If $f:X\to Y$ is *IF slightly* $\zeta-$*continuous*, injective, IF open function from an IF strongly$\zeta-regular$ X onto an IF space Y, then and Y is IF co-regular.

Proof: Let D be an IFζ open set in Y and $y_{(\gamma,\delta)}\notin D$. Take $y_{(\gamma,\delta)}=f(x_{(\alpha,\beta)})$. Since f is IF slightlyζ continuous, $f^{-1}(D)$ is an IFζ –closed set in X. Let $C=f^{-1}(D)$. $x_{(\alpha,\beta)}\notin C$. Since X is IF strongly $\zeta-regular$, there exists intuitionistic fuzzy open sets U and V such that $C\subseteq U, x_{(\alpha,\beta)}\in B$ and $U\cap V=0$. Hence, we have $D=f(C)\subseteq f(A)$ and $y_{(\gamma,\delta)}=f(x_{(\alpha,\beta)})\in f(B)$ such that $f(A)$ and $f(B)$ are disjoint IF open sets. Hence Y is IFζ regular.

Definition 5.7: An IFTS (X,τ) is called IF strongly$\zeta-normal$ if for each IF clopen sets C_1 and C_2 in X such that IFP set C and intuitionistic fuzzy point $x_{(\alpha,\beta)}\notin C$, there exists intuitionistic fuzzy open sets U and V in X such that $C_1 \cap C_2 = 0_\sim$ there exists IFζ –open sets U, V such that $C_1 \subseteq U$ and $C_2 \subseteq V$ and $U \cap V = 0$.

Theorem 5.8: : If $f : X \to Y$ is *IF slightly* $\zeta -$*continuous*, injective, IF open function from an IF strongly $\zeta - normal$ space X onto an IF space Y, then and Y is IF co-normal.

Proof: Let C_1 and C_2 be disjoint IF clopen sets in Y. Since f is IF slightly ζ continuous, $f^{-1}(C_1)$ and $f^{-1}(C_2)$ are IF ζ closed sets in X. Let us take $C = f^{-1}(C_1)$ and $D = f^{-1}(C_2)$. We have $C \cap D = 0_\sim$. Since X is IF strongly $\zeta - normal$, there exists disjoint IF open sets U and V such that $C \subseteq U$ and $D \subseteq V$. Thus $C_1 = f(C) \subseteq f(U)$ and $C_2 = f(D) \subseteq f(V)$ such that $f(U)$ and $f(V)$ are disjoint IF open sets. Hence Y is IF ζ normal.

6. INTUISTIONISTIC FUZZY COVERING PROPERTIES

Definition 6.1: Let X be an IFTS. A family of $\{< x, \mu_{G_i}(x), \upsilon_{G_i}(x) >; i \in J\}$ intuitionistic fuzzy open sets (*intuitionistic fuzzy* $\zeta -$*open sets*) in X satisfies the condition $1_\sim = \bigcup\{< x, \mu_{G_i}(x), \upsilon_{G_i}(x) > i \in J\}$ is called an intuitionistic fuzzy open cover (*intuitionistic fuzzy* $\zeta -$*open cover*) of X. A finite subfamily of an intuitionistic fuzzy open cover (*intuitionistic fuzzy* $\zeta -$*open cover*) $\{< x, \mu_{G_i}(x), \upsilon_{G_i}(x) >; i \in J\}$ of X which is also an intuitionistic fuzzy open cover (*intuitionistic fuzzy* $\zeta -$*open cover)* is called a finite subcover of $\{< x, \mu_{G_i}(x), \upsilon_{G_i}(x) >; i \in J\}$.

Definition 6.2: A space X is called an intuitionistic fuzzy ζ *-compact(*ζ *-Lindelof)* if every *intuitionistic fuzzy* $\zeta -$*open cover* of X has a finite (countable) subcover.

Definition 6.3: An IFTS X is said to be

(i) *IF*ζ *-compact* if every ζ -open cover of X has a finite subcover.
(ii) *IF countably* ζ *-compact* if every ζ -open countably cover of X has a finite subcover.
(iii) *IF*ζ *-Lindelof* if every cover of X by IFζ -open sets has a countable subcover.
(iv) IF mildly compact if every IFζ cover of X has a finite subcover.
(v) IF mildly countably compact if every *IF*$\zeta -$*countably cover* of X has a finite subcover.
(vi) IF mildly Lindelof if every cover of X has IFζ -open sets has a countable subcover.

Theorem 6.4: Let $f : X \to Y$ be an IF slightlyζ continuous surjection. Then the following statements hold:

(i) If X is IFζ -compact, then Y is IF mildly compact.
(ii) If X is IFζ -Lindelof, then Y is IF mildly Lindelof.
(iii) If X is IF countably ζ -compact, then Y is IF mildly countably compact.

Proof: (i) Let $\{A_\alpha : \alpha \in I\}$ be any IF clopen cover of Y. Since f is IF slightly ζ continuous, then $\{f^{-1}(A_\alpha) : \alpha \in I\}$ is IFζ -open cover of X. Since X is IFζ -compact, there exists a finite subset *I₀* of I such that $1_{\sim X} = \bigcup\{f^{-1}(A_\alpha); \alpha \in I_0\}$. Thus we have $1_{\sim y} = \bigcup\{A_\alpha; \alpha \in I_0\}$ and Y is IF mildly compact.

(ii) Let $\{A_\alpha : \alpha \in I\}$ be any IF clopen cover of Y. Since f is *IF slightly* $\zeta -$*continuous*, then $\{f^{-1}(A_\alpha) : \alpha \in I\}$ is IF IFζ -open cover of X. Since X is IFζ -Lindelof, there exists a countable subset I_0 of I such that $1_{\sim X} = \bigcup\{f^{-1}(A_\alpha); \alpha \in I_0\}$. Thus we have $1_{\sim y} = \bigcup\{A_\alpha; \alpha \in I_0\}$ and Y is IF mildly Lindelof.

(iii) Let $\{A_\alpha : \alpha \in I\}$ be any IF clopen cover of Y. Since f is *IF slightly* $\zeta -$*continuous*, then $\{f^{-1}(A_\alpha) : \alpha \in I\}$ is IF IF ζ -open cover of X. Since X is is IF countably ζ -compact, subset I_0 of I such that $1_{\sim X} = \bigcup\{f^{-1}(A_\alpha); \alpha \in I_0\}$. Thus we have $1_{\sim y} = \bigcup\{A_\alpha; \alpha \in I_0\}$ and Y is IF midly compact.

Definition 6.5: An IFTS X is said to be

(i) IF ζ *–closed compact* if every ζ –closed of X has a finite subcover.

(ii) IF ζ *–closed Lindelof* if ever y cover of X by ζ –closed sets has a countable subcover.

(iii) IF *countably* ζ *–closed compact* if every countable cover of X by ζ –closed sets has a finite subcover.

Theorem 6.6: Let $f : X \to Y$ be an *IF slightly* $\zeta -$*continuous , surjection*. Then the following statements hold:

(i) If X is *IF* ζ *-closed compact*, then Y is *mildly compact.*

(ii) If X is *IF* ζ *-closed Lindelof*, then Y is *mildly Lindelof.*

(iii) If X is *IF countably* ζ *-closed compact*, then Y is *mildly countably compact.*

Proof: (i) Let $\{A_\alpha : \alpha \in I\}$ be any IF clopen cover of Y. Since *f* is *IF slightly* $\zeta -$*continuous*, then $\{f^{-1}(A_\alpha) : \alpha \in I\}$ is IF ζ -closed cover of X. Since X is IF ζ -closed compact, there exists a finite subset I_0 of I such that $1_{\sim X} = \bigcup\{f^{-1}(A_\alpha); \alpha \in I_0\}$. Thus we have $1_{\sim y} = \bigcup\{A_\alpha; \alpha \in I_0\}$ and Y is IF midly compact.

Similarly, we can obtain the proof for (ii) and (iii).

Definition 6.7: An IFTS (X, τ) is said to be *intuitionistic fuzzy* ζ *-disconnected* (*IF* ζ *-disconnected*) if there exists $IF\zeta OS$ U,V in X such that $U \neq 0_\sim, V \neq 0_\sim$ such that $U \cup V = 1_\sim$ and $U \cap V = 0_\sim$. If X is not *IF* ζ *-disconnected* then it is said to *be intuitionistic fuzzy* ζ *-connected* (*IF* ζ *-connected*).

Theorem 6.8: Let $f : X \to Y$ be an *IF slightly* $\zeta -$*continuous , surjection,* (X, τ) is an *intuitionistic fuzzy* ζ *-connected*, then (Y, κ) is IF connected.

Proof: Assume that (Y, κ) is not IF connected then there exists non-empty intuitionistic fuzzy U and V in (Y, κ) such that $U \cup V = 1_\sim$ and $U \cap V = 0_\sim$. Therefore U and V are intuitionistic fuzzy ζ open sets in Y. Since *f* is *IF slightly* $\zeta -$*continuous,* $C = f^{-1}(A) \neq 0_\sim$, $D = f^{-1}(B) \neq 0_\sim$, which are $IF\zeta OS$ in X. And $f^{-1}(U) \cup f^{-1}(V) = f^{-1}(1_\sim) = 1_\sim$, which implies $C \cap D = 0_\sim$. Thus X is IF ζ -disconnected, which is a contradiction to our hypothesis. Hence Y is IF connected.

Remark 6.9: The following example shows that *IF slightly* $\zeta -$*continuous , surjection* do not necessarily preserve IF hyperconnectedness.

Example 7.0: : Let $X = \{a, b\}$, $Y = \{u, v\}$

$G_1 = \{\langle x, (0.7, 0.6), (0.3, 0.4)\rangle / x \in X\}$, $G_2 = \{\langle x, (0.1, 0.1), (0.9, 0.9)\rangle / x \in X\}$

$G_3 = \{\langle x, (0.9, 0.9), (0.1, 0.1)\rangle / x \in X\}$

Then $\tau = \{0_\sim, 1_\sim, G_1\}$ and $\kappa = \{0_\sim, 1_\sim, G_2, G_3, G_2 \cup G_3, G_2 \cap G_3\}$ are IFT on X and Y respectively.

Define a mapping $f : (X, \tau) \to (X, \kappa)$ by $f(a) = u$ and $f(b) = v$.

Then f is an *IF slightly* $\zeta -$*continuous* surjective. (X, τ) is hyperconnected. But (X, κ) is not hyperconnected.

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