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RESEARCH ARTICLE

ON QUASI NANO RGB-CLOSED MAPS

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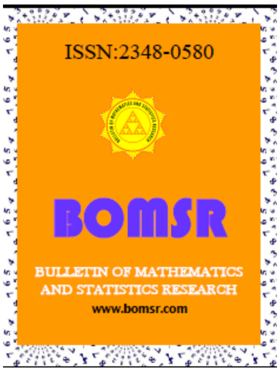

ABSTRACT

The aim of this paper is to explore a new type of maps called quasi nano rgb-closed maps in a nano topological spaces. We also investigate the properties of these maps and compare them with the notion of strongly nano rgb-closed maps.

Key Words: quasi nano rgb-open map, quasi nano rgb-closed map, strongly nano rgb-open map, strongly nano rgb-closed map.

1. INTRODUCTION

In recent years the weak forms of continuities called weak continuity, quasi continuity and almost continuity in topological spaces were studied by various authors [11, 12, 5]. Neubrunnovi [13] showed that semi continuity is equivalent to quasi continuity. In 1973, Popa and Stan [1] introduced weak quasi continuity. Weak quasi continuity is equivalent to weak semi continuity was proved by Arya and Bhamini [18]. And Noiri in [14, 15] investigated the fundamental properties of weakly quasi continuous functions and compared the interrelation among different continuous functions. In 2013 Lellis Thivagar introduced the concept of nano topology and discussed various weak forms of continuities in nano topological spaces.

The purpose of this paper is to obtain some characterizations of quasi nano rgb-open maps and quasi nano rgb-closed maps. We also investigate the relationships between such functions with stronger forms of nano rgb-closed maps.

2. Preliminaries

Definition 2.1[17]: Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$

1. The lower approximation of X with respect to R is the set of all objects, which can be for certainly classified as X with respect to R and it is denoted by $L_R(X)$. That is

$L_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x∈U.

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is

$U_R(X) = \bigcup_{x\in U}\{R(x) : R(x) \cap X \neq \phi\}$

3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is

$B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2[17]: If (U, R) is an approximation space and X, Y $\subseteq$ U, then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$

(ii) $L_R(\phi) = U_R(\phi) = \phi$ and $L_R(U) = U_R(U) = U$

(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$

(iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$

(v) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$

(vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$

(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$

(viii) $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$

(ix) $U_R U_R(X) = L_R U_R(X) = U_R(X)$

(x) $L_R L_R(X) = U_R L_R(X) = L_R(X)$

Definition 2.3[9]: Let U be non-empty, finite universe of objects and R be an equivalence relation on U. Let X$\subseteq$U. Let $\tau_R(X)$ = {U, ϕ, $L_R(X)$, $U_R(X)$, $B_R(X)$}. Then $\tau_R(X)$ is a topology on U, called as the nano topology with respect to X. Elements of the nano topology are known as the nano-open sets in U and $(U, \tau_R(X))$ is called the nano topological space. $[\tau_R(X)]^c$ is called as the dual nano topology of $\tau_R(X)$. Elements of $[\tau_R(X)]^c$ are called as nano closed sets.

Definition 2.4[9]: Let $(U, \tau_R(X))$ be a nano topological space and A$\subseteq$U. Then A is said to be

(i) nano semi-open if A$\subseteq$Ncl(Nint(A))

(ii) nano pre-open if A$\subseteq$Nint(Ncl(A))

(iii) nano α -open if A$\subseteq$Nint(Ncl(Nint(A)))

(iv) nano semi pre-open if A$\subseteq$Ncl(Nint(Ncl(A)))

(v) nano b-open if A$\subseteq$Ncl(NintA) $\cup$ Nint(NclA).

Definition 2.5[7]: A subset A of a nano topological space $(U, \tau_R(X))$ is said to be nano regular generalized closed sets (briefly Nrgb-closed) if Nbcl(A)$\subseteq$G whenever A$\subseteq$G and G is regular open in U.

Definition 2.6[7]: Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(X))$ be nano topological spaces. Then a mapping $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is said to be nano regular generalized b-continuous (nano rgb-continuous) if the inverse image of every closed set in V is nano rgb-closed in U.

Theorem 2.7[7]: A function $f : U \to V$ is nano rgb-irresolute if and only if for every nano rgb-open set F in V, $f^{-1}(F)$ is nano rgb-open in U.

On Quasi nano rgb-open maps

Definition 3.1: A function f: U $\rightarrow$V is said to be quasi nano rgb-open if the image of every nano rgb open set in U is nano open in V.

Example 3.2: Let U = {a, b, c, d} with U/R = {{a}, {b, d}, {c}} and X = {b, d}. Then the nano topology is defined as $\tau_R(X)$ = {U, ϕ, {b, d}} . Let V = {x, y, z, w} with V/R' = {{x}, {z}, {y, w}} and Y = {x, y}. Then $\tau_{R'}(Y)$ = {V, ϕ, {x}, {x, y, w}, {y, w}}. Define $f:(U,\tau_R(X))\rightarrow(V,\tau_{R'}(Y))$ to be a constant function f(a) = f(b)= f(c) = f(d) = x. Then f is quasi nano rgb-open map.

Theorem 3.3: A function f: U $\rightarrow$V is quasi nano rgb-open if and only if for every subset M of U, $f(\mathcal{N}rgb-\text{int}(M)) \subset \mathcal{N}\,\text{int}\, f(M)$.

Proof: Let f be a quasi nano rgb-open function. Now, we have Nint(M) $\subset$M and Nrgb-int(M) is nano rgb-open set. Hence we obtain that $f(\mathcal{N}rgb-\text{int}(M))\subset$f(M). As $f(\mathcal{N}rgb-\text{int}(M))$ is nano open, $f(\mathcal{N}rgb-\text{int}(M)) \subset \mathcal{N}\,\text{int}\, f(M)$.

Conversely, assume that M is a nano rgb-open set in U. Then f(M) = $f(\mathcal{N}rgb-\text{int}(M)) \subset$ $\mathcal{N}\,\text{int}(f(M))$ but $\mathcal{N}\,\text{int}(f(M))\subset$f(M). Consequently, f(M) = Nint(f(M)) and hence f is quasi nano rgb-open.

Theorem 3.4: If a function f: U$\rightarrow$V is quasi nano rgb-open, then $\mathcal{N}rgb-\text{int}(f^{-1}(G)) \subset$ $f^{-1}(\mathcal{N}\,\text{int}(G))$for every subset G of V.

Proof: Let G be any subset of V. Then, $\mathcal{N}rgb-\text{int}(f^{-1}(G))$ is a nano rgb-open set in U and f is quasi nano rgb-open, then $f(\mathcal{N}rgb-\text{int}(f^{-1}(G))) \subset \mathcal{N}\,\text{int}(f(f^{-1}(G))) \subset \text{N}\,\text{int}(G)$. Thus, $\mathcal{N}rgb-\text{int}(f^{-1}(G)) \subset f^{-1}(\mathcal{N}\,\text{int}(G))$.

Definition 3.5: A subset A is called a nano rgb-neighbourhood of a point u of U if there exists a nano rgb-open set M such that u$\in$M $\subset$A.

Theorem 3.6: For a function f: U$\rightarrow$V, the following are equivalent:

(a) f is quasi nano rgb-open.
(b) For each subset M of U, $f(\mathcal{N}rgb-\text{int}(M)) \subset \mathcal{N}\,\text{int}(f(M))$
(c) For each u$\in$U and each nano rgb-neighbourhood M of u in U, there exists a neighbourhood N of f(u) in V such that N$\subset$f(M).

Proof: (a) $\Rightarrow$(b): It follows from Theorem 3.3.

(b) $\Rightarrow$(c):Let u$\in$U and M be an arbitrary nano rgb-neighbourhood of u in U. Then there exists a nano rgb-open set N in U such that u$\in$N$\subset$M. Then by (b), we have f(N) = f(Nrgb-int(N)) $\subset$ Nint(f(N)) and hence f(N) = Nint(f(N)). Therefore, it follows that f(N) is nano open in V such that f(u) $\in$f(N) $\subset$f(M).

(c) $\Rightarrow$(a): Let M be an arbitrary nano rgb-open set in U. Then for each v $\in$f(M), by (c) there exists a neighbourhood N_v of v in V such that N_v $\subset$f(M). As N_v is a neighbourhood of v, there exists an nano open set W_v in V such that v$\in$ $W_v \subset N_v$. Thus, f(M) = $\bigcup\{W_v$:v$\in$f(M)} which is nano open set in V. This implies that f is quasi nano rgb-open function.

Theorem 3.7: A function f: U$\rightarrow$V is quasi nano rgb-open if and only if for any subset B of V and for any nano rgb-closed set F of U containing $f^{-1}(B)$, there exists a nano closed set G of V containing B such that $f^{-1}(G)\subset$F.

Proof: Suppose f is quasi nano rgb-open. Let B$\subset$V and F be a nano rgb-closed set of U containing $f^{-1}(B)$. Now, put G = V – f(U - F). It is clear that $f^{-1}(B) \subset$F implies B$\subset$G. Since f is quasi nano rgb-open, we obtain G as a nano closed set of V. Moreover, we have $f^{-1}(G) \subset$F.

Conversely, let M be a nano rgb-open set of U and put B = V \ f(M). Then U \ M is a nano rgb-closed set in U containing $f^{-1}(B)$. By assumption, there exists a nano closed set F of V such that B$\subset$F and $f^{-1}(F) \subset$U \ M. Hence, we obtain f(M) $\subset$V \ F. On the other hand, it follows that B$\subset$F, V \ F $\subset$V \ B = f(M). Thus, we obtain f(M) = V \ F which is nano open and hence f is quasi nano rgb-open function.

Theorem 3.8: A function f: U$\rightarrow$V is quasi nano rgb-open if and only if $f^{-1}(\mathcal{N}Cl(B) \subset \mathcal{N}rgb-Cl(f^{-1}(B))$ for every subset B of V.

Proof: Suppose that f is quasi nano rgb-open. For any subset B of V, $f^{-1}(B) \subset \mathcal{N}rgb-Cl(f^{-1}(B))$. Therefore by Theorem 3.7, there exists a nano closed set F in V such that B$\subset$F and $f^{-1}(F) \subset \mathcal{N}rgb-Cl(f^{-1}(B))$.

Therefore, we obtain $f^{-1}(\mathcal{N}Cl(B) \subset f^{-1}(F) \subset \mathcal{N}rgb-Cl(f^{-1}(B))$.

Conversely, let B$\subset$V and F be a nano rgb-closed set of U containing $f^{-1}(B)$. Put W =$\mathcal{N}Cl_V(B)$, then we have B$\subset$W and W is nano closed and $f^{-1}(W) \subset \mathcal{N}rgb-Cl(f^{-1}(B)) \subset$F. Then by Theorem 3.7, f is quasi nano rgb-open.

Theorem 3.9: f: U$\rightarrow$V and g: V$\rightarrow$W be two functions and $g \circ f$: U$\rightarrow$W is quasi nano rgb-open. If g is nano continuous, injective, then f is quasi nano rgb-open.

Proof: Let M be a nano rgb-open set in U, then $(g \circ f)(M)$ is nano open in W since $g \circ f$ is quasi nano rgb-open. Again g is injective nano continuous function, f(M) = $g^{-1}(g \circ f(M))$ is nano open in V. This shows that f is quasi nano rgb-open.

Definition 3.10: A function f: U$\rightarrow$V is called strongly nano rgb-open if the image of every nano rgb-open subset of U is nano rgb-open in V.

Example 3.11: Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X = {a, b}. Then the nano topology is defined as $\tau_R(X)$ = {U, ϕ, {a}, {b, d}, {a, b, d}} . Let V = {x, y, z, w} with V/R' = {{x}, {y, w}, {z}} and Y = {y, w}. Then $\tau_{R'}(Y)$ = {V,ϕ, {y, w}}. Define $f:(U,\tau_R(X)) \rightarrow (V,\tau_{R'}(Y))$ to be f(a) = x, f(b)= x, f(c) = y, f(d) = y. Then f is strongly nano rgb-open map.

Theorem 3.12: The composition of two strongly nano rgb-open maps is strongly nano rgb-open.

Proof: Let f: U$\rightarrow$V and g: V$\rightarrow$W be two strongly nano rgb-open maps. Let F be a nano rgb-open set in U. Since f is strongly nano rgb-open map, f(F) is nano rgb-open in V. Also since g is strongly nano rgb-open map, g(f(F)) = $(g \circ f)$(F) is nano rgb-open in W. Thus $g \circ f$: U$\rightarrow$W is strongly nano rgb-open.

Theorem 3.13: The composition of two quasi nano rgb-open maps is quasi nano rgb-open.

Proof: Let f: U$\rightarrow$V and g: V$\rightarrow$W be two quasi nano rgb-open maps. Let F be a nano rgb-open set in U. Since f is quasi nano rgb-open map, f(F) is nano open in V. Since every nano open set is nano rgb-open, f(F) is nano rgb-open in V. Also since g is quasi nano rgb-open map, g(f(F)) = $(g \circ f)$(F) is nano open in W. Thus $g \circ f$: U$\rightarrow$W is quasi nano rgb-open.

Theorem 3.14: If f: U$\rightarrow$V is nano rgb-open and g: V$\rightarrow$W is strongly nano rgb-open, then $g \circ f$: U $\rightarrow$W is nano rgb-open.

Proof: Let A be a nano open set in U. Since f is nano rgb-open, f(A) is nano rgb-open in V. Since g is strongly nano rgb-open, g(f(A)) is nano rgb-open in W. Therefore, g(f(A)) = $(g \circ f)$(A) is nano open in W. Hence $g \circ f$ is nano rgb-open.

Theorem 3.15: If f: U$\rightarrow$V is nano rgb-open and g: V$\rightarrow$W is quasi nano rgb-open, then $g \circ f$: U $\rightarrow$W is nano open.

Proof: Let A be a nano open set in U. Since f is nano rgb-open, f(A) is nano rgb-open in V. Since g is quasi nano rgb-open, g(f(A)) is nano open in W. Therefore, g(f(A)) = $(g \circ f)$(A) is nano open in W. Hence $g \circ f$ is nano open.

Theorem 3.16: If f: U$\rightarrow$V is strongly nano rgb-open and g: V$\rightarrow$W is quasi nano rgb-open, then $g \circ f$: U $\rightarrow$W is quasi nano rgb-open.

Proof: Let A be a nano rgb-open set in U. Since f is strongly nano rgb-open, f(A) is nano open in V. Also since g is quasi nano rgb-open, g(f(A)) is nano open in W. Therefore, g(f(A)) = $(g \circ f)$(A) is nano open in W. Hence $g \circ f$ is quasi nano rgb-open.

Theorem 3.17: If f: U$\rightarrow$V is quasi nano rgb-open and g: V$\rightarrow$W is nano open, then $g \circ f$: U $\rightarrow$W is strongly nano rgb-open.

Proof: Let A be a nano rgb-open set in U. Since f is quasi nano rgb-open, f(A) is nano open in V. Since g is nano rgb-open, g(f(A)) is nano rgb-open in W. Therefore, g(f(A)) = $(g \circ f)$(A) is nano rgb-open in W. Hence $g \circ f$ is strongly nano rgb-open.

Theorem 3.18: If f: U$\rightarrow$V is quasi nano rgb-open and g: V$\rightarrow$W is nano open, then $g \circ f$: U $\rightarrow$W is quasi nano rgb-open.

Proof: Let A be a nano rgb-open set in U. Since f is quasi nano rgb-open, f(A) is nano open in V. Since g is nano open, g(f(A)) is nano open in W. Therefore, g(f(A)) = $(g \circ f)$(A) is nano open in W. Hence $g \circ f$ is quasi nano rgb-open.

Theorem 3.19: For any bijection f: U$\rightarrow$V, the following are equivalent:

(i) f^{-1}: V$\rightarrow$U is nano rgb-irresolute

(ii) f is strongly nano rgb-open map

(iii) f is strongly nano rgb-closed map

Proof: (i) $\Rightarrow$ (ii): Let A be nano rgb-open set in U. By assumption we have, $(f^{-1})^{-1}(A)$ = f(A) is nano rgb-open in V. Hence f is strongly nano rgb-open.

(ii) $\Rightarrow$(iii): Let A be nano rgb-closed set in U. Then A^c is nano rgb-open in U. By (ii),f(A^c) is nano rgb-open in U. Therefore f(A^c) = $(f(A))^c$ is nano rgb-open in V. Thus, f(A) nano rgb-closed in V, which implies that f is strongly nano rgb-closed.

(iii) $\Rightarrow$(i): Let F be a nano rgb-closed set in U. By (iii), f(F) is nano rgb-closed in V. But f(F) = $(f^{-1})^{-1}(F)$ and therefore f^{-1} is nano rgb-irresolute.

4. Quasi nano rgb-closed maps

Definition 4.1: A function f: U$\rightarrow$V is said to be quasi nano rgb-closed if the image of each nano rgb-closed set in U is nano closed in V.

Theorem 4.2: Let $f:(U,\tau_R(X))\rightarrow(V,\tau_{R'}(Y))$ be a function. Then,

(i) Every quasi nano rgb-closed function is nano rgb-closed

(ii) Every quasi nano rgb-closed function is nano closed

(iii) Every quasi nano rgb-closed map is strongly nano rgb-closed

(iv) Every strongly nano rgb-closed map is nano rgb-closed.

Proof: (i) Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$be a quasi nano rgb-closed map. Let F be a nano closed set in$(U,\tau_R(X))$, which implies F is nano rgb-closed in$(U,\tau_R(X))$. Since f is quasi nano rgb-closed, f(F) is nano closed in $(V,\tau_{R'}(Y))$. Which implies f(F) is nano rgb-closed in $(V,\tau_{R'}(Y))$. Hence f is nano rgb-closed.

(ii) Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$be a quasi nano rgb-closed map. Let F be a nano closed set in $(U,\tau_R(X))$. Then F is nano rgb-closed in$(U,\tau_R(X))$. Since f is quasi nano rgb-closed, f(F) is nano closed in $(V,\tau_{R'}(Y))$, which implies f(F) is nano closed in $(V,\tau_{R'}(Y))$. Hence f is nano closed.

(iii) Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$be a quasi nano rgb-closed map. Let F be a nano rgb-closed set in$(U,\tau_R(X))$.Since f is quasi nano rgb-closed, f(F) is nano closed in $(V,\tau_{R'}(Y))$. Since every nano closed set is nano rgb-closed, f(F) is nano rgb-closed in $(V,\tau_{R'}(Y))$. Hence f is strongly nano rgb-closed.

(iv) Let $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$be a strongly nano rgb-closed map. Let F be a nano closed set in$(U,\tau_R(X))$. Which implies F is nano rgb-closed in$(U,\tau_R(X))$. Since f is strongly nano rgb-closed, f(F) is nano rgb-closed in $(V,\tau_{R'}(Y))$. Which implies f(F) is nano rgb-closed in $(V,\tau_{R'}(Y))$. Hence f is nano rgb-closed.

Remark 4.3: Converse of the above theorem need not be true as can be seen from the following examples.

Example 4.4: Let U = {x, y, z} with U/R = {{x}, {y, z}} and X = {x, z}. Then the nano topology is defined as $\tau_R(X)$= {U, ϕ, {x}, {y, z}} . Let V = {x, y, z, w} with V/R' = {{a}, {b, c}, {d}} and Y = {a, c}. Then $\tau_{R'}(Y)$ = {V,ϕ, {a}, {a, b, c}, {b, c}}. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ to be f(x) = b, f(y)= a, f(z) = c, f(w) = d. Then f is nano rgb-closed map but not quasi nano rgb-closed because f({y, z}) = {a, c} is not nano closed in V.

Example 4.5: Let U = V = {a, b, c, d} with U/R = {{a, d}, {b}, {c}} = $V/R'$and X = Y = {a, c}. Then $\tau_R(X)$= $\tau_{R'}(Y)$ = {V,ϕ, {c}, {a, c, d}, {a, d}}. Define $f:U\to V$ as f(a) = a, f(b) = d, f(c) = c and f(d) = d. Then the function f is nano closed but not quasi nano rgb-closed because f({a, c}) = {a, c} is not nano closed in V.

Example 4.6: Let U = {a, b, c, d} with U/R = {{a, d}, {b}, {b, c}} and X = {a, c}. Then the nano topology is defined as $\tau_R(X)$= {U, ϕ, {c}, {a, c, d}, {a, d}} . Let V = {x, y, z, w} with V/R' = {{x, z}, {y}, {w}} and Y = {x, w}. Then $\tau_{R'}(Y)$ = {V,ϕ, {w}, {x, z, w}, {x, z}}. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ to be f(a) = w, f(b)= x, f(c) = x, f(d) = y. Then f is strongly nano rgb-closed map but not quasi nano rgb-closed because f({a}) = {w} is not nano closed in V.

Example 4.7: Let U = {a, b, c, d} with U/R = {{a}, {b, d}, {c}} and X = {b, d}. Then the nano topology is defined as $\tau_R(X)$= {U, ϕ, {b, d}} . Let V = {x, y, z, w} with V/R' = {{x, w}, {y}, {z}} and Y = {x, z}. Then $\tau_{R'}(Y)$ = {V,ϕ, {z}, {x, z, w}, {x, w}}. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ to be f(a) = x, f(b)= y, f(c) = x, f(d) = y. Then f is nano rgb-closed map but not strongly nano rgb-closed because f({b}) = {z} is not nano rgb-closed in V.

Remark 4.8: The following examples show that strongly nano rgb-closed maps and nano closed maps are independent to each other.

Example 4.9: Let U = {a, b, c, d} with U/R = {{a, d}, {b}, {b, c}} and X = {a, c}. Then the nano topology is defined as $\tau_R(X)$= {U, ϕ, {c}, {a, c, d}, {a, d}} . Let V = {x, y, z, w} with V/R' = {{x}, {y, w}, {z}} and Y

= {y, w}. Then $\tau_{R'}(Y)$ = {V, ϕ, {y, w}}. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ to be f(a) = w, f(b)= x, f(c) = x, f(d) = y. Then f is strongly nano rgb-closed map but not nano closed because f({a, b, d}) = {w, x, y} is not nano closed in V.

Example 4.10: Let U = {a, b, c, d} with U/R = {{a, c}, {b}, {d}} and X = {a, d}. Then the nano topology is defined as $\tau_R(X)$ = {U, ϕ, {d}, {a, c, d}, {a, c}} . Let V = {x, y, z, w} with V/R' = {{x, w}, {y}, {z}} and Y = {x, z}. Then $\tau_{R'}(Y)$ = {V, ϕ, {z}, {x, z, w}, {x, w}}. Define $f:(U,\tau_R(X))\to(V,\tau_{R'}(Y))$ to be f(a) = x, f(b)= y, f(c) = w, f(d) = z. Then f is nano closed map but not strongly nano rgb-closed because f({d}) = {z} is not nano rgb-closed in V.

Theorem 4.11: If a function f: U$\to$V is quasi nano rgb-closed, then $f^{-1}(\mathcal{N}\operatorname{int}(B))\subset \mathcal{N}rgb-\operatorname{int}(f^{-1}(B))$ for every subset B of V.

Proof is similar to Theorem 3.4.

Theorem 4.12: A function f: U$\to$V is quasi nano rgb-closed if and only if for any subset B of V and for any nano rgb-open set G of U containing $f^{-1}(B)$, there exists a nano open set N of V containing B such that $f^{-1}(N)\subset$G.

Proof: Suppose f is quasi nano rgb-closed. Let B$\subset$V and G be nano rgb-open set of U containing $f^{-1}(B)$. Now put N = $(f(U^c))^c$. Then N is nano open set of V containing B such that $f^{-1}(N)\subset$G. Conversely, let F be a nano rgb-closed set of U and put B = $(f(F))^c$. Then we have $f^{-1}(B)\subset F^c$ and F^c is nano rgb-open set in U containing $f^{-1}(B)$. By assumption, there exists a nano open set N of V containing B and $f^{-1}(N)\subset F^c$. And so F$\subset$ $(f^{-1}(N))^c = f^{-1}(N^c)$. Hence we obtain f(F) $\subset N^c$. Since N^c is nano closed, f(F) is nano closed in V. This implies that f is quasi nano rgb-closed.

Theorem 4.13: A function f: U $\to$V is quasi nano rgb-closed if and only if for every subset M of U, $\mathcal{N}Clf(M)\subset f(\mathcal{N}rgb-Cl(M))$.

Proof: Let f be a quasi nano rgb-closed. Now, we have M $\subset$Nrgb-Cl(M) and Nrgb-Cl(M) is nano rgb-closed set. Hence we obtain that f(M) $\subset f(\mathcal{N}rgb-Cl(M))$. As $f(\mathcal{N}rgb-Cl(M))$ is nano closed, $\mathcal{N}Clf(M)\subset f(\mathcal{N}rgb-Cl(M))$

Conversely, assume that M is a nano rgb-closed set in U. Then $\mathcal{N}Clf(M)\subset f(\mathcal{N}rgb-Cl(M))$ = f(M). Consequently, f(M) = NCl(f(M)). Hence f(M) is nano closed. Hence f is quasi nano rgb-closed.

Definition 4.14: A function f: U$\to$V is called strongly nano rgb-closed if the image of every nano rgb-closed subset of U is nano rgb-closed in V.

Theorem 4.15: A function f: U $\to$V is strongly nano rgb-closed if and only if for every subset M of U, $\mathcal{N}rgb-Clf(M)\subset f(\mathcal{N}rgb-Cl(M))$.

Proof: Let f be a strongly nano rgb-closed. Now, we have M $\subset$Nrgb-Cl(M) and Nrgb-Cl(M) is nano rgb-closed set. Hence we obtain that f(M) $\subset f(\mathcal{N}rgb-Cl(M))$. As $f(\mathcal{N}rgb-Cl(M))$ is nano closed, $\mathcal{N}rgb-Clf(M)\subset f(\mathcal{N}rgb-Cl(M))$

Conversely, assume that M is a nano rgb-closed set in U. Then $\mathcal{N}rgb-Clf(M)\subset f(\mathcal{N}rgb-Cl(M))$ = f(M). Consequently, f(M) = NCl(f(M)). Hence f(M) is nano rgb-closed. Hence f is strongly nano rgb-closed.

Theorem 4.16: The composition of two strongly nano rgb-closed maps are strongly nano rgb-closed.

Proof: Let f: U$\to$V and g: V$\to$W be any two strongly nano rgb-closed maps. Let F be a nano rgb-closed set in U. Since f is strongly nano rgb-closed map, f(F) is nano rgb-closed in V. Also since g is strongly nano rgb-closed map, g(f(F)) = $(g \circ f)(F)$ is nano rgb-closed in W. Thus $g \circ f$:U$\rightarrow$W is strongly nano rgb-closed.

Theorem 4.17: If f: U$\rightarrow$V and g: V$\rightarrow$W be any two functions. Then,

(i) If f is nano rgb-closed and g is quasi nano rgb-closed, then $g \circ f$ is nano closed.

(ii) If f is quasi nano rgb-closed and g is nano rgb-closed, then $g \circ f$ is strongly nano rgb-closed.

(iii) If f is strongly nano rgb-closed and g is quasi nano rgb-closed, then $g \circ f$ is quasi nano rgb-closed.

Proof is obvious.

Theorem 4.18: If f: U$\rightarrow$V and g: V$\rightarrow$W be any two functions such that $g \circ f$: U$\rightarrow$W is strongly nano rgb-closed, then

(i) f is nano rgb-irresolute surjection implies that g is strongly nano rgb-closed.

(ii) If g is nano rgb-irresolute injective, then f is strongly nano rgb-closed.

Proof: (i)Suppose that F is nano rgb-closed set in V. As f is nano rgb-irresolute, $f^{-1}(F)$ is nano rgb-closed in U. Since $g \circ f$ is strongly nano rgb-closed and f is surjective, $(g \circ f(f^{-1}(F))$ = g(F), which is nano rgb-closed in W. Thus g is strongly nano rgb-closed function.

(ii) Suppose F is any nano rgb-closed set in U. Since $g \circ f$ is strongly nano rgb-closed, $(g \circ f)(F)$ is nano rgb-closed in W. Again g is nano rgb-irresolute injective function, $g^{-1}(g \circ f)(F))$ = f(F), which is nano rgb-closed in V. This shows that f is strongly nano rgb-closed map.

Theorem 4.19: If f: U$\rightarrow$V and g: V$\rightarrow$W be any two functions such that $g \circ f$: U$\rightarrow$W is quasi nano rgb-closed.

(iii) If f is nano rgb-irresolute surjection, then g is nano closed.

(iv) If g is quasi nano rgb-continuous injective, then f is strongly nano rgb-closed.

Proof: (i)Suppose that F is an arbitrary nano closed set in V. As f is nano rgb-irresolute, $f^{-1}(F)$ is nano rgb-closed in U. Since $g \circ f$ is quasi nano rgb-closed and f is surjective, $(g \circ f(f^{-1}(F))$ = g(F), which is nano closed in W. This implies that g is nano closed function.

(ii) Suppose F is any nano rgb-closed set in U. Since $g \circ f$ is quasi nano rgb-closed, $(g \circ f)(F)$ is nano closed in W. Again g is nano rgb-continuous injective function, $g^{-1}(g \circ f)(F))$ = f(F), which is nano rgb-closed in V. This shows that f is strongly nano rgb-closed.

Theorem 4.20: Let U and V be nano topological spaces. Then the function f: U$\rightarrow$V is a quasi nano rgb-closed if and only if f(U) is nano closed in V and f(N) \ f(U \ N) is nano open in f(U) whenever N is nano rgb-open in U.

Proof: Necessity: Suppose f: U$\rightarrow$V is quasi nano rgb-closed function. Since U is nano rgb-closed, f(U) is nano closed in V and f(N) \ f(U \ N) = f(N) $\cap$ f(U) \ f(U \ N) is nano open in f(U) when N is nano rgb-open in U.

Sufficiency: Suppose f(U) is nano closed in V, f(N)\ f(U \ N) is nano open in f(U) when N is nano rgb-open in U, and P be nano closed in U. Then f(P) = f(U) \ (f(U \ P) \ f(P)) is nano closed in f(U) and hence nano closed in V.

Corollary 4.21: Let U and V be nano topological spaces. Then a surjective function f: U$\rightarrow$V is quasi nano rgb-closed if and only if f(N) \ f(U \ N)is nano open in V whenever M is nano rgb-open in U.

Proof is obvious.

Corollary 4.22: Let U and V be nano topological spaces and let f: U $\rightarrow$ V be a nano rgb-continuous quasi nano rgb-closed surjective function. Then the topology on V is {f(N) \ f(U\N) : N is nano rgb-open in U}.

Proof: Let A be nano open in V. Then $f^{-1}(A)$ is nano rgb-open in U, and $f(f^{-1}(A) \setminus f(U \setminus f^{-1}(A))$ = A. Hence all nano open sets in V are of the form f(N) \ f(U \ N), N is nano rgb-open in U. On the other hand, all the sets of the form f(N) \ f(U \ N), N is nano rgb-open in U, are nano open in V from the corollary 4.21.

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