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RESEARCH ARTICLE

ACQUISITION OF PERSONAL ASSETS THROUGH A SET OF REGULAR INSTALLMENTS  
VERSUS DEFERRED REGULAR INSTALLMENTS.WHAT IS THE BEST OPTION?

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ABSTRACT

The aim of this essay is to demonstrate by means of using a mathematical model, the relevance of paying a debt through a regular payments scheme and a scheme with deferment over time; this last one, in order to pay the first of the overdue installments with  $k-1$ . The hypothetical scenario focuses on the acquisition of an automobile in both forms of payment. The final result lead us to believe that although the payment form may seem similar, with deferred payment instead of supporting the debtor, it becomes an additional charge of hidden interests that in the final payment is reflected on the total amount paid for such operation.

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1. INTRODUCTION

Nowadays, people may use a model of annuities to pay a set of periodical payments or otherwise, to perform a set of deposits for the creation of an investment fund. Therefore, it is very important to identify which is the best option for both, paying off a debt or saving money.

In the financial mathematics field, the theory proposes several theorems that allow us to value money over time, as noted above, either for investment or savings, as well as for the acquisition of credit or financing among many other operations that are performed in financial institutions within the Mexican Financial System.

In commercial practice, we may observe that the automobile agencies offer several ways to purchase a vehicle, one of this, is attractive discounts by cash payment or by including insurance payment. Other forms may be without interest payments and probably with past due payments schemes and preferential rates.

Finally, there is a scheme of regular installments but with a deferral of time, i.e., the automobile is acquired and begins paying months later. Here what is being discussed is; what is the option that really favors those buyers who purchase the vehicle? In theory, there are several mathematics proposals which can be applied to calculate this kind of financial operations.

Therefore, the objective is to demonstrate if the deferral of the payments which automobile agencies offer to the clients is favorable or not, and if they include hidden interest, which at the end is reflected in the total amount paid for the automobile.

## 2. REVIEW OF LITERATURE

As noted in the introduction of this essay, in the field of mathematics, there is a branch within this discipline that provides several theorems which allow us to value money over time, such as financial mathematics, whose purpose is assessing operations where money is present.

Some recent work in this field has tried to explain with mathematical models some of the benefits of a deposit scheme that involves different interest rates (Moreno-García, García-Santillán, Guerra-Castro; García-Zárate and Manríquez-Gallardo, 2016), with anticipated annuities and floating interest rate (García-Santillán, Gutiérrez-Delgado, Cristóbal-Hernández and Catalayud-Gutiérrez, 2015), with the modality of simple ordinary annuities and his modality of annuities with geometric gradients (García-Santillán, Moreno-García, Escalera-Chávez, Peña-Osorio and Guerra-Hernández, 2015; García-Santillán, Moreno-García, Saco-Baschkir, Ramos-Hernández, 2015).

Also, mathematical models have demonstrated the acquisitions of real estate through bank loans and how they can be compared with an investment fund, i.e. as a scheme of debt amortization where at the end a high interest payment for the loan obtained is reflected. If on the contrary, instead of paying it is deposited in an investment fund, at the end of the same time, the savings allow us to obtain sufficient resources to acquire real property (Moreno-García, García-Santillán, Abascal-Sánchez, González-Zarco and Galindo-Martínez, 2015).

And finally, once acquired the debt for the acquisition of the automobile, if the debtor does not have the economic resources to pay the fees and falls in the past due portfolio, models for debt restructuring have also been proposed through models of equivalent equations in order to redesign a new scheme to pay the debt. This model of equivalent equations applies to both, overdue promissory notes and for those not yet expired. All this can be done by means of a new scheme with equal payments or with different amounts and different maturity dates (Moreno-García, García-Santillán, Bermúdez-Pérez and Almeida-Fernández, 2015; García-Santillán, Venegas-Martínez and Escalera-Chávez, 2014).

In order to explain theoretically the proposed scenario about purchasing a vehicle, through a series of regular past due payments, we follow the proposal of Ayres (1988), Lopez-Haro (2000), García-Santillán (2007, 2010, 2014), García-Santillán, Edel-Navarro, Escalera-Chavez (2010) whom in their works have proposed to apply an annuities models to identify the periodic payments to be made for each particular case. Furthermore, financial mathematic provides formulas for evaluating several financial operations performed in financial institutions, an example of this is: when a commitment is acquired through the purchase of some real state or personal property, or when establishing a fund of investment is desired.

In summary, for better understanding of the issue regarding past due and deferred annuities, we can refer to what García-Santillán (2014) says about it: "they are those annuities for

which payment, deposit, rent and interest payment is made at the end of each period, in comparison with the deferred annuities, which start with the first payment after a time extension, also known as deferral".

Therefore, we developed a financial calculation through payment options in the form of overdue payments and with a deferred first payment, and thereby be able to determine what is the best option.

### Development of the case

The hypothetical case analyzed here is about the acquisition of a vehicle, which is acquired with overdue payments. Such payments are constant over time "n" with an interest rate "i" with "m" capitalization, a net value of the transaction that we call "NPV" with fixed amounts  $Rp_1 \dots j$ , and the second case with the same data, except that the variable of the deferral "k-1" is added.

In the first scenario, the automobile agency offers their clients the option of paying the car as follows: The cash value of the transaction is \$200,000.00, or otherwise, 24 equal payments with a nominal interest rate compound monthly of 18%

Theoretically we know that:

$$NPV = Rp_1 \left[ \frac{1 - (1 + i/m)^{-n}}{i/m} \right] \quad (1)$$

Hence, to know the value of each installment,  $Rp_1$  is derived from the formula (1)

Obtaining:

$$Rp_1 = \frac{NPV}{\left[ \frac{1 - (1 + i/m)^{-n}}{i/m} \right]} \quad (1.1)$$

For the development of the case, we have the following:

**DATA:**  $n = 24$  payments;  $m =$  monthly;  $i = 18\%$ ;  $NPV = \$200,000.00$ ;  $Rp_1 = ?$

$$Rp_1 = \frac{\$200,000.00}{\left[ \frac{1 - (1 + (.18/12))^{-24}}{.18/12} \right]} = \frac{\$200,000.00}{\left[ \frac{1 - (1 + .015)^{-24}}{.015} \right]} \quad (1.2)$$

$$Rp_1 = \frac{\$200,000.00}{\left[ \frac{1 - 0.06995439}{.015} \right]} = \frac{\$200,000.00}{\left[ \frac{.3004561}{.015} \right]} = \frac{\$200,000.00}{20.0304066} = \$9,984.81$$

$$\$9,984.81 \times 24 = \$239,635.44$$

The verification with amortization schedule is:

**Table 1.** Amortization chart

Number of payment	Annuity	Interest	Capital	Balance
<b>0</b>				<b>\$200,000.00</b>
1	\$9,984.82	\$3,000.00	\$6,984.82	\$193,015.18
2	9,984.82	2,895.23	7,089.59	185,925.59
3	9,984.82	2,788.88	7,195.94	178,729.65
4	9,984.82	2,680.94	7,303.88	171,425.77
5	9,984.82	2,571.39	7,413.43	164,012.34
6	9,984.82	2,460.19	7,524.64	156,487.71
7	9,984.82	2,347.32	7,637.50	148,850.20
8	9,984.82	2,232.75	7,752.07	141,098.13
9	9,984.82	2,116.47	7,868.35	133,229.79
10	9,984.82	1,998.45	7,986.37	125,243.41
11	9,984.82	1,878.65	8,106.17	117,137.24
12	9,984.82	1,757.06	8,227.76	108,909.48
13	9,984.82	1,633.64	8,351.18	100,558.30
14	9,984.82	1,508.37	8,476.45	92,081.86
15	9,984.82	1,381.23	8,603.59	83,478.26
16	9,984.82	1,252.17	8,732.65	74,745.62
17	9,984.82	1,121.18	8,863.64	65,881.98
18	9,984.82	988.23	8,996.59	56,885.39
19	9,984.82	853.28	9,131.54	47,753.85
20	9,984.82	716.31	9,268.51	38,485.34
21	9,984.82	577.28	9,407.54	29,077.80
22	9,984.82	436.17	9,548.65	19,529.14
23	9,984.82	292.94	9,691.88	9,837.26
24	9,984.82	147.56	9,837.26	0.00
Total	\$239,635.68	\$39,635.69	\$200,000.00	

Source: own

For the second scenario, the car dealership gives the option to start their payments after the seventh month in a regular payment modality (k-1).

Theoretically we know that:

$$NPV = Rp_1 \left[ \frac{1 - (1 + i / m)^{-n}}{i / m(1 + i / m)^{k-1}} \right] \tag{2}$$

Hence, to know the value of each installment with deferral,  $Rp_1$  is derived from the formula (2)

Obtaining:

$$Rp_1 = \frac{NPV}{\left[ \frac{1 - (1 + i / m)^{-n}}{i / m(1 + i / m)^{k-1}} \right]} \tag{2.1}$$

For the development of the case, we have the following:

**DATA:**  $n= 24$  payments;  $m=$  monthly;  $i=18\%$ ;  $NPV= \$200,000.00$ ;  $k-1=7$ ;  $Rp_1= ?$

$$Rp_1 = \frac{\$200,000.00}{\left[ \frac{1 - (1 + (.18 / 12))^{-24}}{.18 / 12(1 + (.18 / 12))^{7-1}} \right]} = \frac{\$200,000.00}{\left[ \frac{1 - (1 + (.015))^{-24}}{.015(1 + (.015))^{7-1}} \right]} \tag{2.2}$$

$$Rp_1 = \frac{\$200,000.00}{\left[ \frac{1 - .6995439}{.015(1.0934432)} \right]} = \frac{\$200,000.00}{\left[ \frac{.3004561}{.0164016} \right]} = \$10,917.80$$

$$Rp_1 = \frac{\$200,000.00}{18.3187067} = \$10,917.80$$

$$\$10,917.80 \times 24 = \$262,027.20$$

The verification with amortization schedule is:

**Table 2.** Amortization chart

Number of payment	Annuity	Interest	Capital	Balance
0				\$200,000.00
1	\$0.00	\$3,000.00		203,000.00
2	0.00	3,045.00		206,045.00
3	0.00	3,090.68	<i>k-1</i>	209,135.68
4	0.00	3,137.04		212,272.71
5	0.00	3,184.09		215,456.80
6	0.00	3,231.85		218,688.65
7	10,917.83	3,280.33	7,637.50	211,051.15
8	10,917.83	3,165.77	7,752.07	203,299.08
9	10,917.83	3,049.49	7,868.35	195,430.73
10	10,917.83	2,931.46	7,986.37	187,444.36
11	10,917.83	2,811.67	8,106.17	179,338.19
12	10,917.83	2,690.07	8,227.76	171,110.43
13	10,917.83	2,566.66	8,351.18	162,759.25
14	10,917.83	2,441.39	8,476.45	154,282.80
15	10,917.83	2,314.24	8,603.59	145,679.21
16	10,917.83	2,185.19	8,732.65	136,946.56
17	10,917.83	2,054.20	8,863.64	128,082.93
18	10,917.83	1,921.24	8,996.59	119,086.34
19	10,917.83	1,786.30	9,131.54	109,954.80
20	10,917.83	1,649.32	9,268.51	100,686.29
21	10,917.83	1,510.29	9,407.54	91,278.75
22	10,917.83	1,369.18	9,548.65	81,730.09
23	10,917.83	1,225.95	9,691.88	72,038.21
24	10,917.83	1,080.57	9,837.26	62,200.95
25	10,917.83	933.01	9,984.82	52,216.13
26	10,917.83	783.24	10,134.59	42,081.53
27	10,917.83	631.22	10,286.61	31,794.92
28	10,917.83	476.92	10,440.91	21,354.01
29	10,917.83	320.31	10,597.52	10,756.49
30	10,917.83	161.35	10,756.49	0.00
Total	\$262,028.03	\$62,028.03	\$200,000.00	

Source: own

**DISCUSSION AND CONCLUSION**

As a result of the analysis for both hypothetical scenarios through modalities of due and deferred annuities, the following results were obtained:

It is clearly evident that both formulas in theory, give us different results, as we can see in Table 3, whether for the ordinary payment, as for ordinary payments with deferral regarding the time the first payment starts.

Table 3 Summary of calculus

Total	Interest payment	NPV	Concept
\$239,635.68	\$39,635.69	\$200,000.00	Regular payment
\$262,028.03	\$62,028.03	\$200,000.00	Regular payment with deferral ( $k-1$ )
\$22,392.35	\$22,392.35	\$0.00	More interest payment within deferred scheme

Source: own

The idea of stating it into this mathematical essay is to show what in theory indicates about this form of payment deferred: that although it is an attractive scheme to acquire any kind of asset, it is also clear that when an extension is obtained over time, the creditor will necessarily gain interest in the time granted since the financed capital is not recovered during the time period in which the payments are deferred. These assertions are consistent with the several papers presented by García-Santillán *et al* (2007, 2010, 2013 and 2014).

In commercial practice, namely in real life, it may seem that people, when buying a car –if they were given an extension for payment–, think it is without charge, but the fact is it is not, as this document proved. In order to do this, the theoretical guideline that sets out the steps to calculate both proposed scenarios were followed at all times (Moreno-García *et al* 2014 and García-Santillán *et al* 2014). Also, we can say that this kind of behavior in some customers, who purchase any personal asset, is associated with a low or zero financial knowledge regarding this kind of operation.

We can think as well that the seller of the assets, probably do not explains to the customer the implications and differences than a payment scheme has over the other, or otherwise, the customer does know this from the beginning of the operation and even then, chooses the second scheme ( $k-1$ ), which is valid and acceptable.

Finally, in this paper the theoretical arguments that support this mathematical essay were stated. Furthermore, this essay aims to be a consultation document, which could help regulate some behaviors and to serve as a support to decision making regarding the acquisition of personal assets.

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