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VAGUE IDEALS OF A LATTICE

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ABSTRACT

The aim of this paper is to introduce the concept of Vague Lattice and Vague Ideals. We discuss some characterizations of Vague sublattices and Vague Ideals in terms of its level subsets. Further, we define the notion of Ψ-invariant class of Vague ideals and investigate their properties.

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1. INTRODUCTION

In 1993 W.L.Gau and D.J.Buehrer[3] Proposed the theory of Vague sets as an improvement of theory of Fuzzy sets in approximating the real life situation. Vague sets are higher order Fuzzy sets. A Vague set A in the universe of discourse U is a Pair $(t_A, 1-f_A)$ where t_A and f_A are Fuzzy subsets of U satisfying the condition $t_A(u) \leq 1 - f_A(u)$ for all u ∈U. R.Biswas[1] initiated the study of Vague algebra by introducing the concepts of Vague groups, Vague normal groups. H.Khan , M.Ahmad and R.Biswas[6] introduced the notion of Vague relations and studied some properties of them. N.Ramakrishna[7] continued this study by studying Vague Cosets, Vague Products and several properties related to them. In 2008, Y.B.Jun and C.H.Park[5] introduced the notion of Vague Ideals in Substraction algebra. T.Eswarlal[2] had introduced the notion of Vague ideals and normal Vague ideals in Semirings in 2008. In 2005 K.Hur et.al[4] studied in detail the notion of intuitionistic Fuzzy Ideals of a ring and established their characterization in terms of level subsets. Moreover they studied the Lattice structure of intuitionistic Fuzzy Ideals of a ring and their Modularity. In this Paper we introduce the concept of Vague sublattices and Ideals in a Lattice. Their characterizations in terms of level subsets are provided and their homomorphic images under various conditions are obtained.

2.Preliminaries

Definition 2.1: [3]

A Vague set A in the universe of discourse S is a Pair (t_A, f_A) where t_A : S → [0,1] and f_A : S → [0,1] are mappings (called truth membership function and false membership function respectively) where t_A(x) is a lower bound of the grade of membership of x derived from the evidence for x and

f_A(x) is a lower bound on the negation of x derived from the evidence against x and t_A(x) + f_A(x) ≤ 1 ∀x∈ S.

Definition 2.2: [3]

The interval [t_A(x),1- f_A(x)] is called the Vague value of x in A, and it is denoted by V_A(x). That is V_A(x) = [t_A(x),1- f_A(x)].

Definition 2.3: [3]

A Vague set A of S is said to be contained in another Vague set B of S. That is A $\subseteq$ B, if and only if V_A(x) $\leq$ V_B(x). That is t_A(x)≤ t_B(x) and $1 - f_A$(x)≤ $1 - f_B$(x) ∀x ∈ S.

Definition 2.4: [3]

Two Vague sets A and B of S are equal (i.e) A = B, if and only if A⊆B and B⊆A. (i.e) V_A(x)≤ V_B(x) and V_B(x)≤ V_A(x) ∀x∈ S, which implies t_A(x) = t_B(x) and $1 - f_A$(x) = $1 - f_B$(x).

Definition 2.5 :[3]

The Union of two vague sets A and B of S with respective truth membership and false membership functions t_A, f_A and t_B, f_B is a Vague set C of S, written as C = A ∪ B, whose truth membership and false membership functions are related to those of A and B by t_C = max{t_A, t_B} and $1 - f_C$ = max {$1 - f_A$, $1 - f_B$ }=1-min{f_A, f_B}.

Definition 2.6: [3]

The Intersection of two vague sets A and B of S with respective truth membership and false membership functions t_A, f_A and t_B, f_B is a Vague set C of S, written as C = A ∩ B, whose truth membership and false membership functions are related to those of A and B by t_C = min{t_A, t_B} and $1 - f_C$ = min {$1 - f_A$, $1 - f_B$ }=1-max{f_A, f_B}.

Definition 2.7: [3]

A Vague set A of S with t_A(x) = 1 and f_A(x) = 0 ∀x∈ S, is called the unit vague set of S.

Definition 2.8: [3]

A Vague set A of S with t_A(x) = 0 and f_A(x) = 1 ∀x∈ S, is called the zero vague set of S.

Definition 2.9: [3]

Let A be a Vague set of the universe S with truth membership function t_A and false membership function f_A, for α,β ∈ [0,1] with α≤β, the (α, β) cut or Vague cut of the Vague set A is a crisp subset $A_{(\alpha,\beta)}$ of S given by $A_{(\alpha,\beta)}$ ={x∈S: V_A(x)≥ (α, β)}, (i.e) $A_{(\alpha,\beta)}$ = {x∈S: t_A(x)≥α and 1-f_A(x)≥ β}

Definition 2.10: [3] The α-cut, A_α of the Vague set A is the (α,α) cut of A and hence it is given by A_α = { x∈S : t_A(x) ≥ α}.

Definition 2.11: [4]

Let (X,≤) be a Poset, if ∀ a,b∈S ⇒ a∨b, a∧b ∈ X. Then (X,≤) or (X,∨,∧) is called a Lattice where a∨b = ∨{a,b}=sup{a,b} , a∧b = ∧{a,b} = inf{a,b}.

Definition 2.12: [4]

Let (X,∨,∧) be a Lattice, if it satisfied following distributivity Laws, then it is called a distributive Lattice i) a∨(b∧c) = (a∨b)∧(a∨c) ii) a∧(b∨c) = (a∧b)∨(a∧c) , ∀a,b,c ∈L

Definition 2.13: [4]

A Fuzzy subset μ of L is called a Fuzzy Sublattice of L if i) μ$(x \vee y) \geq$ min{$\mu(x), \mu(y)$}

ii) μ$(x \wedge y) \geq$ min{$\mu(x), \mu(y)$} ∀ x,y ∈L

3. Vague Sublattices and Vague Ideals:

In this section we introduce the concept of Vague sublattices and Ideals and discuss their properties.

Definition 3.1 :

Let L be a Lattice A be a Vague set over L. Then A is said to be Vague Sublattice over L if each x,y∈L

i) $V_A(x\vee y) \geq$ min $\{V_A(x), V_A(y)\}$

ii) $V_A(x\wedge y) \geq$ min $\{V_A(x), V_A(y)\}$ ∀ x,y ∈L where V_A = $[t_A$,1- $f_A]$.

The set of all Vague Sublattice of L is denoted as VSL(L).

Example 3.2 :

Consider the Lattice L of 'divisors of 10' that is L = {1,2,5,10}

Let A = {x, [t_A(x),1- f_A(x)] > / x ∈ L} be given by

{<1, [0.5,0.9] >, <2, [.4,.5] >,<5,[.4,.7] >, <10, [.7,.7] > }. Then A is a Vague Sublattice of L.

Definition 3.3 :

A Vague set A of L is called Vague Ideal of L , if the following conditions hold:

i) $V_A(x\vee y) \geq$ min $\{V_A(x), V_A(y)\}$

ii) $V_A(x\wedge y) \geq$ max $\{V_A(x), V_A(y)\}$ ∀ x,y ∈L where V_A = $[t_A$,1- $f_A]$.

The set of all Vague Ideals of L is denoted as VI(L)

Example 3.4 :

Consider the Lattice L = {1,2,3,4,6,12} of divisors of 12

We define A = {<x, [t_A(x),1- f_A(x)] > / x ∈ L} by

{<1, [0.7,0.8] >, <2, [.5,.5] >,<3,[.6,.7] >, <4, [.4,.5] >, <6, [.5,.5] >,<12,[.4,.5] > }. Then A is a Vague Ideal of L.

Definition 3.5 :

A Vague set A of L is called Vague Prime Ideal , if $V_A(x\wedge y) =$max $\{V_A(x), V_A(y)\}$ ∀ x,y ∈L

Theorem 3.6:

If A and B are two VSLs (VIs) of a Lattice L then A∩B is a VSL(VI) of L.

Proof:

A = {<x, [t_A(x),1- f_A(x)] > / x ∈ L} and B = {x, [t_B(x),1- f_B(x)] > / x ∈ L} be two Vague sublattice of L. Then A∩B = {x, [$t_{A\cap B}$(x), 1- $f_{A\cap B}$(x)] > / x ∈ L},

where $t_{A\cap B}$(x) = min{t_A(x), t_B(x)} and 1- $f_{A\cap B}$(x) = min {1- f_A(x), 1- f_B(x)}.

So that $t_{A\cap B}$(x∨y) = min {t_A(x∨y) , t_B(x∨y)} ≥ min {min{t_A(x), t_A(y)}, min{t_B(x), t_B(y)}}= min{min{t_A(x), t_B(x)}, min{t_A(y), t_B(y)}}=min{$t_{A\cap B}$(x),$t_{A\cap B}$(y)},∀ x,y∈L.

Similarly we get $t_{A\cap B}$(x∧y)≥ min{$t_{A\cap B}$(x),$t_{A\cap B}$(y)}, ∀ x,y∈L.

Also 1- $f_{A\cap B}$(x∨y) = min{1- f_A(x∨y), 1- f_B(x∨y) } ≥ min{min{1- f_A(x) , 1- f_A(y) },min{1- f_B(x) ,1- f_B(y) }} = min{min{1- f_A(x) , 1- f_B(x) }, min{1- f_A(y), 1- f_B(y) }} = min {1- $f_{A\cap B}$(x) , 1- $f_{A\cap B}$(y) }, ∀ x,y∈L.

Similarly we get 1- $f_{A\cap B}$(x∧y)≥ min {1- $f_{A\cap B}$(x) , 1- $f_{A\cap B}$(y) }, ∀ x,y∈L. Hence A∩B is VSL(L). Similarly we can prove that A∩B is VI(L).

Proposition 3.7:

Suppose A is a VSL(VI) of L if and only if [A] and <A> are VSLs(VIs) of L.

Proof:

We will prove the case of VSL.

Let A be a Vague sublattice of L. We have [A] = {x, [t_A(x),1- t_A(x)] > / x ∈ L}.

Then ∀x,y∈L, t_A(x∨y)≥min{t_A(x), t_A(y)}and t_A(x∧y) ≥min{t_A(x), t_A(y)}. Now 1- t_A(x∨y)≤1-min{t_A(x), t_A(y) } = max{1-t_A(x), 1-t_A(y)} ∀x,y∈L. Similarly 1- t_A(x∧y)≤ max{1-t_A(x), 1-t_A(y)} ∀x,y∈L. Hence [A] is a Vague sublattice of L. Next to Prove <A> is a Vague sublattice of L. Here <A> = {x, [f_A(x),1- f_A(x)] > / x ∈ L}. Then ∀x, y∈L 1- f_A(x∨y) ≥ min {1- f_A(x), 1- f_A(y)} and 1- f_A(x∧y) ≥min{1- f_A(x), 1- f_A(y)}.

f_A(x∨y) ≤ 1- min{1- f_A(x), 1- f_A(y)} = max{ f_A(x), f_A(y)}. Hence <A> is a Vague sublattice of L. Conversely assume that [A] and <A> are Vague sublattices of L. Then A is a VSL(L) follows from the definition.

Remark 3.8:

The Union of two VSLs need not be a VSL which may be seen by the following Example.
Consider the Lattice given in example 3.2, we define
A = {<1, [0.7,0.8] >, <2, [.4,.5] >,<5,[.1,.5] >, <10, [.2,.6] > } and
B = {<1, [0.6,0.7] >, <2, [.1,.5] >,<5,[.3,.7] >, <10, [.2,.7] > }. Here A and B are Vague sublattice of L.
Also A∪B = {<1, [0.7,0.8] >, <2, [.4,.5] >,<5,[.3,.7] >, <10, [.2,.7] > }
$t_{A\cup B}$(10) = $t_{A\cup B}$(5∨2) = 0.2 < min {$t_{A\cup B}$(5), $t_{A\cup B}$(2)} = .3. Hence A∪B need not be a Vague sublattice of L.

Proposition 3.9:

Every VI(L) is a VSL(L) but not conversely.

Proof: Follows from definitions

Example 3.10:

Consider the Lattice given in Example 3.2.
Define A = {<1, [0.5,0.9] >, <2, [.4,.7] >,<5,[.4,.5] >, <10, [.7,.7] > }.
Then A is a Vague sublattice of L but not a Vague ideal of L because
t_A(2) = t_A(2∧10) = .4<max{t_A(2), t_A(10)} = 0.7.

Remark 3.11:

The union of two Vague ideal of L need not be a Vague ideal, which follows from Remark 3.8 and 3.10.

Proposition 3.12:

If A is a VI(L) and B a VSL(L) then A∩B is a VSL but not a VI by the following example.

Proof:

Consider the Lattice given in example 3.4.
Let A = {<1, [0.7,0.8] >, <2, [.5,.5] >,<3,[.6,.7] >, <4, [.4,.5] >, <6, [.5,.5] >,<12,[.4,.5] > } and
B = {<1, [0.2,0.3] >, <2, [.4,.6] >,<3,[.2,.5] >, <4, [.3,.4] >, <6, [.5,.5] >,<12,[.6,.7] > }
here A is a VI(L) and B is a VSL(L).
Then A∩B = {<1, [0.2,0.3] >, <2, [.4,.5] >,<3,[.2,.5] >, <4, [.3,.4] >, <6, [.5,.5] >,<12,[.4,.5] > }. Clearly A∩B is a VSL(L) but not a VI(L) because $t_{A\cap B}$(1) = $t_{A\cap B}$(2∧3) = .2 < max{$t_{A\cap B}$(2), $t_{A\cap B}$(3)}= 0.4.

Proposition 3.12:

Let A∈VS(L). Then A∈VI(L)[VSL(L)] if and only if all non empty level sets $A^{(\alpha,\beta)}$ is an ideal [sublattice] of L for each (α,β) ∈[0,1] with α+β≤1.

Proof:

Firstly, suppose that A∈VI(L). Let x,y ∈$A^{(\alpha,\beta)}$. Then t_A(x∨y) ≥min{t_A(x), t_A(y)}≥ α and 1- f_A(x∨y) ≥ min {1- f_A(x), 1- f_A(y)}≥ β, since A∈VI(L). Hence x∨y∈$A^{(\alpha,\beta)}$.
Also t_A(x∧y) ≥max{t_A(x), t_A(y)}≥ α and 1- f_A(x∧y) ≥max{1- f_A(x), 1- f_A(y)} ≥ β. Hence x∧y∈$A^{(\alpha,\beta)}$. Thus $A^{(\alpha,\beta)}$ is a Vague ideal of L. Conversely, assume that $A^{(\alpha,\beta)}$ is an ideal of L. Let x,y∈L such that [t_A(x),1- f_A(x)] =[α,β] and [t_A(y),1- f_A(y)] = [s,t] such that α≤s and β≤t. Then we have the following cases.

Case 1:

Suppose α=0 and β=1. Then obviously, t_A(x∨y) ≥ α = min{t_A(x), t_A(y)} and 1- f_A(x∨y) ≥ β = min {1- f_A(x), 1- f_A(y)}. Now y∈$A^{(s,t)}$ implies x∧y∈$A^{(s,t)}$,since $A^{(s,t)}$ is a Vague ideal of L. Hence t_A(x∧y) ≥s = max{t_A(x), t_A(y)} and 1- f_A(x∧y) ≥ t = max{1- f_A(x), 1- f_A(y)}. Thus A∈VI(L).

Case 2:

$\alpha \neq 0$ and $\beta \neq 1$, Choose $\varepsilon>0$ such that $\varepsilon<\alpha$, Then we have t_A(y) > s-$\varepsilon \geq \alpha$-ε, 1- f_A(y) > t+$\varepsilon \geq \beta$+ε and t_A(x) > α-ε, 1- f_A(x) > β+ε. Thus x,y$\in A^{(\alpha-\varepsilon,\ \beta+\varepsilon)}$. By our assumption $A^{(\alpha-\varepsilon,\ \beta+\varepsilon)}$ is a Vague ideal of L. so x$\vee$y $\in A^{(\alpha-\varepsilon,\ \beta+\varepsilon)}$ and x$\wedge$y$\in A^{(s-\varepsilon,\ t+\varepsilon)}$. Thus t_A(x$\vee$y) > α -ε , 1- f_A(x$\vee$y) > β+ε and t_A(x$\wedge$y) >s-ε and 1- f_A(x$\wedge$y) > t+ε. Since ε>o is arbitrary, we obtain t_A(x$\vee$y) > α = min{t_A(x), t_A(y)} , 1- f_A(x$\vee$y) > β = min {1- f_A(x), 1- f_A(y)} and t_A(x$\wedge$y) $\geq$s = max{t_A(x), t_A(y)} and 1- f_A(x$\wedge$y) $\geq$ t = max{1- f_A(x), 1- f_A(y)} $\forall$x,y$\in$L. Hence A$\in$VI(L). Proof is similar for VSL(L).

4. Vague ideals and Homomorphism:

Definition 4.1:

If Ψ : L$\rightarrow L'$ be a mapping from a Lattice L to another Lattice L' and A = {<x, [t_A(x),1- f_A(x)] > / x $\in$ L} be Vague set of L then the image Ψ(A) is defined by

Ψ(A) = {<y,[Ψ(V_A)(y)]>/y$\in L'$} where Ψ(V_A)(y) = Sup {V_A(x) / x $\in \Psi^{-1}(y)$} , $\Psi^{-1}(y) \neq \phi$

= ϕ , $\Psi^{-1}(y) = \phi$

where V_A = [t_A, 1-f_A]. Similarly if A' = {<y, [$t_{A'}$(y),1- $f_{A'}$(y)] > / y $\in L'$} is a Vague set of L' then $\Psi^{-1}(A')$ = {<x, [Ψ^{-1} ($t_{A'}$) (x), Ψ^{-1}(1- $f_{A'}$)(x)] > / x $\in$ L},

where $\Psi^{-1}(t_{A'}(x))$ = $t_{A'}$(Ψ(x)) and Ψ^{-1}(1- $f_{A'}$)(x) = (1- $f_{A'}$)Ψ(x)

Proposition 4.2:

Let A be a Vague ideal of L then Ψ(A) is a Vague ideal of L' .

Proof:

Let A be a Vague ideal of L. Then Ψ(A) = {<y, [Ψ(V_A)(y)]>/y$\in L'$} Let y,z $\in L'$. Then Ψ(V_A)(y$\vee$z) = Sup {V_A(x) / x $\in \Psi^{-1}(y\vee z)$}$\geq$ Sup {V_A(u$\vee$v) / u $\in \Psi^{-1}(y)$, v$\in \Psi^{-1}(z)$} $\geq$ Sup {min{(V_A(u), V_A(v)}/ u $\in \Psi^{-1}(y)$, v$\in \Psi^{-1}(z)$} = min{Sup V_A(u) / u $\in \Psi^{-1}(y)$, SupV_A(v) / v$\in \Psi^{-1}(z)$} = min{Ψ(V_A)(y) , Ψ(V_A)(z)}. Also Ψ(V_A)(y$\wedge$z) = Sup {V_A(x) / x $\in \Psi^{-1}(y\wedge z)$}$\geq$ Sup {V_A(u$\cap$v) / u $\in \Psi^{-1}(y)$, v$\in \Psi^{-1}(z)$} $\geq$Sup {max{(V_A(u), V_A(v)}/ u $\in \Psi^{-1}(y)$, v$\in \Psi^{-1}(z)$} = max{Sup V_A(u) / u $\in \Psi^{-1}(y)$, SupV_A(v) / v$\in \Psi^{-1}(z)$} = max{Ψ(V_A)(y) , Ψ(V_A)(z)}. Hence Ψ(A) is a Vague ideal of L'.

Theorem4.3:

If Ψ : L$\rightarrow L'$ is a Lattice homomorphism and A' is a Vague Lattice of L', then $\Psi^{-1}(A')$ is a Vague ideal of L.

Proof:

Let A' be a Vague ideal of L'. Then $\Psi^{-1}(A')$ = {<x, [Ψ^{-1} ($V_{A'}$)(x)]>/x$\in$L}.

Let x,y $\in$L . Then Ψ^{-1} ($V_{A'}$)(x$\vee$y) = $V_{A'}$[Ψ (x$\vee$y)] = $V_{A'}$[Ψ(x)$\vee\Psi$(y)] $\geq$ min{$V_{A'}$(Ψ(x), $V_{A'}$(Ψ(y)} = min{Ψ^{-1} (($V_{A'}$)(x), Ψ^{-1} (($V_{A'}$)(y))}. Also Ψ^{-1} ($V_{A'}$)(x$\wedge$y) = $V_{A'}$[Ψ (x$\wedge$y)]

= $V_{A'}$[Ψ(x)$\wedge\Psi$(y)] $\geq$ max {$V_{A'}$(Ψ(x), $V_{A'}$(Ψ(y)} = max {Ψ^{-1} (($V_{A'}$)(x), Ψ^{-1} (($V_{A'}$)(y))}. Hence $\Psi^{-1}(A')$ is a Vague ideal of L.

Theorem 4.4:

If Ψ : L$\rightarrow L'$ is an onto mapping and A , A' are Vague set of the Lattices L and L' respectively. Then i) Ψ[$\Psi^{-1}(A')$] = A' ii) A $\subseteq \Psi^{-1}$[Ψ(A)].

Proof:

i)Let y$\in L'$. Then we have Ψ[Ψ^{-1} ($V_{A'}$)(y)] = Sup{Ψ^{-1} ($V_{A'}$)(x) / x$\in \Psi^{-1}$(y) } = Sup{$V_{A'}$(Ψ(x) / x$\in$L , Ψ(x) = y} = $V_{A'}$(y). Since Ψ is an onto mapping for every y$\in L'$ there exist x$\in$L such that Ψ(x) = y. Hence Ψ[$\Psi^{-1}(A')$] = A'.

ii) Let x$\in$L. Then we have Ψ^{-1} (Ψ(V_A)(x)) = Ψ(V_A)(Ψ(x)) = Sup{V_A(x) / x$\in$ Ψ^{-1} (Ψ (x)} = V_A(x). Hence A $\subseteq \Psi^{-1}$[Ψ(A)]

Definition 4.5:

If Ψ : S→T be any function from a set S to another set T and {<x, [t_A(x),1- f_A(x)] > / x ∈ S} be a Vague set of S. Then A is said to be Ψ- invariant if x,y∈S such that Ψ(x) = Ψ(y) ⇒ V_A(x) = V_A(y).

Proposition 4.6:

If a Vague set A is Ψ- invariant then Ψ^{-1}[Ψ(A)] = A.

Proof: follows from theorem 4.4 and Definition 4.5.

Theorem 4.7:

If Ψ : S→T be any function from a set S to another set T and A,B are Vague sets of S, A', B' are Vague sets of T then i) A⊆B ⇒ Ψ(A)⊆Ψ(B) and ii) $A' \subseteq B' \Rightarrow \Psi^{-1}(A') \subseteq \Psi^{-1}(B')$.

Proof:

Let A = {<x, [t_A(x),1- f_A(x)] > / x ∈ L} and B = {x, [t_B(x),1- f_B(x)] > / x ∈ L}. Then A⊆B ⇒ V_A(x) ≤ V_B(x). Also Ψ(A) = {<y,[Ψ(V_A)(y)]>/y∈T} and Ψ(B) = {<y,[Ψ(V_B)(y)]>/y∈T}. Now ∀ y∈S, we have $\Psi(V_A)$(y) = Sup{V_A(x) / x∈ Ψ^{-1}(y)}≤ Sup{V_B(x) / x∈ Ψ^{-1}(y)} = Ψ(V_A(y)). Hence Ψ(A)⊆Ψ(B). Similarly we can prove ii.

Theorem 4.8:

If Ψ : L→ L' is a Lattice epimorphism,then there is one to one order preserving correspondence between the Vague ideals of L' and those of L which are Ψ-invariant.

Proof:

Let I(L') denote the set of all Vague ideals of L' and I(L) denote the set of all Vague ideals of L which are Ψ- invariant. Define ∑ : I(L) →I(L') and ∑′ : I(L')→I(L) such that ∑(A) = Ψ(A) and $\sum'(A') = \Psi^{-1}(A')$ by theorem 4.2 and 4.3 ∑,Ψ are well-defined. Also by theorem 4.4 and 4.6 , ∑ and Ψ inverse to each other which gives the one to one correspondence. Also by theorem 4.7 we have A⊆B ⇒ Ψ(A)⊆Ψ(B). Thus the correspondence is order preserving.

Definition 4.9:

Let A∈VS(L) . Then A is said to have Sup –Property if for each subset S⊆L there exist x_0 ∈S such that , $\sup_{x \in S}${ V_A(x)} = $V_A(x_0)$.

Lemma 4.10:

Let Ψ : L→ L' be an onto mapping and Let A be a Vague Lattice with Sup-Property in L. Then U($\Psi(t_A)$,α) = Ψ(U(t_A, α)) and L($\Psi(1-f_A)$,β) = Ψ(L($1-f_A$, β)), ∀α,β∈[0,1] with α+β≤1.

Proof:

Let y∈ U($\Psi(t_A)$,α). Then $\Psi(t_A)$(y) = $\sup_{x \in \Psi^{-1}(y)} t_A$(x) ≥α. Since A has Sup property, there exist $x_0 \in \Psi^{-1}$(y) such that $t_A(x_0)$ = $\sup_{x \in \Psi^{-1}(y)} t_A$(x) and $\Psi(x_0)$ = y. Hence $t_A(x_0)$ ≥α. So x_0∈U(t_A, α) and y∈Ψ(U(t_A, α)). Thus U($\Psi(t_A)$,α) ⊆ Ψ(U(t_A, α)).

For reverse inclusion , Let y∈Ψ(U(t_A, α)), then there exist x_0∈Ψ(U(t_A, α)), such that $\Psi(x_0)$ = y. conversely, Let y∈Ψ(U(t_A, α)), then there exist x_0∈(U(t_A, α)) such that $\Psi(x_0)$ = y. Now $t_A(x_0)$≥ α ⇒ $\Psi(t_A)$(y) = $\sup_{x \in \Psi^{-1}(y)} t_A$(x) ≥α. Therefore y∈ U($\Psi(t_A)$,α). Hence Ψ(U(t_A, α)) ⊆ U($\Psi(t_A)$,α).

Therefore U($\Psi(t_A)$,α) = Ψ(U(t_A, α)). Similarly we can Prove that if A has sup property, then L($\Psi(1-f_A)$,β) = Ψ(L($1-f_A$, β)).

Theorem 4.11:

Ψ : L→ L' be a Lattice epimorphism and Let A be a Ψ-invariant Vague Prime Ideal of L, then Ψ(A) is a Vague prime ideal of L'.

Proof:

Suppose Ψ(A) is not a Vague Prime ideal of L'. Then for some Ψ(a), Ψ(b) $\in L'$, we have $\Psi(t_A)$[Ψ(a)$\wedge\Psi$(b)] $\neq \Psi(t_A)$[Ψ(a)] and $\Psi(t_A)$[Ψ(a)$\wedge\Psi$(b)] $\neq \Psi(t_A)$[Ψ(b)], $\Psi(1-f_A)$[Ψ(a)$\wedge\Psi$(b)] $\neq \Psi(1-f_A)$[Ψ(a)] and $\Psi(1-f_A)$[Ψ(a)$\wedge\Psi$(b)] $\neq \Psi(1-f_A)$[Ψ(b)]. Since Ψ(A) is Vague ideal of L', $\Psi(t_A)$[Ψ(a)$\wedge\Psi$(b)] $>\Psi(t_A)$[Ψ(a)] and $\Psi(t_A)$[Ψ(a)$\wedge\Psi$(b)] $>\Psi(t_A)$[Ψ(b)], $\Psi(1-f_A)$[Ψ(a)$\wedge\Psi$(b)] $>\Psi(1-f_A)$[Ψ(a)] and $\Psi(1-f_A)$[Ψ(a)$\wedge\Psi$(b)] $>\Psi(1-f_A)$[Ψ(b)]. Now by definition of Pre-image and the fact that Ψ is a Lattice homomorphism we get, $\Psi^{-1}(\Psi(t_A))$[a$\wedge$b] $>$ $\Psi^{-1}(\Psi(t_A))$[a] and $\Psi^{-1}(\Psi(t_A))$[a$\wedge$b] $>$ $\Psi^{-1}(\Psi(t_A))$[b], $\Psi^{-1}(\Psi(1-f_A))$[a$\wedge$b] $>$ $\Psi^{-1}(\Psi(1-f_A))$[a] and $\Psi^{-1}(\Psi(1-f_A))$[a$\wedge$b] $>$ $\Psi^{-1}(\Psi(1-f_A))$[b]. Since A is Ψ-invariant by Proposition 4.6 we get V_A(a$\wedge$b)$>V_A$ (a) and V_A(a$\wedge$b)$>V_A$ (b) . This contradicts the fact that A is a Vague Prime ideal of L'.

Theorem 4.12:

Let Ψ : L$\rightarrow L'$ be a Lattice homomorphism then $\Psi^{-1}(A')$ is a Vague Prime ideal of L if A' is a Vague Prime ideal of L'.

Proof:

Let A' is a Vague Prime ideal of L'. Then we have $\forall$a,b$\in$L, $\Psi^{-1}(V_{A'})$(a$\wedge$b) = $V_{A'}\Psi$(a$\wedge$b) = $V_{A'}$[Ψ(a)$\wedge\Psi$(b)], Since A' is a Vague Prime ideal of L', $V_{A'}$[Ψ(a)$\wedge\Psi$(b)] = $V_{A'}$[Ψ(a)] (or) Ψ(b). This implies $\Psi^{-1}(V_{A'})$(a$\wedge$b) = $\Psi^{-1}(V_{A'})$(a) (or) $\Psi^{-1}(V_{A'})$(b). Hence $\Psi^{-1}(A')$ is a Vague Prime ideal of L'.

Theorem 4.13:

Let Ψ : L$\rightarrow L'$ be a Lattice epimorphism . Then Ψ(a) is a Vague prime ideal of L', if A is a Vague Prime ideal of L with Supremum Property in L.

Proof:

Let A be a Vague Prime ideal with Supremum Property in L, then Ψ(A) is a Vague ideal of L' by theorem 4.2. This implies Ψ(A) is Vague Prime ideal of L', if each nonempty upper and lower level sets of Ψ(A) are Prime ideals of L'. Now let $\alpha,\beta\in$[0,1] with $\alpha+\beta\leq 1$ and U($\Psi(t_A)$,α) = Ψ(U(t_A, α)) and L($\Psi(1-f_A)$,β) = Ψ(L($1-f_A$, β)). But U(t_A,α) and L($1-f_A$,β) are Prime ideals of L'. So Ψ(U(t_A, α)) and Ψ(L($1-f_A$), β)) are prime ideals of L'. Therefore U($\Psi(t_A)$,α) and L($\Psi(1-f_A)$,β) are prime ideals of L'. Hence Ψ(A) is a Vague prime ideal of L'.

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