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RESEARCH ARTICLE

COST-PROFIT OPTIMAL JOINT POLICY FOR ORDERING AND PRICING FOR A CLASS OF DECAYING INVENTORY SYSTEMS

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ABSTRACT

This paper deals with an optimal inventory systems wherein profit maximization as well as, total cost minimization is simultaneously carried out under conditions of exponential decay patterns of the inventory holding. These patterns are studied under different situations of Constancy, Linearity and Quadratic forms. Usual assumptions are made about instant replenishment, time horizon being infinite with shortages being allowed and so on. All the theoretical work and results are extensively supported by empirical work as a comparison study to bring out the qualitative aspects.

Keywords: Optimal Price; Optimal cycle length; Optimal Order rate; optimal cost; Optimal Profit.

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1. INTRODUCTION

In recent times, we consider the class of Inventory systems: (a) provided with shortages and (b) suffering decay with some general, say, quadratic rate. First, we observe that inventory models allowing for shortages are: (i) more pragmatic in character. For, in most physical conditions of marketing, there invariably exists some time lapse before replenishment through fresh inventory. Further, (ii) these systems are more generally, in as much as the results for systems with 'no shortages' case can be recovered from systems 'with shortages' by rendering the time interval degenerate, corresponding to the "excess demand situation". In the following, we therefore devote our attention to inventory systems provided with 'shortages'. Nextly, the inventory modeling formulated by us relates to perishable commodities, like, for example, food grains, blood-stocks and so on where it is assumed that the deterioration takes place exponentially. In fact, Cohen [1] initiated some work in this direction, while Mukherjee [2] and Sumalatha [3] further added some contributions in [1,2].

However, we observe that still a good scope exists for further work in this line. Motivated thus, we obtained results in this direction and report these in the following.

The analytic results are obtained using an approximation technique. The practical use of the results is illustrated with empirical work, carried out in some detail. Specifically, we considered inventory models for item that perish exponentially with decay rates that are: (a) constant, that is, time independent (b) linear and the dependent and (c) quadratic and time dependent. Some demand and pricing patterns are assumed. We then developed cost-profit optimal joint policy for ordering and pricing. That is, analytic expressions for optimal order rates and optimal cycle lengths are obtained.

Discussion leading to operational policy for inventory management is also given in the end. First we give the notation and nomenclature.

2. NOTATION AND NOMENCLATURE

Following Cohen [1], the notation and nomenclature is adopted.

P	Selling price
d (p)	Known demand rate
I (t)	Inventory storage at time ‘ t ’
λ	Stock decay rate (constant, time-independent)
λ (t)	Time dependent decay rate
Z (t)	Stock loss due to decay in the closed time interval [0,t]
T	Cycle length, i.e., I (NT-) = 0 and I (NT+) = Q_T , N = 0,1,2,3….
Q	Order quantity in [0,T]
η	Fraction in [0,T], when there is no excess demand, i.e I(N ηT+) = 0, N=0,1,2,3….
Z(T)	Stock loss due to decay in closed [0,T]
K	Order cost
h	Holding cost per unit
C_1	Purchase cost per unit
S	Shortage cost rate per unit thus in the no shortage case, S = + ∞
T^*	Optimal cycle length (or order interval)
p^*	Optimal price
Q_0	Optimal order rate = Q/T*
C	$\overline{C}$ (T, T_1, p), total cost
C^*	C (T^*, η , p^*), optimal cost (per unit time)
Π^*	π (T^*, η , p^*), optimal profit.

In consistent with most physical situations, we stipulate that $0 < \lambda, \lambda(t) << 1$.

We now explain the model and present our results.

3. THE MODEL AND RESULTS

As indicated earlier, we consider the inventory model for items that exponentially perish with progress of time. Shortages are allowed. The inventory replenishment is assumed to be instantaneously. First the case of constant decay rate is dealt with.

The basic purpose is to obtain analytic expressions for T^*, Q_0, and P^* by minimizing cost and or profit functions. An important aspect of our work consists in deriving joint optimal pairs (T^* , p^*), employing algorithms suitably developed by us.

3a. Constant Decay Rate

Allowing for shortages implies backlogging of excess demand in (T_1 , T), where T_1 = η T.

Following Cohen [1], we have:

The differential equation explaining the time behavior of the inventory system, described above is given by:

$\frac{d\,I(t)}{d\,t}$ = - λ I (t) – d (p) (1)

Using known results from the theory of differential equations, the solution to (1) is given by:

I (t) = I (o) $e^{-\lambda t}$ - d (p) [1- $e^{-\lambda t}$] / λ (2)

The stock lost due to decay in [0, T] is

Z (T_1) = the difference in the stock with no decay to that when decay exists.

Z (T_1) = I (o) – d (p) T_1 – I (T_1)

= d (p) T_1 + d (p) [$e^{\lambda T1}$ - 1] / λ (3)

Now the order quantity comprises of the sum of demands met and back-logged and stocks that decayed.

Therefore, Q_0 = Z(T_1) + T d(p) (4)

The total cost equation for $\overline{C}$ (T, T_1, p) I given by:

C= $\overline{C}$ (T, T_1, p) = K + C_1 Q + $\int_0^{T1} I\,(t)dt$ + S d(p) + $\int_0^{(T-T1)} t\,dt$ (5)

Using (2), (3) and (4) in (5) we obtain after some calculations:

$\overline{C}$ (T, T_1, p) / T = $\frac{K}{T}$ + C_1 d (p) { $\frac{T-T1}{T}$+ $\frac{e^{\lambda T1}-1}{\lambda T}$ }+ h d(p) { $e^{\lambda T1} - 1 - \lambda T_1$ } / λ^2T + S d(p) $(T-T_1)^2$ / 2T (6)

Truncating the Taylor's expansion for $e^{\lambda T1}$ to second term (that is , ignoring terms with λ^j , j ≥ 3) in (6) and recasting (6) in terms of η, we obtain, after some calculations:

C (T, η, p) = C_1 d(p) + K / T + T d (p) [λ C_1 η^2 + h η^2 + s $(1- \eta)^2$] / 2 (7)

T^* is now obtained by minimizing C (T, η, p), with respect to T, that is, using the two conditions:

$\left.\frac{\partial C\,(T,\eta,p)}{\partial T}\right|_{T=T^*}$ = 0 and (7a)

$\left.\frac{\partial^2 C\,(T,\eta,p)}{\partial T^2}\right|_{T=T^*}$ > 0 (7b)

We notice that the later condition (of positivity) is really satisfied. Thus we obtain :

T^* = [2K / d (p) (C_1λ η^2 + h η^2 + s $(1- \eta)^2$) $]^{1/2}$ (8)

Notice that, in the case with 'no shortages', that is when η = 1 (8) reduces to :

T^* = [2K / d (p) [λ C_1 + h] $]^{½}$

Agreeing with an earlier result (see also [1]). Now p^* is obtained by maximizing the profit function π (T, η , p), with respect to ' p ' , where

π (T, η , p) = p d(p) - C (T, η , p) (9)

Proceeding on similar lines as above (see also [1]), we obtain

p^* = C_1(1+ λ η^2 T / 2) + h η^2 T / 2 + s $(1- \eta)^2$ T / 2 – d(p) / d' (p) ……..(10)

The maximum of π (T, η , p) with respect to p, is obtained through the conditions

$\left.\frac{d\,\pi\,(T,\eta,p)}{d\,T}\right|_{p=p^*}$ = 0 and (10a)

$\left.\frac{d^2\pi\,(T,\eta,p)}{d\,T^2}\right|_{p=p^*}$ > 0 (10b)

Now, (8) and (10) can be solved simultaneously for obtaining the joint optimal (T^* , p^*). Also the optimal orders quantity Q_0 can be obtained as

Q_0 = d (p) [1+ λ η^2 T^* / 2] (11)

We now propose the following algorithm and hence obtain the optimal policy jointly for order cycle and pricing, namely (T^* , p^*).

STEP I : Use (8) to obtain T^*, for some assumed 'p' in d(p).

STEP II : Use T^* obtained in step I, in (10) and obtain the p^*, yielding the pair (T^*, p^*).

STEP III: Continue the procedure in the above two steps till we get the same pair (T^*, p^*), that is finally stabilizing.

Even though no theoretical justification is established assuring a guaranteed termination (convergence) of the algorithmic procedure, we remark that in practice, final results (in step III) are usually obtained fairly fast after a few iterative steps are performed.

Using (6) and (9) we obtain the optimal cost (C^*) and optimal profit (π^*). Numerical results are given in Table 1, Table 2, based on computer programmes.

3B. LINEAR AND TIME-DEPENDENT DECAY RATE

We now add to the results un earlier literature by considering linear decay rate λ (t). We recall that, Cohen [1] considered only constant decay rate while Mukherjee [2] had not investigated the case with shortages being allowed.

We stipulate: $\lambda(t) = a + bt,\ 0 < a,\ b << 1$ (12)

For the time dependent decay rate, λ (t), we have the following versions of the results given in section 3a.

$$Z(T_1) = d(p)\,T_1 + I(T_1)\,[\,\exp(\int_0^{T1}\lambda(x)dx - 1] + d(p)\int_0^{T1}\exp(\int_0^{t}\lambda(x)dx)\,dy \quad (13)$$

Inserting (12) for λ (t) in (13) and using Taylor's approximation for exp ($\int_0^{T1}\lambda(x)dx$) upto the first term in T_1 (justified, as 0 < a, b<<1), we obtain after calculations the following:

$$Q_T = Z(T_1) + d(p)\,T$$

$$= (T-T_1)\,d(p) + d(p)\int_0^{T1}\exp(\int_0^{t}(a+bx)dx)\,dy$$

$$= d(p)\,\{(T-T_1) + T_1 + \frac{aT1^2}{2} + \frac{bT1^3}{6}\} \quad (14)$$

$$I(t) = d(p)\exp(-\int_0^{t}(a+bx)dx)\,(\int_t^{T1}\exp(\int_0^{y}(a+bx)dx)\,dy)$$

$$\int_0^{T1} I(t)dt = d(p)\,\{\frac{T1^2}{2} + \frac{aT1^3}{6} - \frac{a^2T1^4}{8} - \frac{abT1^5}{12} + \frac{bT1^4}{12} - \frac{b^2T1^6}{72}\} \quad (15)$$

Now, $C(T, T_1, p) = \bar{C}(T, T_1, p) / T$

$$= \frac{K}{T} + \frac{C1\,d(p)}{T}\{(T-T_1) + T_1 + \frac{aT1^2}{2} + \frac{bT1^3}{6}\} + \frac{h\,d(p)}{T}\{\frac{T1^2}{2} + \frac{aT1^3}{6} - \frac{a^2T1^4}{8} - \frac{abT1^5}{12} + \frac{bT1^4}{12} - \frac{b^2T1^6}{72}\} + \frac{s\,d(p)}{2T}(T-T_1)^2 \quad (16)$$

Casting (16) in terms of η, we have

$$C(T, \eta, p) = \frac{K}{T} + C_1\,d(p)\,\{(1-\eta) + (\eta + \frac{a\,\eta^2 T}{2} + \frac{b\,\eta^3T^2}{6}\} + h\,d(p)\,\{\frac{\eta^2 T}{2} + \frac{a\eta^3T^2}{6} + \frac{b\eta^4T^3}{12} - \frac{a^2\eta^4T^3}{8} - \frac{ab\,\eta^5T^4}{12} - \frac{b^2\eta^6T^5}{72}\} + s\,d(p)\,T(1-\eta)^2/2 \quad (17)$$

Now we have:

$$\Pi(T, \eta, p) = pd(p) - C(T, \eta, p) \quad (18)$$

The optimal: T^* and p^* are obtained by minimizing C (T, η, p) with respect to T and maximizing π(T, η, p) with respect to 'p'.

From (17) we have

$$\left.\frac{d\,C(T,\eta,p)}{d\,T}\right|_{T=T^*} = -K + [\frac{C1\,d(p)a\,\eta^2}{2} + \frac{h\,d(p)\,\eta^2}{2}]\,T^2 + [\frac{C1\,d(p)b\,\eta^3}{3} + \frac{h\,d(p)a\,\eta^3}{3}]\,T^3 + [\frac{h\,d(p)b\,\eta^4}{4} - \frac{3hd(p)\,a^2\eta^4}{8}]\,T^4 - [\frac{h\,d(p)ab\,\eta^5}{3}]\,T^5 - [\frac{5hd(p)\,b^2\eta^6}{72}]\,T^6 + [sd(p)(1-\eta)^2/2]T^2 = 0 \quad (19)$$

From (18) we obtain

$$\left.\frac{d\,\pi(T,\eta,p)}{d\,p}\right|_{p=p^*} = \frac{-d(p)}{d'(p)} + C_1\{1 + \frac{a\,\eta^2 T}{2} + \frac{b\,\eta^3T^2}{6}\} + h\,\{\frac{\eta^2 T}{2} + \frac{a\,\eta^3T^2}{6} + \frac{b\,\eta^4T^3}{12} - \frac{a^2\eta^4T^3}{8} - \frac{ab\,\eta^5T^4}{12} - \frac{b^2\eta^6T^5}{72}\} + ST(1-n)^2/2 = 0$$

If d(p) =X+YP; X>0 and Y<0, the above equation becomes:

$$\left.\frac{d\,\pi(T,\eta,p)}{d\,p}\right|_{p=p^*} = -\frac{-X}{Y} + C_1\{1 + \frac{a\,\eta^2 T}{2} + \frac{b\,\eta^3T^2}{6}\} + h\,\{\frac{\eta^2 T}{2} + \frac{a\,\eta^3T^2}{6} + \frac{b\,\eta^4T^3}{12} - \frac{a^2\eta^4T^3}{8} - \frac{ab\,\eta^5T^4}{12} - \frac{b^2\eta^6T^5}{72}\} + ST(1-n)^2/2 = 0 \quad (20)$$

From (19) and (20), we obtain optimal values T^* and p^*. For this T^* satisfies the conditions (7a) and (7b) and also for the p^*, the conditions (10a) and (10b) should be satisfied. Now (19) and (20) can be solved simultaneously for obtaining the joint optimal values of ordering interval and price, that is, (T^*, p^*). Also using (14), (17) and (18) we respectively obtain the optimal quantity (Q_0), optimal cost (C^*) and optimal profit (π^*). Numerical results are given in Table 3 and Table 4 based on computer programmes.

As before, the algorithm suggested in the earlier section will lead to the optimal pair (T^*, p^*). Further, for given price-pattern, we obtain T^* and the other characteristic of the system, namely, Q_0.

3C. QUADRATIC AND TIME DEPENDENT DECAY RATE

We now consider the decay rate as:

$\lambda(t) = a+bt+ct^2$, o<a, b, c <<1 (21)

we observe that it is worthwhile to consider decay rates even upto second degree (quadratic), as certain food commodities (like certain type of special fruits like mangoes, bananas, and so on) decay not only exponentially but also at a faster rate than just linear rate. Such considered as these, motivated us to investigate this situation in detail. Further, we observe that decay rates: constant and linear or not just special cases of (21) in terms of respectively obtaining results by setting b=c=0 and b ≠ 0 but c=0 (this is so because of the truncated Taylor's expansion, yielding different results).

We now give the theoretical results. Substituting (21) in (16) and using Taylor's series expansion for exponential function (truncated upto the second term), we get

$$C(T, T_1, p) = \frac{K}{T} + \frac{C1\, d(p)}{T}\left[\int_0^{T1}\left(1+\int_0^t (a+bx+cx^2)\,dx\right)dt\right] + \frac{h\, d(p)}{T}\left[\int_0^{T1}\left(1-\int_0^t (a+bx+cx^2)\,dx\int_t^{T1}\left(1+\int_0^y (a+bx+cx^2)\,dx\right)dy\right]dt + \frac{s\, d(p)}{2T}(T-T_1)^2$$

After some calculations, we have

$$C^* = \bar{C}(T, \eta, p) = \frac{K}{T} + C_1 d(p)\left[1+\frac{a\,\eta^2 T}{2} + \frac{b\,\eta^3 T^2}{6} + \frac{c\,\eta^4 T^3}{12}\right. + h\, d(p)\left[\frac{\eta^2 T}{2} + \frac{a\,\eta^3 T^2}{6} + \left(\frac{b}{12} - \frac{a^2}{8}\right)\eta^4 T^3 + \left(\frac{c}{20} - \frac{ab}{12}\right)\eta^5 T^4 + \left(\frac{-ac}{24} - \frac{b^2}{72}\right)\eta^6 T^5 - \frac{bc\,\eta^7 T^6}{72} - \frac{c^2\eta^8 T^7}{288}\right] + s\, d(p)\left[\frac{T}{2} + \frac{\eta^2 T}{2} - \eta T\right] \quad \ldots\ldots (22)$$

Minimizing (22) with respect to T, we obtain optimal interval, T^*

$$C^* = \bar{C}(T, \eta, p) = \frac{K}{T} + C_1 d(p)\left[1+\frac{a\,\eta^2 T}{2} + \frac{b\,\eta^3 T^2}{6} + \frac{c\,\eta^4 T^3}{12}\right. + h\, d(p)\left[\frac{\eta^2 T}{2} + \frac{a\,\eta^3 T^2}{6} + \left(\frac{b}{12} - \frac{a^2}{8}\right)\eta^4 T^3 + \left(\frac{c}{20} - \frac{ab}{12}\right)\eta^5 T^4 + \left(\frac{-ac}{24} - \frac{b^2}{72}\right)\eta^6 T^5 - \frac{bc\,\eta^7 T^6}{72} - \frac{c^2\eta^8 T^7}{288}\right] + s\, d(p)\left[\frac{T}{2} + \frac{\eta^2 T}{2} - \eta T\right] \quad (22)$$

Minimizing (22) with respect to T, we obtain optimal interval, T^*

$$\left.\frac{\partial C(T,\eta,p)}{\partial T}\right|_{T=T^*} = -K + C_1 d(p)\left[\frac{a\,\eta^2}{2} + \frac{b\,\eta^3 T}{3} + \frac{c\,\eta^4 T^2}{4}\right]T^2 + h\, d(p)\left[\frac{\eta^2}{2} + \frac{a\,\eta^3 T}{3} + 3\left(\frac{b}{12} - \frac{a^2}{8}\right)\eta^4 T^2 + 4\left(\frac{c}{20} - \frac{ab}{12}\right)\eta^5 T^3 + 5\left(\frac{-ac}{24} - \frac{b^2}{72}\right)\eta^6 T^4 - \frac{bc\,\eta^7 T^5}{12} - \frac{7c^2\eta^8 T^6}{288}\right]T^2 + s\, d(p)\left[\frac{1}{2} + \frac{\eta^2}{2} - \eta\right]T^2 \quad \ldots\ldots (23)$$

$$= 0$$

Now T* is obtained from (23) which must also satisfy (7a) and (7b).

Again maximizing the profit function $\pi(T, \eta, p)$ with respect to p, we get

$$\frac{d\pi(T,\eta,p)}{dp}\Big|_{p=p^*} = \frac{-d(p)}{d'(p)} + C_1\left[1+\frac{a\,\eta^2 T}{2}+\frac{b\,\eta^3 T^2}{6}+\frac{c\,\eta^4 T^3}{12}\right] + h\left[\frac{\eta^2 T}{2}+\frac{a\,\eta^3 T^2}{6}+\left(\frac{b}{12}-\frac{a^2}{8}\right)\eta^4 T^3 + \left(\frac{c}{20}-\frac{ab}{12}\right)\eta^5 T^4+\left(\frac{-ac}{24}-\frac{b^2}{72}\right)\eta^6 T^5-\frac{bc\,\eta^7 T^6}{72}-\frac{c^2\eta^8 T^7}{288}\right] + s\left[\frac{T}{2}+\frac{\eta^2 T}{2}-\eta T\right] \quad \ldots(24)$$
$$= 0$$

Setting d(p) = X+Yp, Y<0 in (24), we obtain

$$\frac{d\pi(T,\eta,p)}{dp}\Big|_{p=p^*} = \frac{-X}{Y} + C_1\left[1+\frac{a\,\eta^2 T}{2}+\frac{b\,\eta^3 T^2}{6}+\frac{c\,\eta^4 T^3}{12}\right] + h\left[\frac{\eta^2 T}{2}+\frac{a\,\eta^3 T^2}{6}+\left(\frac{b}{12}-\frac{a^2}{8}\right)\eta^4 T^3 + \left(\frac{c}{20}-\frac{ab}{12}\right)\eta^5 T^4+\left(\frac{-ac}{24}-\frac{b^2}{72}\right)\eta^6 T^5-\frac{bc\,\eta^7 T^6}{72}-\frac{c^2\eta^8 T^7}{288}\right] + s\left[\frac{T}{2}+\frac{\eta^2 T}{2}-\eta T\right] \ldots. (25)$$
$$= 0$$

Now P^* is obtained from (25) which must also satisfy (10a) and (10b).

Now the maximum profit is given by

$$\pi^* = \pi(T, \eta, p) - C(T, \eta, p)$$
$$= p^* d(p) - C^* \quad (26)$$

The joint optimal pair (p^*, T^*) can be obtained from (23) and (25). Further Q_0 is given by

$$Q_0 = d(p)\left[1+\frac{a\,\eta^2 T}{2}+\frac{b\,\eta^3 T^2}{6}+\frac{c\,\eta^4 T^3}{12}\right] \quad (27)$$

Finally, optimal cost, C^* and optimal profit π^* are obtained using (22) and (26). Further, from (27) we can obtain the optimal order rate. Illustrate empirical work is presented in the tables 6 and 7 using computer programmes.

In the following section, we present some numerical work.

4. NUMERICAL WORK

We choose the following parametric values K=Rs.250; C_1 =Rs.1/unit; h=Rs. 0.5/unit; d(p)= X- Yp; s = 0.7.

Table 1: OPTIMAL VALUES; constant decay rate

K=250; C_1=1; h=0.5; X=25; Y=- 0.5: s=0.7; $\eta = 0.5$

	Optimal Price (p^*)	Optimal interval (T^*)	Optimal order rate (Q_0)	Optimal Cost (C^*)	Optimal Profit (π^*)
0.05	25.89	11.52	12.92	55.45	256.65
0.10	26.44	11.43	13.46	55.54	255.93
0.15	26.46	11.22	14.24	56.33	255.10
0.20	26.48	11.02	14.99	57.12	254.28
0.25	26.50	10.83	15.73	57.90	253.48
0.30	26.52	10.66	16.43	58.66	252.69
0.35	26.55	10.49	17.12	59.41	251.91
0.40	26.55	10.32	17.78	60.15	251.15
0.45	26.56	10.17	18.42	60.88	250.40
0.50	26.58	10.02	19.05	61.59	249.66

5. DISCUSSION

A perusal of the expression for T^* in (8) and p^* in (10) reveals that theoretically as λ increases, T^* decreases while p^* increases and this is also observed in the computed values in table 1. However, the knowledge of the extent of increase (or decrease) is important in physical situations for policy and management of inventory. Empirical exercises such as are indicated in the above bring to view these operational aspects. Using (11), we compute Q_0 while C^* and π^* are obtained on the basis of T^*

and p^* using (7) and (9). From the tabulated values, we also notice that as increases, Q_0 and C^* increases while it is otherwise for π^*, that is profits drop down, which is consistent with experience.

However, if price variable is not controllable or it is prefixed or predetermined, we can only obtain optimal order cycle T^*, using (8) for various and given 'p'. it will be useful to observe the extent of increases or decreases in (T^*, Q_0)'s for increases or decreases in (T^*, p^*)'s for operational purposes.

In the following we report further empirical work

TABLE 2: OPTIMAL VALUES: CONSTANT DECAY RATE ('p' fixed) (T^*)

K=250; C_1=1; h=0.5; X=25; Y=- 0.5: s=0.7; $\eta = 0.5$

P	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.0	8.00 (26.25)	7.84 (27.45)	7.70 (28.61)	7.56 (29.72)	7.43 (30.82)	7.30 (31.85)	7.18 (32.86)	7.07 (33.84)	6.96 (34.79)	6.86 (35.72)
1.0	8.08 (25.74)	7.92 (26.93)	7.78 (28.07)	7.64 (29.18)	7.50 (30.24)	7.38 (31.28)	7.26 (32.28)	7.14 (33.25)	7.03 (34.19)	6.93 (35.11)
3.0	8.25 (24.71)	8.09 (25.88)	7.94 (26.99)	7.80 (28.08)	7.66 (29.13)	7.53 (30.14)	7.41 (31.12)	7.29 (32.07)	7.18 (32.99)	7.08 (33.89)
5.0	8.43 (23.69)	8.27 (24.83)	8.11 (25.92)	7.97 (26.98)	7.83 (28.01)	7.70 (28.99)	7.57 (29.95)	7.45 (30.89)	7.34 (31.79)	7.23 (32.67)
10.0	8.94 (21.12)	8.77 (22.10)	8.61 (23.23)	8.45 (24.23)	8.30 (25.19)	8.16 (26.12)	8.03 (27.03)	7.91 (27.91)	7.78 (28.69)	7.67 (29.59)
15.0	9.56 (18.55)	9.38 (19.55)	9.20 (20.52)	9.04 (21.45)	8.87 (22.36)	8.73 (23.23)	8.59 (24.07)	8.45 (24.90)	8.32 (25.69)	8.20 (26.47)
20.0	10.33 (15.97)	10.13 (16.90)	9.94 (17.80)	9.76 (18.66)	9.59 (19.49)	9.43 (20.30)	9.27 (21.09)	9.13 (21.85)	8.99 (22.58)	8.86 (23.30)

TABLE 3: OPTIMAL VALUES: LINEAR DECAY RATE

K=250; C_1=1; h=0.5; a=0.02 X=25; Y=- 0.5: s=0.7; $\eta = 0.5$

λ	Optimal Price (p^*)	Optimal Time (T^*)	Optimal order rate (Q_0)	Optimal Cost (C^*)	Optimal Profit (π^*)
0.04	26.24	17.23	12.57	44.01	267.72
0.08	26.27	16.88	12.74	44.84	266.86
0.12	26.29	16.59	12.94	45.61	266.07
0.16	26.31	16.33	13.13	46.30	265.34
0.20	26.33	16.08	13.32	46.97	264.64
0.24	26.35	15.83	13.51	47.63	263.97
0.28	26.36	15.55	13.69	48.28	263.29
0.32	26.38	15.24	13.85	48.95	262.60
0.36	26.39	14.86	14.07	49.67	261.86
0.40	26.40	14.37	14.26	50.52	260.99
0.44	26.42	13.69	14.45	51.63	259.87
0.52	26.43	10.89	14.83	56.74	254.74

TABLE 4: OPTIMAL VALUES; LINEAR DECAY RATE

K=250; C_1=1; h=0.5; a= 0.02; s=0.7; $\eta = 0.5$

P \ λ	0.05	0.09	0.13	0.16	0.20	0.25	0.30	0.35	0.40	0.45
0.0	11.75 (25.89)	11.39 (26.13)	11.09 (26.35)	10.98 (26.45)	10.72 (26.66)	10.45 (26.92)	10.21 (27.45)	10.07 (27.33)	9.91 (27.51)	9.79 (27.68)
1.0	11.87 (25.39)	11.50 (25.62)	11.19 (25.84)	11.09 (25.94)	10.83 (26.13)	10.54 (26.40)	10.30 (26.66)	10.16 (26.82)	10.01 (27.00)	9.89 (27.15)
2.0	11.99 (24.87)	11.61 (25.11)	11.32 (25.34)	11.15 (25.43)	10.93 (25.62)	10.63 (25.89)	10.39 (26.15)	10.25 (26.31)	10.13 (26.46)	10.01 (26.59)
3.0	12.11 (24.37)	11.72 (24.61)	11.43 (24.85)	11.27 (24.92)	11.03 (25.11)	10.73 (25.38)	10.48 (25.63)	10.34 (25.81)	10.21 (25.94)	10.08 (26.93)
5.0	12.36 (23.35)	11.95 (23.59)	11.67 (23.90)	11.48 (23.90)	11.24 (24.09)	10.93 (24.36)	10.76 (24.53)	10.55 (24.83)	10.42 (24.94)	10.31 (25.86)
10.0	13.07 (20.81)	12.64 (21.00)	12.42 (21.25)	12.12 (21.40)	11.82 (21.54)	11.60 (21.72)	11.40 (21.88)	11.16 (22.13)	11.00 (22.38)	10.83 (22.41)
15.0	13.92 (18.26)	13.12 (18.41)	13.18 (18.60)	12.82 (18.79)	12.67 (18.89)	12.40 (19.06)	12.13 (19.25)	11.90 (19.69)	11.74 (19.63)	11.61 (19.79)
20.0	14.96 (15.71)	14.26 (15.83)	14.11 (16.02)	13.90 (16.14)	13.54 (16.32)	13.29 (16.61)	13.02 (16.70)	12.82 (16.83)	12.45 (16.99)	12.50 (17.14)

Note: The values in the brackets indicate the optimal order rate.

TABLE 5: OPTIMAL VALUES; QUADRATIC DECAY RATE

K=250; C_1=1; h=0.5; c= 0.002; X=25; Y=- 0.5: s=0.7; $\eta = 0.5$

λ	Optimal Price (p^*)	Optimal interval (T^*)	Optimal order rate (Q_0)	Optimal Cost (C^*)	Optimal Profit (π^*)
0.05	26. 28	11.66	12.26	51.69	259.98
0.10	26.29	11.45	12.41	52.33	259.33
0.15	26.30	11.26	12.55	52.94	258.72
0.20	26.31	11.09	12.68	53.50	258.14
0.25	26.32	10.95	12.80	54.04	257.59
0.30	26.33	10.81	12.92	54.54	257.08
0.35	26.34	10.69	13.04	55.01	256.59
0.40	26.35	10.59	13.15	55.45	256.15
0.45	26.35	10.49	13.26	55.86	255.72

TABLE 6: OPTIMAL VALUES[$\frac{T^*}{Q^*}$]; QUADRATIC DECAY RATE

K=250; C_1=1; h=0.5; a= 0.02; s=0.7;$\eta = 0.5$; c=0.0002

P \ λ	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45
0	9.57 (25.34)	9.50 (25.76)	9.46 (25.87)	9.40 (26.02)	9.35 (26.15)	9.31 (26.28)	9.27 (26.45)	9.25 (26.59)	9.22 (26.75)
1	8.06 (25.00)	8.02 (25.23)	7.92 (25.38)	7.82 (25.55)	7.74 (25.69)	7.68 (25.82)	7.60 (25.98)	7.54 (26.13)	7.45 (26.34)
2	8.13 (24.57)	8.05 (24.78)	7.95 (24.95)	7.86 (25.12)	7.77 (25.31)	7.70 (25.46)	7.62 (25.66)	7.55 (25.85)	7.46 (26.01)
3	8.21 (24.06)	8.08 (24.28)	7.98 (24.45)	7.89 (24.62)	7.80 (24.80)	7.72 (24.96)	7.63 (25.16)	7.56 (25.33)	7.48 (25.69)
5	8.39 (23.05)	8.25 (23.26)	8.17 (23.41)	8.05 (23.60)	7.95 (23.79)	7.89 (23.92)	7.80 (24.12)	7.73 (24.26)	7.66 (25.03)

10	8.88 (20.52)	8.73 (20.71)	8.65 (20.85)	8.53 (21.05)	8.43 (21.20)	8.36 (21.33)	8.25 (21.53)	8.18 (21.67)	8.00 (21.99)
15	9.45 (17.90)	9.36 (18.14)	9.24 (18.29)	9.09 (18.48)	9.00 (18.62)	8.92 (18.74)	8.81 (18.92)	8.73 (19.06)	8.54 (19.96)
20	10.15 (15.53)	10.07 (15.62)	9.96 (15.73)	9.81 (15.89)	9.71 (16.03)	9.59 (16.18)	9.48 (16.32)	9.41 (16.44)	9.02 (16.98)

We can now tabulate the optimal values of the three yard sticks namely: (i.) π^*/Q_0 (ii.) C^*/Q_0 and (iii.) Q_0 / T^* (that is, (i.) optimal profit / optimal order Quantity, (ii.) optimal cost / optimal Order Quantity and (iii.) optimal order Quantity / optimal ordering time unit) in the following table 7.

The purpose is to comprehensively put all the empirical results together at the place for ready comparison.

TABLE 7: COMPARISON OF OPTIMAL VALUES

λ	π^*/Q_0			C^*/Q_0			Q_0/T^*		
	Constant	Linear	Quadratic	Constant	Linear	Quadratic	Constant	Linear	Quadratic
0.05	19.06	21.29	21.21	4.29	3.50	4.22	1.12	0.73	1.05
0.10	19.01	20.75	20.89	4.12	3.57	4.22	1.17	0.76	1.08
0.15	17.91	20.20	20.61	3.95	3.52	4.22	1.26	0.78	1.11
0.20	16.96	19.86	20.34	3.89	3.52	4.22	1.36	0.82	1.14
0.25	16.11	19.53	20.12	3.68	3.52	4.22	1.45	0.86	1.17
0.30	15.38	19.03	19.89	3.57	3.53	4.22	1.54	0.90	1.19
0.35	14.71	18.53	19.68	3.47	3.53	4.22	1.63	0.94	1.22
0.40	14.12	16.06	19.48	3.38	3.54	4.22	1.72	0.99	1.24
0.45	13.59	14.71	19.28	3.30	3.56	4.22	1.87	1.05	1.26

5. CONCLUSIONS

We first observe that empirical work such as is reported in Tables 1 – 4 has this use of enabling one to perform qualitative analysis or interpretation of the quantitative results. For example, comparison of optimal values from Tables 1 and 3 reveals that for linear decay rate (as compared to constant decay rate – see table 1) T^* and π^* (optimal) values show an increasing pattern, while p^* , Q_0 and c^* decrease. The qualitative analysis points to a desirable or preferred inventory system in this case, especially as the more realistic pattern of linear decay rate is also incorporated. However the extent of increases and decreases in the relevant optimal values corresponding to the same changes in numerical values of λ (note that in the case of Table3, the linear decay rate pattern is incorporated) would help the policy maker or business executive to suitably design the system as per his requirements or choice. For similar purposes, table 4 values are also presented to facilitate comparison with optimal values corresponding to the constant decay rate setup given in table 2. Earlier works (see [1]) had not considered this case of constant decay rate corresponding to various alternative values of p. An attempt is thus made by us now to profitability bring into comprehensive focus the qualitative behavior of the different inventory systems (with regard to the decay patterns and price-structures) based on the quantitative (analytic) results derived by us.

These points, in fact are brought out in a better focus through Table 7. The models involving linear decay rates appear to be preferable yard sticks (i), (ii) and (iii), even though an certain other minor counts inventory models suffering quadratic decay rate seem to be better (for example, π^*/Q_0 are higher where λ is > 0.35). A deeper analysis through more detailed empirical work may lead to clearer insight into the qualitative aspects, further.

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