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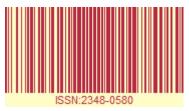


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NOTES ON MULTI FUZZY RW-CLOSED, MULTI FUZZY RW-OPEN SETS IN MULTI FUZZY TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we study some of the properties of multi fuzzy rw-closed and multi fuzzy rw-open sets in multi fuzzy topological spaces and prove some results on these.

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KEY WORDS: fuzzy subset, multi fuzzy subset, multi fuzzy topological spaces, multi fuzzy rw-closed, multi fuzzy rw-open.

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INTRODUCTION

The concept of a fuzzy subset was introduced and studied by L.A.Zadeh [15] in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. The following papers have motivated us to work on this paper. C.L.Chang [4] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces. Many researchers like R.H.Warren [14], K.K.Azad [1], G.Balasubramanian and P.Sundaram [2, 3], S.R.Malghan and S.S.Benchalli [10, 11] and many others have contributed to the development of fuzzy topological spaces. We introduce the concept of multi fuzzy rw-closed and multi fuzzy rw-open sets in multi fuzzy topological spaces and established some results.

1.PRELIMINARIES:

1.1 Definition[15]:Let X be a non-empty set. A fuzzy subset A of X is a function

 $A: X \rightarrow [0, 1].$

1.2 Definition: A multi fuzzy subset A of a set X is defined as an object of the form

 $A = \{ \langle x, A_1(x), A_2(x), A_3(x), ..., A_n(x) \rangle / x \in X \}, \text{ where } A_i : X \rightarrow [0, 1] \text{ for all } i. \text{ It is denoted as } A = \langle A_1, A_2, A_3, ..., A_n \rangle.$

1.3 Definition: Let A and B be any two multi fuzzy subsets of a set X. We define the following relations and operations:

(i) $A \subseteq B$ if and only if $A_i(x) \le B_i(x)$ for all i and for all x in X.

(ii) A = B if and only if $A_i(x) = B_i(x)$ for all i and for all x in X.

(iii) $A^{c} = 1-A = \langle 1-A_{1}, 1-A_{2}, 1-A_{3}, ..., 1-A_{n} \rangle$.

 $(iv) \ A \cap B = \{ \langle x, \min\{A_1(x), B_1(x) \}, \min\{A_2(x), B_2(x)\}, ..., \min\{A_n(x), B_n(x)\} \rangle \ / \ x \in X \}.$

 $(v) \ A \cup B = \{ \langle x, \max \{A_1(x), B_1(x) \}, \max\{A_2(x), B_2(x)\}, ..., \max\{A_n(x), B_n(x)\} \rangle \ / \ x \in X \}.$

1.4 Definition: Let X be a set and \Im be a family of multi fuzzy subsets of X. The family \Im is called a multi fuzzy topology on X if and only if \Im satisfies the following axioms

(i) $\overline{0}, \overline{1} \in \mathfrak{I},$

(ii) If { A_i; i \in I } $\subseteq \mathfrak{I}$, then $\bigcup_{i \in I} A_i \in \mathfrak{I}$,

(iii) If A₁, A₂, A₃,...., A_n $\in \mathfrak{I}$, then $\bigcap_{i=1}^{i=n} A_i \in \mathfrak{I}$.

The pair (X, \Im) is called a multi fuzzy topological space. The members of \Im are called multi fuzzy open sets in X. A multi fuzzy set A in X is said to be multi fuzzy closed set in X if and only if A^c is a multi fuzzy open set in X.

1.5 Definition: Let (X, \mathfrak{I}) be a multi fuzzy topological space and A be amulti fuzzy set in X. Then \cap { B : $B^c \in \mathfrak{I}$ and $B \supseteq A$ } is called multi fuzzy closure of A and is denoted by mfcl(A).

1.6 Theorem: Let A and B be two multi fuzzy sets in multi fuzzy topological space (X, \Im). Then the following results are true,

- I. mfcl(A) is a multi fuzzy closed set in X.
- II. mfcl(A) is the least multi fuzzy closed set containing A.
- III. A is a multi fuzzy closed if and only if A = mfcl(A).
- IV. mfcl($\overline{0}$) = $\overline{0}$, $\overline{0}$ is the empty multi fuzzy set
- V. mfcl(mfcl(A)) = mfcl(A).
- VI. $mfcl(A \cup B) = mfcl(A) \cup mfcl(B)$.

VII. $mfcl(A) \cap mfcl(B) \supseteq mfcl(A \cap B)$.

1.7 Definition: Let (X, \Im) be a multi fuzzy topological space and A be amulti fuzzy set in X. Then $\cup \{B : B \in \Im$ and $B \subseteq A$ is called multi fuzzy interior of A and is denoted by mfint(A).

1.8 Theorem: Let (X, \Im) be a multi fuzzy topological space, A and B be two multi fuzzy sets in X. The following results hold good,

- I. mfint (A) is a multi fuzzy open set in X.
- II. mfint(A) is the largest multi fuzzy open set in X which is less than or equal to A.
- III. A is a multi fuzzy open set if and only if A = mfint (A).
- IV. $A \subseteq B$ implies mfint (A) \subseteq mfint(B).
- V. mfint (mfint (A))= A.
- VI. $mfint(A \cap B) = mfint(A) \cap mfint(B)$.
- VII. mfint(A) \cup mfint(B) mfint(A \cup B).

VIII. mfint($\overline{1}$ –A)= $\overline{1}$ – mfcl(A).

IX. mfcl($\overline{1} - A$) = $\overline{1} - mfint(A)$.

1.9 Definition: Let (X, \Im) be a multi fuzzy topological space and A bemulti fuzzy set in X. Then A is said to be

- I. multi fuzzy semiopen if and only if there exists a multi fuzzy open set V in X such that $V \subseteq A \subseteq mfcl(V)$.
- II. multi fuzzy semiclosed if and only if there exists a multi fuzzy closed set V in X such that $mfint(V) \subseteq A \subseteq V$.

- III. multi fuzzy regular open set of X if mfint(mfcl(A)) = A.
- IV. multi fuzzy regular closed set of X if mfcl(mfint(A)) = A.
- V. multi fuzzy regular semiopen set of X if there exists a multi fuzzy regular open set V in X such thatV⊆ A ⊆mfcl(V). We denote the class of multi fuzzy regular semiopen sets in multi fuzzy topological space X by MFRSO(X).
- VI. multi fuzzy generalized closed (mfg-closed) if mfcl(A) \subseteq V whenever A \subseteq V and V is multi fuzzy open set and A is multi fuzzy generalized open if $\overline{1}$ A is multi fuzzy generalized closed.

1.10 Definition: An multi fuzzy set A of a multi fuzzy topological space (X, \Im) is called:

- I. multi fuzzy g-closed if mfcl(A) \subseteq V whenever A \subseteq V and V is multi fuzzy open set in X.
- II. multi fuzzy g-open if its complement A^c is multi fuzzy g-closed set in X.
- III. multi fuzzy rg-closed if mfcl(A) \subseteq V whenever A \subseteq V and V is multi fuzzy regular open set in X.
- IV. multi fuzzy rg-open if its complement A^c is multi fuzzy rg-closed set in X.
- V. multi fuzzy w-closed if mfcl(A) \subseteq V whenever A \subseteq V and V is multi fuzzy semi open set in X.
- VI. multi fuzzy w-open if its complement A^c is multi fuzzy w-closed set in X.
- VII. multi fuzzy gpr-closed if $pcl(A) \subseteq V$ whenever $A \subseteq V$ and V is multi fuzzy regular open set in X.
- VIII. multi fuzzy gpr-open if its complement A^c is multi fuzzy gpr-closed set in X.

1.11 Definition: Let (X, \Im) be a multi fuzzy topological space. A multi fuzzy set A of X is called multi fuzzy regular w-clsoed(briefly, multi fuzzy rw-closed) if mfcl(A) \subseteq U whenever A \subseteq U and U is multi fuzzy regular semiopen in multi fuzzy topological space X.

1.12 Definition: A multi fuzzy set A of a multi fuzzy topological space X is called a multi fuzzy regular w-open (briefly, multi fuzzy rw-open) set if its complement A^c is a multi fuzzyrw-closed set in multi fuzzy topological space X.

2. SOME PROPERTIES:

2.1 Theorem: Every multi fuzzy closed set is a multi fuzzyrw-closed set in a multi fuzzy topological space X.

2.2 Theorem: If A and B are multi fuzzy rw-closed sets in multi fuzzy topological space X, then union of A and B is multi fuzzy rw-closed set in multi fuzzy topological space X.

2.3 Theorem: If a multi fuzzy subset A of a multi fuzzy topological space X is both multi fuzzy regular semiopen and multi fuzzy rw-closed, then A is a multi fuzzy closed set in multi fuzzy topological space X.

2.4 Theorem: Let A be a multi fuzzy rw-closed set of a multi fuzzy topological space X and suppose A $\subseteq B \subseteq$ mfcl(A). Then B is also a multi fuzzy rw-closed set in multi fuzzy topological space X.

Proof: Let $A \subseteq B \subseteq mfcl(A)$ and A be a multi fuzzy rw-closed set of multi fuzzy topological space X. Let E be any multi fuzzy regular semiopen set such that $B \subseteq E$. Then $A \subseteq E$ and A is multi fuzzy rw-closed, we have $mfcl(A) \subseteq E$. But $mfcl(B) \subseteq mfcl(A)$ and thus $mfcl(B) \subseteq E$. Hence B is a multi fuzzy rw-closed set in multi fuzzy topological space X.

2.5 Theorem: In a multi fuzzy topological space X if MFRSO(X) = { $\overline{0}$, $\overline{1}$ }, where MFRSO(X) is the family of all multi fuzzy regular semiopen sets then every multi fuzzy subset of X is multi fuzzy rw-closed.

Proof: Let X be a multi fuzzy topological space and MFRSO(X) = { $\overline{0}$, $\overline{1}$ }. Let A be any multi fuzzy subset of X. Suppose A = $\overline{0}$. Then $\overline{0}$ is a multi fuzzyrw-closed set in multi fuzzy topological space X. Suppose A $\neq \overline{0}$. Then $\overline{1}$ is the only multi fuzzy regular semiopen set containing A and so mfcl(A) $\subseteq \overline{1}$. Hence A is a multi fuzzyrw-closed set in multi fuzzy topological space X.

2.1 Remark: The converse of the above Theorem 2.5 need not be true in general.

Proof: Consider the example: Let X = { 1, 2, 3 } and the multi fuzzy sets A, B be defined as A = { <1, 1, 1, 1 >, < 2, 0, 0, 0 >, < 3, 0, 0, 0 > } and B = { <1, 0, 0, 0 >, <2, 1, 1, 1 >, <3, 1, 1, 1 > }. Consider \Im = { $\overline{0}$, $\overline{1}$, A, B }. Then (X, \Im) is a multi fuzzy topological space. In this multi fuzzy topological space X, every multi fuzzy subset of X is a multi fuzzyrw-closed set in multi fuzzy topological space X, but MFRSO = { $\overline{0}$, $\overline{1}$, A, B }.

2.6 Theorem: If A is a multi fuzzyrw-closed set of multi fuzzy topological space X and mfcl(A) \cap ($\overline{1} - \text{mfcl}(A)$) = $\overline{0}$, then mfcl(A) – A does not contain any non-zero multi fuzzy regular semiopen set in multi fuzzy topological space X.

Proof: Suppose A is a multi fuzzyrw-closed set of multi fuzzy topological space X and mfcl(A) \cap ($\overline{1}-$ mfcl(A)) = $\overline{0}$. We prove the result by contradiction. Let B be a multi fuzzy regular semiopen set such that mfcl(A) – A \supseteq B and B $\neq \overline{0}$. Now B \subseteq mfcl(A) –A, i.e B $\subseteq \overline{1}$ –A which implies A $\subseteq \overline{1}$ – B. Since B is a multi fuzzy regular semiopen set, by Theorem 2.3, $\overline{1}$ –B is also multi fuzzy regular semiopen set in multi fuzzy topological space X. Since A is a multi fuzzyrw-closed set in multi fuzzy topological space X. So B $\subseteq \overline{1}$ – mfcl(A). Therefore B \subseteq mfcl(A) \cap ($\overline{1}$ – mfcl(A)) = $\overline{0}$, by hypothesis. This shows that B = $\overline{0}$ which is a contradiction. Hence mfcl(A) –A does not contain any non-zero multi fuzzy regular semiopen set in multi fuzzy topological space X.

2.2 Corollary: If A is a multi fuzzyrw-closed set of multi fuzzy topological space X and mfcl(A) \cap ($\overline{1} - \text{mfcl}(A)$) = $\overline{0}$, then mfcl(A) – A does not contain any non-zero multi fuzzy regular open set in multi fuzzy topological space X.

Proof: Follows from the Theorem 2.6 and the fact that every multi fuzzy regular open set is a multi fuzzy regular semiopen set in multi fuzzy topological space X.

2.3 Corollary: If A is a multi fuzzyrw-closed set of multi fuzzy topological space X and mfcl(A) \cap ($\overline{1} - \text{mfcl}(A)$) = $\overline{0}$, then mfcl(A) – A does not contain any non-zero multi fuzzy regular closed set in multi fuzzy topological space X.

Proof: Follows from the Theorem 2.6 and the fact that every multi fuzzy regular closed set is a multi fuzzy regular semiopen set in multi fuzzy topological space X.

2.7 Theorem: Let A be a multi fuzzy rw-closed set of multi fuzzy topological space X and mfcl(A) \cap ($\overline{1} - \text{mfcl}(A)$) = $\overline{0}$ Then A is a multi fuzzy closed set if and only if cl(A) – A is a multi fuzzy regular semiopen set in multi fuzzy topological space X.

Proof: Suppose A is a multi fuzzy closed set in multi fuzzy topological space X. Then mfcl(A) = A and so mfcl(A) – A = $\overline{0}$, which is a multi fuzzy regular semiopen set in multi fuzzy topological space X.

Conversely, suppose mfcl(A) –A is a multi fuzzy regular semiopen set in multi fuzzy topological space X. Since A is multi fuzzy rw-closed, by Theorem 2.6 mfcl(A) –A does not contain any-non zero multi fuzzy regular open set in multi fuzzy topological space X. Then mfcl(A) –A = $\overline{0}$. That is mfcl(A) = A and hence A is a multi fuzzy closed set in multi fuzzy topological space X.

2.8 Theorem: If a multi fuzzy subset A of a multi fuzzy topological space X ismulti fuzzy open, then it is multi fuzzy rw-open but not conversely.

Proof: Let A be a multi fuzzy open set of multi fuzzy topological space X. Then A^C is multi fuzzy closed. Now by Theorem 2.1,A^C is multi fuzzy rw-closed. Therefore A is a multi fuzzyrw-open set in multi fuzzy topological space X.

2.4 Remark: The converse of the above Theorem 2.8 need not be true in general.

Proof:Consider the example: Let X = { 1, 2, 3 }. Define a multi fuzzy subset A in X by A = { < 1, 1, 1, 1 >, < 2, 1, 1, 1 >, < 3, 0, 0, 0 > }. Let $\Im = \{\overline{0}, \overline{1}, A\}$. Then (X, \Im) is a multi fuzzy topological space. Define a multi fuzzy set B in X by B ={ < 1, 0, 0, 0 >, < 2, 1, 1, 1 >, < 3, 0, 0, 0 > }. Then B is a multi fuzzyrw-open set but it is not multi fuzzy open set in multi fuzzy topological space X.

2.9 Theorem: A multi fuzzy subset A of a multi fuzzy topological space X is multi fuzzy rw-open if and only if $D \subseteq mfint(A)$, whenever $D \subseteq A$ and D is a multi fuzzy regular semiopen set in multi fuzzy topological space X.

Proof: Suppose that D \subseteq mfint(A), whenever D \subseteq A and D is a multi fuzzy regular semiopen set in multi fuzzy topological space X. To prove that A is multi fuzzy rw-open in multi fuzzy topological space X. Let $A^C \subseteq B$ and B is any multi fuzzy regular semiopen set in multi fuzzy topological space X. Then $B^C \subseteq A$. By Theorem 2.3, B^C is also multi fuzzy regular semiopen set in multi fuzzy topological space X. Then $B^C \subseteq A$. By Theorem 2.3, B^C is also multi fuzzy regular semiopen set in multi fuzzy topological space X. By hypothesis, $B^C \subseteq$ mfint(A) which implies (mfint(A)) $^C \subseteq B$. That is mfcl(A^C) $\subseteq B$, since mfcl(A^C) = (mfint(A)) C . Thus A^C is a multi fuzzyrw-closed and hence A is multi fuzzy rw-open in multi fuzzy topological space X.

Conversely, suppose that A is multi fuzzy rw-open. Let $B \subseteq A$ and B is any multi fuzzy regular semiopen in multi fuzzy topological space X. Then $A^C \subseteq B^C$. By Theorem 2.3, B^C is also multi fuzzy regular semiopen. Since A^C is multi fuzzy rw-closed, we have mfcl(A^C) $\subseteq B^C$ and so B \subseteq mfint(A), since mfcl(A^C) = (mfint(A))^C.

2.10 Theorem: If A and B are multi fuzzy rw-open sets in a multi fuzzy topological space X, then $A \cap B$ is also a multi fuzzy rw-open set in multi fuzzy topological space X.

Proof: Let A and B be two multi fuzzy rw-open sets in a multi fuzzy topological space X. Then A^c and B^c are multi fuzzy rw-closed sets in multi fuzzy topological space X. By Theorem 2.2, $A^c \cup B^c$ is also a multi fuzzy rw-closed set in multi fuzzy topological space X. That is $(A^c \cup B^c) = (A \cap B)^c$ is a multi fuzzyrw-closed set in X. Therefore $A \cap B$ is also a multi fuzzy rw-open set in multi fuzzy topological space X.

2.11 Theorem: The union of any two multi fuzzy rw-open sets in a multi fuzzy topological space X is generally not a multi fuzzy rw-open set in multi fuzzy topological space X.

Proof: Consider the multi fuzzy topological space (X, \Im) defined as in Remark 2.4. In this multi fuzzy topological space X, the multi fuzzy sets D_1 , D_2 are defined by $D_1 = \{<1, 1, 1, 1>, <2, 0, 0, 0>, <3, 0, 0, 0>\}$ and $D_2 = \{<1, 0, 0, 0>, <2, 0, 0, 0>, <3, 1, 1, 1>\}$. Then D_1 and D_2 are the multi fuzzy rwopen sets in multi fuzzy topological space X. Let $E = D_1 \cup D_2$. Then $E = \{<1, 1, 1, 1>, <2, 0, 0, 0>, <3, 1, 1, 1>\}$. Then $E = D_1 \cup D_2$ is not a multi fuzzy rwopen set in multi fuzzy topological space X.

2.12 Theorem: If mfint(A) \subseteq B \subseteq A and A is a multi fuzzyrw-open set in a multi fuzzy topological space X, then B is also a multi fuzzy rw-open set in multi fuzzy topological space X.

Proof: Suppose mfint(A) \subseteq B \subseteq A and A is a multi fuzzyrw-open set in a multi fuzzy topological space X. To prove that B is a multi fuzzyrw-open set in multi fuzzy topological space X. Let F be any multi fuzzy regular semiopen set in multi fuzzy topological space X such that F \subseteq B. Now F \subseteq B \subseteq A. That is F \subseteq A. Since A is multi fuzzy rw-open set of multi fuzzy topological space X, F \subseteq mfint(A), by Theorem 2.9. By hypothesis mfint(A) \subseteq B. Then mfint(mfint(A)) \subseteq mfint(B). That is mfint(A) \subseteq mfint(B). Then F \subseteq mfint(B). Again by Theorem 2.9, B is a multi fuzzyrw-open set in multi fuzzy topological space X.

2.13 Theorem: If a multi fuzzy subset \overline{A} of a multi fuzzy topological space X is multi fuzzy rw-closed and mfcl(A) \cap ($\overline{1} - mfcl(A)$) = $\overline{0}$, then mfcl(A) -A is a multi fuzzyrw-open set in multi fuzzy topological space X.

Proof: Let A be a multi fuzzy rw-closed set in an multi fuzzy topological space X and mfcl(A) \cap ($\overline{1} - \text{mfcl}(A)$) = $\overline{0}$. Let B be any multi fuzzy regular semiopen set of multi fuzzy topological space X such that B \subseteq (mfcl(A) – A). Then by Theorem 2.6, mfcl(A) –A does not contain any non-zero multi fuzzy regular semiopen set and so B = $\overline{0}$. Therefore B \subseteq mfint(mfcl(A) –A). By Theorem 2.9, mfcl(A) –A is multi fuzzy rw-open.

2.14 Theorem: Let A and B be two multi fuzzy subsets of a multi fuzzy topological space X. If B is a multi fuzzyrw-open set and A \supseteq mfint(B), then A \cap B is a multi fuzzyrw-open set in multi fuzzy topological space X.

Proof: Let B be a multi fuzzy rw-open set of a multi fuzzy topological space X and A \supseteq mfint(B). That is mfint(B) \subseteq (A \cap B). Also mfint(B) \subseteq (A \cap B) \subseteq B and B is amulti fuzzyrw-open set. By Theorem 2.12, A \cap B is also a multi fuzzy rw-open set in multi fuzzy topological space X.

2.5 Remark: Every multi fuzzy w-open set is multi fuzzyrw-open but its converse may not be true.

Proof: Consider the example: Let X = { 1, 2 } and \Im = { $\overline{1}$, $\overline{0}$, A } be a multi fuzzy topology on X, where A = { < 1, 0.7, 0.7, 0.7 >, < 2, 0.6, 0.6, 0.6 >}. Then the multi fuzzy set B = {<1, 0.2, 0.2, 0.2 >, < 2, 0.1, 0.1, 0.1 > } is multi fuzzyrw-open in (X, \Im) but it is not multi fuzzy w-open in (X, \Im).

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