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RESEARCH ARTICLE

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APPLICATION OF DIFFERENTIAL EQUATIONS TO HUMAN HEIGHT: A CASE STUDY OF XYZ SCHOOL

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ABSTRACT

Background: This study describes an application of simple differential equations in the field of biology particularly the height of an organism (Humans).

Objective: The study was to predict the mean height of adult Males at full growth which helps in addressing why knowing the height of one contributes to productivity and ergonomic design.

Methods: The study employed a quantitative methodology which emphasizes the objective measurement and analysis of data collected through surveys. A modified first order differential equation (Malthusian growth model) was used to model the data collected and used in the data analysis.

Results: It was found that the calculated value obtained from the model and the observed value from a template containing average height of people (males) around the world converged to the same value.

Conclusion: Our study gained an insight into the importance of predicting the height of a person which helps ergonomic professionals to apply an understanding of human factors to the design of equipment and systems in order to improve comfort, safety, health and productivity.

Keywords: differential equations, human height, biological growth, mathematical model, nutrition, environmental factors

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INTRODUCTION

Differential equations (Dontwi & Obeng-Denteh, 2011; Dontwi, *et al,* 2013; Dontwi, *et al,* 2014; Obeng-Denteh, Twumasi, Barnes & Mari, 2014; Obeng-Denteh, Kyei & Eghan, 2014; Andam , Obiri-Apraku, Agyei & Obeng-Denteh, 2015) play major roles in mathematical modeling. Emphasis on problem solving and mathematical modeling (Charalampos, 2004) has gained considerable attention in the last few years. Connecting mathematics to other subjects and to the real world outside the classroom has received increased attention in the mathematics programmes.

Human height or stature is the distance from the bottom of the feet to the top of the head in a human body, standing erect. It is measured using a stadiometer, usually in centimetres when using the metric system, and feet and inches when using the imperial system. Adult human height has varied from under 60 centimetres (2 ft 0 in) to over 260 centimeters (8 ft 6 in). On average, males are taller than females.

Human height varies greatly between individuals and across populations for a variety of complex biological, genetic, and environmental factors, among others. The average height in genetically and environmentally homogeneous populations is often proportional across a large number of individuals. Exceptional height variation (around 20 deviation from a population's average) within such a population is sometimes due to gigantism or dwarfism, which are caused by specific genes or endocrine abnormalities.

METHODOLOGY

The study employed a quantitative methodology (Obeng-Denteh *et al.,* 2011**)** which emphasizes the objective measurement and statistical analysis of data collected through surveys. Our research focuses on gathering numerical data and generalizing it across groups of people to explain a particular phenomenon. Our aim in conducting this quantitative research study is to determine the relationship between time (independent variable) and height (dependent variable) The data was collected from XYZ School, in Kumasi. In all forty-five (45) pupils (males) were measured using structured research instrument calibrated in inches. A modified first order differential equation (Malthusian growth model) was used to model the data collected and used in the data analysis.

LIMITATIONS OF STUDY

i. There was no information of an actual data on heights from some health institutions.
ii. Some of the pupils did not position themselves well during the measurement of their height.
iii. Genetics.
iv. Environmental factors.
v. Diet / Nutrition

DATA COLLECTION AND ANA LYSIS

DATA COLLECTION

The data was collected from Nursery to Basic three (3) pupils in XYZ School, Kumasi(Actual name of School hidden) during their Third Term of the 2014/2015 academic year. In all, forty five male pupils were measured.

The observations were carried out for a day and the data were documented. The collected data was then used to calculate the mean height at various ages. These calculated mean heights were used in the model to predict the average mean height at full growth. The mean heights in inches of male children at various ages are shown in the table below.

Table 1.0: calculated values of mean height of male children at various ages

AGE	HEIGHT (INCHES)
1 year	32.0
2 year	34.8
3 year	35.5
4 year	40.8
5 year	46.0
6 year	46.3

7 year	48.6
8 year	52.5
9 year	54.6

BIOLOGICAL GROWTH MODEL

A fundamental problem in biology involves that of growth, (Spiegel, 1967) whether it be the growth of a cell, an organ, a human, aplant or a population. The fundamental differential equation was:

$$\frac{dy}{dt} = \propto y \qquad (1)$$

Having solution

$$Y = C\, e^{\alpha t} \qquad (2)$$

Where c is an arbitrary constant. From this we see that growth occurs if$\alpha>0$ while decay (or shrinkage) occurs if $\alpha<0$.

One obvious defect of equation (1) and the corresponding solution (2) is that if $\alpha>0$ then we have $Y \to \infty$ as $t\to \infty$ so that as time the growth is unlimited.

This conflicts with reality, for after a certain time elapses we know that a cell or individual stops growing, having attained a maximum size. The question naturally arises as to whether we can modify equation (1) so as to correspond to these biological facts. Let us see if we can formulate the problem mathematically.

Mathematical Formulation

To fix our ideas, let us suppose that y denotes the height of a human being. It is natural to assume that (Spiegel, 1967) the rate of change of height depends on the height in a more complicated manner than simple proportionality as in (1). Thus, we would have

$$\frac{dy}{dt} = F(y) \qquad (3)$$

$$y = y_0 \text{ for } t = 0.$$

Where y_0 represents the heights at some specified time $t = 0$, and where F is some suitable but as yet unknown function . Since the linear function

$F(y) = \alpha y$ is unsuitable , we are led to try next order of approximation provided by a quadratic function $F(y) = \alpha y - \beta y^2$, where , where we choose the constant $\beta>0$ in order to inhibit the growth of y as demanded by reality . the differential equation (3) thus becomes

$$\frac{dy}{dt} = \propto y - \beta y^2 \qquad (4)$$

$$y = y_0 \text{ for } t = 0$$

Since equation (4) is one in which variables are separable , we have

$$\frac{dy}{\propto y - \beta y^2} = dt$$

Or

$$\int \frac{dy}{\propto y - \beta y^2} = t + C$$

that is

$$\int \frac{1}{\propto}\left[\frac{1}{y} + \frac{\beta}{\alpha - \beta y}\right] = t + C \ \text{ or } \ \frac{1}{\propto}[\text{In } y - \text{In}(\alpha - \beta y)] = t + c \qquad (5)$$

Using the condition $y = y_0$ at $t = 0$, we see that $C = \frac{1}{\alpha}[\text{In } y - \text{In}(\alpha - \beta y)]$.

Thus (5) becomes

$$\frac{1}{\propto}[\text{In } y - \text{In}(\alpha - \beta y)] = t + \frac{1}{\propto}[\text{In} y_0 - \text{In}(\alpha - \beta y_0)]$$

solving for y yields

$$y = \frac{\frac{\alpha}{\beta}}{1 + \left(\frac{\frac{\alpha}{\beta}}{y0} - 1\right)e^{-xt}} \tag{6}$$

If we take the limit of (6) as $t \to \infty$ we see, since α>0 that

$$Y_{max} = \lim_{t\to\infty} y = \frac{\alpha}{\beta} \tag{7}$$

This shows that there is a limit to the growth of y as required by the biological facts . As indicated in (7), this maximum is denoted by *Ymax*

To apply the result (6), let us suppose that the values of y corresponding to the times t=1 and t=2 (where we use some specific unit of time) are given respectively by y_1 and y_2. The from (6) we see that Yields

$$\frac{\frac{\alpha}{\beta}}{1 + \left(\frac{\frac{\alpha}{\beta}}{y0} - 1\right)e^{-\alpha}} = y_1 , \quad \frac{\frac{\alpha}{\beta}}{1 + \left(\frac{\frac{\alpha}{\beta}}{y0} - 1\right)e^{-2\alpha}} = y_2$$

or

$$\frac{\beta}{\alpha}(1 - e^{-\alpha}) = \frac{1}{y1} - \frac{e^{-\alpha}}{y0} . \frac{\beta}{\alpha}(1 - e^{-2\alpha}) = \frac{1}{y2} - \frac{e^{-2\alpha}}{y0} \tag{8}$$

This yield

$$1 + e^{-\alpha} = \frac{\frac{1}{y2} - \frac{e^{-2\alpha}}{y0}}{\frac{1}{y1} - \frac{e^{-\alpha}}{y0}} \tag{9}$$

Using some simple algebra

$$e^{-\alpha} = \frac{yo\,(y2 - y1)}{y2(y1 - y0)} \tag{10}$$

If this now substituted into the first equations (8), we find

$$\frac{\beta}{\alpha} = \frac{y1^2 - y0\,y2}{y1\,(yoy\,2 - 2y0y2 + y1y2\,)} \tag{11}$$

$$Y_{max} = \quad lim_{t\to\infty} \frac{y_1(y_0y_1 - 2y_0y_2 + y_1y_2}{y^2{}_1 - y_0y_2} \tag{12}$$

Solution

To cover the full set of data given in the table, let t= 0, 1, 2 correspond to the ages at 1year, 5years and 9 years respectively

Then we have

$$Y_0 = 32.0, \quad Y_1 = 46.0, \quad Y_2 = 54.6$$

Substituting these values into equation (12)

$$y_{max} = lim_{t\to\infty} \frac{y_1(y_0y_1 - 2y_0y_2 + y_1y_2}{y^2{}_1 - y_0y_2} \tag{13}$$

$$y_{max} = \frac{46.0((32*46) - 2(32*54.6) + (46*54.6))}{(46)\,2 - (32*54.6)}$$

$$= \frac{46.0(1472 - 3494.4 + 2511.6)}{2116.0 - 1747.2}$$

$$= \frac{22503.2}{368.8}$$

= 60.017 inches

≅5 feet 1 inch.

ANALYSIS

Using the observed values from the average world height in 2008 and our calculated values from Table 1.0, We realized that the values were almost the same which renders our model as a very good one (Bulator, Kpabitey, & Aboagye, 2015).

The observed value for average height of male adults in Ghana (2008) = 5 feet 2 inches. The Calculated value from our table for average height of male adult = 5 feet 1 inch. The above illustration serves to show that the mathematical model (4) as expressed by equation (6) does have a potential as a possible law of biological growth (or even growth in other fields, such as physics, chemistry, and economics).

$$Y_{max} = lim_{t\to\infty} \frac{y_1(y_0y_1 - 2y_0y_2 + y_1y_2}{y^2{}_1 - y_0y_2}$$

CONCLUSION

Our formulated model was used to predict the average mean height of adult male at full growth XYZ school, Kumasi.

The predicted height of the male children were obtained by using this Differential Equation model and were then compared with a template of data (Average height around the world in 2008) containing the average height of adult males across the world.

It was seen that the actual average height from the template and the predicted average height of male at full growth almost converged to the same values which signified that the model is go o d for the prediction of average height at full growth.

Furthermore, our study was also to gain an insight into the importance of predicting one's height. Today, ergonomics professionals apply an understanding of human factors to the design of equipment, systems and working methods in order to improve comfort, health, safety, and productivity.

Finally , our study also realized that variation in heights were caused by the following : genetic , nutrition and environmental factors, among others.

The average height in genetically and environmental homogeneous populations is often proportional across a large number of individuals. Exceptional height variation (around 20% deviation from a population's average) within such a population is sometimes due to gigantism or dwarfisms, which are caused by specific genes or endocrine abnormalities.

RECOMMENDATIONS

- Heights of children must be recorded at birth and various intervals to help make researches.
- Government can determine the maximum population of a country using our model and can also determine the expected population in a certain year.
- Researchers can further use our model in a case study against an actual data collected from statistics offices and various hospitals.
- Knowledge about the average height of a certain country can help various companies to know how to manufacture certain equipment.

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