Vol.4.Issue.1.2016 (January-March)

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

http://www.bomsr.com
Email:editorbomsr@gmail.com

ISSN:2348-0580

RESEARCH ARTICLE

INTERVAL VALUED INTUITIONISTIC FUZZY SUBSEMIRINGS OF A SEMIRING

K.MURUGALINGAM[1], K.ARJUNAN[2]

[1]Department of Mathematics, Dr.Zakir Hussain College, Ilayangudi,Tamilnadu, India.
Email:kp.murugalingam@rediffmail.com

[2]Department of Mathematics, H.H.The Rajahs College, Pudukkottai,Tamilnadu, India.
Email: arjunan.karmegam@gmail.com

ABSTRACT

In this paper, we study some of the properties of interval valued intuitionistic fuzzy subsemiring of a semiring and prove some results on these.

2000 AMS SUBJECT CLASSIFICATION: 03F55, 08A72, 20N25.

KEY WORDS: Interval valued fuzzy subset, interval valued fuzzy subsemiring, interval valued intuitionistic fuzzy subset, interval valued intuitionistic fuzzy subsemiring.

INTRODUCTION

Interval-valued fuzzy sets were introduced independently by Zadeh [13], Grattan-Guiness [5], Jahn [7], in the seventies, in the same year. An interval valued fuzzy set (IVF) is defined by an interval-valued membership function. Jun.Y.B and Kin.K.H[8] defined an interval valued fuzzy R-subgroups of nearrings. Solairaju.A and Nagarajan.R[10] defined the charactarization of interval valued anti fuzzy Left h-ideals over Hemirings. M.G.Somasundara Moorthy and K. Arjunan[11] have defined an interval valued fuzzy subring of a ring under homomorphism. We introduce the concept of interval valued intuitionistic fuzzy subsemiring of a semiring and established some results.

1.PRELIRMINARIES

1.1 Definition: Let X be any nonempty set. A mapping [M] : X $\rightarrow$ D[0, 1] is called an interval valued fuzzy subset (briefly, IVFS) of X, where D[0,1] denotes the family of all closed subintervals of [0,1] and $[M](x) = [M^{-}(x), M^{+}(x)]$, for all x in X, where M^{-} and M^{+} are fuzzy subsets of X such that $M^{-}(x) \leq M^{+}(x)$, for all x in X. Thus $M^{-}(x)$ is an interval (a closed subset of [0,1]) and not a number from the interval [0,1] as in the case of fuzzy subset. Note that [0] = [0, 0] and [1] = [1, 1].

1.2 Definition: Let (R, +, $\cdot$) be a semiring. An interval valued fuzzy subset [M] of R is said to be an **interval valued fuzzy subsemiring** of R if the following conditions are satisfied:

(i) $[M](x+y) \geq \text{rmin}\{ [M](x), [M](y) \}$

(ii) $[M](xy) \geq \text{rmin}\{ [M](x), [M](y) \}$ for all x and y in R.

1.3 Definition: An **interval valued intuitionistic fuzzy subset** (IVIFS) [A] in X is defined as an object of the form [A] = { < x, $\mu_{[A]}(x)$, $\nu_{[A]}(x)$ > / x in X}, where

$\mu_{[A]} : X \to D[0, 1]$ and $\nu_{[A]} : X \to D[0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and for every $x \in X$ satisfying $\mu_{[A]}^{+}(x) + \nu_{[A]}^{+}(x) \leq 1$.

1.4 Definition: Let (R, +, .) be a semiring. An interval valued intuitionistic fuzzy subset [A] of R is said to be an interval valued intuitionistic fuzzy subsemiring of R if it satisfies the following axioms:

(i) $\mu_{[A]}(x+y) \geq \text{rmin}\{\mu_{[A]}(x), \mu_{[A]}(y)\} = [\min\{\mu_{[A]}^{-}(x), \mu_{[A]}^{-}(y)\}, \min\{\mu_{[A]}^{+}(x), \mu_{[A]}^{+}(y)\}]$

(ii) $\mu_{[A]}(xy) \geq \text{rmin}\{\mu_{[A]}(x), \mu_{[A]}(y)\} = [\min\{\mu_{[A]}^{-}(x), \mu_{[A]}^{-}(y)\}, \min\{\mu_{[A]}^{+}(x), \mu_{[A]}^{+}(y)\}]$

(iii) $\nu_{[A]}(x+y) \leq \text{rmax}\{\nu_{[A]}(x), \nu_{[A]}(y)\} = [\max\{\nu_{[A]}^{-}(x), \nu_{[A]}^{-}(y)\}, \max\{\nu_{[A]}^{+}(x), \nu_{[A]}^{+}(y)\}]$

(iv) $\nu_{[A]}(xy) \leq \text{rmax}\{\nu_{[A]}(x), \nu_{[A]}(y)\} = [\max\{\nu_{[A]}^{-}(x), \nu_{[A]}^{-}(y)\}, \max\{\nu_{[A]}^{+}(x), \nu_{[A]}^{+}(y)\}]$, for all x and y in R.

1.5 Definition: Let [A] be an interval valued intuitionistic fuzzy subsemiring of a semiring R and a in R. Then the **pseudo interval valued intuitionistic fuzzy coset** $(aA)^p$ is defined by $((a\mu_{[A]})^p)(x) = p(a)\mu_{[A]}(x)$ and $((a\nu_{[A]})^p)(x) = p(a)\nu_{[A]}(x)$ for every x in R and for some p in P.

1.6 Definition: Let [A] and [B] be interval valued intuitionistic fuzzy subsets of sets G and H, respectively. The **product** of [A] and [B], denoted by [A]×[B], is defined as [A]×[B] = {⟨ (x, y), $\mu_{[A]\times[B]}(x,y)$, $\nu_{[A]\times[B]}(x,y)$ ⟩ / for all x in G and y in H }, where

$\mu_{[A]\times[B]}(x, y) = \text{rmin}\{\mu_{[A]}(x), \mu_{[B]}(y)\}$ and $\nu_{[A]\times[B]}(x, y) = \text{rmax}\{\nu_{[A]}(x), \nu_{[B]}(y)\}$.

1.7 Definition: Let [A] be an interval valued intuitionistic fuzzy subset in a set S. The **strongest interval valued intuitionistic fuzzy relation** on S, that is an interval valued intuitionistic fuzzy relation on [A] is [V] given by $\mu_{[V]}(x, y) = \text{rmin}\{\mu_{[A]}(x), \mu_{[A]}(y)\}$ and $\nu_{[V]}(x, y) = \text{rmax}\{\nu_{[A]}(x), \nu_{[A]}(y)\}$ for all x and y in S.

1.8 Definition: Let [A] be an interval valued intuitionistic fuzzy subset of X. Then the following operations are defined as

(i) ?(*[A]*) = { ⟨ x, rmin { [½, ½], $\mu_{[A]}(x)$ }, rmax { [½, ½], $\nu_{[A]}(x)$ }⟩ / for all $x \in X$ }.

(ii) !(*[A]*) = { ⟨ x, rmax { [½, ½], $\mu_{[A]}(x)$ }, rmin { [½, ½], $\nu_{[A]}(x)$ }⟩/ for all $x \in X$ }.

(iii) $Q_{\alpha,\beta}$(*[A]*) ={⟨x, rmin{α, $\mu_{[A]}(x)$}, rmax{β, $\nu_{[A]}(x)$}⟩ /for all $x \in X$ and α, β in D[0, 1]}.

(iv) $P_{\alpha,\beta}$(*[A]*)={⟨x, rmax{α, $\mu_{[A]}(x)$}, rmin{β, $\nu_{[A]}(x)$}⟩ / for all $x \in X$ and α, β in D[0, 1]}.

(v) $G_{\alpha,\beta}$(*[A]*) = { ⟨ x, α $\mu_{[A]}(x)$, β $\nu_{[A]}(x)$ } ⟩ / for all $x \in X$ and α, β in [0, 1] }.

2. SOME PROPERTIES

2.1 Theorem: Intersection of any two interval valued intuitionistic fuzzy subsemirings of a semiring R is an interval valued intuitionistic fuzzy subsemiring of R.

Proof: Let [A] and [B] be any two interval valued intuitionistic fuzzy subsemirings of a semiring R and x, y in R. Let [A] = {(x, $\mu_{[A]}(x)$, $\nu_{[A]}(x)$) / $x \in R$} and [B] = { (x, $\mu_{[B]}(x)$, $\nu_{[B]}(x)$) / $x \in R$} and also let [C] = $[A] \cap [B]$ = { (x, $\mu_{[C]}(x)$, $\nu_{[C]}(x)$) / $x \in R$}, where

$\mu_{[C]}(x) = \text{rmin}\{\mu_{[A]}(x), \mu_{[B]}(x)\}$ and $\nu_{[C]}(x) = \text{rmax}\{\nu_{[A]}(x), \nu_{[B]}(x)\}$. Now $\mu_{[C]}(x+y) = \text{rmin}\{\mu_{[A]}(x+y), \mu_{[B]}(x+y)\} \geq \text{rmin}\{\text{rmin}\{\mu_{[A]}(x), \mu_{[A]}(y)\}, \text{rmin}\{\mu_{[B]}(x), \mu_{[B]}(y)\}\} = \text{rmin}\{\text{rmin}\{\mu_{[A]}(x), \mu_{[B]}(x)\}, \text{rmin}\{\mu_{[A]}(y), \mu_{[B]}(y)\}\} = \text{rmin}\{\mu_{[C]}(x), \mu_{[C]}(y)\}$. Therefore $\mu_{[C]}(x+y) \geq \text{rmin}\{\mu_{[C]}(x), \mu_{[C]}(y)\}$ for all x, y in R. And $\mu_{[C]}(xy) = \text{rmin}\{\mu_{[A]}(xy), \mu_{[B]}(xy)\} \geq \text{rmin}\{\text{rmin}\{\mu_{[A]}(x), \mu_{[A]}(y)\}, \text{rmin}\{\mu_{[B]}(x), \mu_{[B]}(y)\}\} = \text{rmin}\{\text{rmin}\{\mu_{[A]}(x), \mu_{[B]}(x)\}, \text{rmin}\{\mu_{[A]}(y), \mu_{[B]}(y)\}\} = \text{rmin}\{\mu_{[C]}(x), \mu_{[C]}(y)\}$. Therefore $\mu_{[C]}(xy) \geq \text{rmin}\{\mu_{[C]}(x), \mu_{[C]}(y)\}$ for all x, y in R. Now $\nu_{[C]}(x+y) = \text{rmax}\{\nu_{[A]}(x+y), \nu_{[B]}(x+y)\} \leq \text{rmax}\{\text{rmax}\{\nu_{[A]}(x), \nu_{[A]}(y)\}, \text{rmax}\{\nu_{[B]}(x), \nu_{[B]}(y)\}\} = \text{rmax}\{\text{rmax}\{\nu_{[A]}(x), \nu_{[B]}(x)\}, \text{rmax}\{\nu_{[A]}(y), \nu_{[B]}(y)\}\} = \text{rmax}\{\nu_{[C]}(x), \nu_{[C]}(y)\}$. Therefore $\nu_{[C]}(x+y) \leq \text{rmax}\{\nu_{[C]}(x), \nu_{[C]}(y)\}$ for all x, y in R. And $\nu_{[C]}(xy) = \text{rmax}\{\nu_{[A]}(xy), \nu_{[B]}(xy)\} \leq \text{rmax}\{\text{rmax}\{\nu_{[A]}(x), \nu_{[A]}(y)\}, \text{rmax}\{\nu_{[B]}(x), \nu_{[B]}(y)\}\} = \text{rmax}\{\text{rmax}\{\nu_{[A]}(x), \nu_{[B]}(x)\}, \text{rmax}\{\nu_{[A]}(y), \nu_{[B]}(y)\}\} = \text{rmax}\{\nu_{[C]}(x), \nu_{[C]}(y)\}$. Therefore $\nu_{[C]}(xy) \leq \text{rmax}\{\nu_{[C]}(x), \nu_{[C]}(y)\}$, for all x, y in R. Therefore [C] is an interval valued intuitionistic fuzzy subsemiring of R. Hence the intersection of any two interval valued intuitionistic fuzzy subsemirings of a semiring R is an interval valued intuitionistic fuzzy subsemiring of R.

2.2 Theorem: The intersection of a family of interval valued intuitionistic fuzzy subsemirings of a semiring R is an interval valued intuitionistic fuzzy subsemiring of R.

Proof: It is trivial.

2.3 Theorem: If [A] and [B] are any two interval valued intuitionistic fuzzy subsemirings of the semirings R_1 and R_2 respectively, then [A]×[B] is an interval valued intuitionistic fuzzy subsemiring of $R_1 \times R_2$.

Proof: Let [A] and [B] be two interval valued intuitionistic fuzzy subsemirings of the semirings R_1 and R_2 respectively. Let x_1, x_2 in R_1 and y_1, y_2 in R_2. Then (x_1, y_1) and (x_2, y_2) are in $R_1 \times R_2$. Now $\mu_{[A]\times[B]}[(x_1, y_1)+(x_2, y_2)] = \mu_{[A]\times[B]}(x_1+x_2, y_1+y_2) = \text{rmin}\{ \mu_{[A]}(x_1+x_2), \mu_{[B]}(y_1+y_2) \} \geq \text{rmin}\{ \text{rmin}\{ \mu_{[A]}(x_1), \mu_{[A]}(x_2) \}, \text{rmin}\{ \mu_{[B]}(y_1), \mu_{[B]}(y_2)\}\} = \text{rmin}\{ \text{rmin}\{\mu_{[A]}(x_1), \mu_{[B]}(y_1) \}, \text{rmin}\{\mu_{[A]}(x_2), \mu_{[B]}(y_2)\}\} = \text{rmin}\{ \mu_{[A]\times[B]}(x_1, y_1), \mu_{[A]\times[B]}(x_2, y_2)\}$. Therefore $\mu_{[A]\times[B]}[(x_1, y_1)+(x_2, y_2)] \geq \text{rmin}\{ \mu_{[A]\times[B]}(x_1, y_1), \mu_{[A]\times[B]}(x_2, y_2)\}$. Also $\mu_{[A]\times[B]}[(x_1, y_1)(x_2, y_2)] = \mu_{[A]\times[B]}(x_1x_2, y_1y_2) = \text{rmin}\{ \mu_{[A]}(x_1x_2), \mu_{[B]}(y_1y_2) \} \geq \text{rmin}\{\text{rmin}\{ \mu_{[A]}(x_1), \mu_{[A]}(x_2) \}, \text{rmin}\{ \mu_{[B]}(y_1), \mu_{[B]}(y_2)\}\} = \text{rmin}\{ \text{rmin}\{ \mu_{[A]}(x_1), \mu_{[B]}(y_1) \}, \text{rmin}\{ \mu_{[A]}(x_2), \mu_{[B]}(y_2)\}\} = \text{rmin}\{ \mu_{[A]\times[B]}(x_1, y_1), \mu_{[A]\times[B]}(x_2, y_2) \}$. Therefore $\mu_{[A]\times[B]}[(x_1, y_1)(x_2, y_2)] \geq \text{rmin}\{ \mu_{[A]\times[B]}(x_1, y_1), \mu_{[A]\times[B]}(x_2, y_2)\}$. Now $\nu_{[A]\times[B]}[(x_1, y_1)+(x_2, y_2)] = \nu_{[A]\times[B]}(x_1+x_2, y_1+y_2) = \text{rmax}\{ \nu_{[A]}(x_1+x_2), \nu_{[B]}(y_1+y_2) \} \leq \text{rmax}\{ \text{rmax}\{ \nu_{[A]}(x_1), \nu_{[A]}(x_2)\}, \text{rmax}\{\nu_{[B]}(y_1), \nu_{[B]}(y_2)\}\} = \text{rmax}\{ \text{rmax}\{ \nu_{[A]}(x_1), \nu_{[B]}(y_1) \}, \text{rmax}\{ \nu_{[A]}(x_2), \nu_{[B]}(y_2)\}\} = \text{rmax}\{ \nu_{[A]\times[B]}(x_1, y_1), \nu_{[A]\times[B]}(x_2, y_2) \}$. Therefore $\nu_{A\times B}[(x_1, y_1)+(x_2, y_2)] \leq \text{rmax}\{\nu_{[A]\times[B]}(x_1, y_1), \nu_{[A]\times[B]}(x_2, y_2)\}$. Also $\nu_{[A]\times[B]}[(x_1, y_1) (x_2, y_2)] = \nu_{[A]\times[B]}(x_1x_2, y_1y_2) = \text{rmax}\{ \nu_{[A]}(x_1x_2), \nu_{[B]}(y_1y_2) \} \leq \text{rmax}\{ \text{rmax}\{ \nu_{[A]}(x_1), \nu_{[A]}(x_2) \}, \text{rmax}\{ \nu_{[B]}(y_1), \nu_{[B]}(y_2)\}\} = \text{rmax}\{ \text{rmax}\{ \nu_{[A]}(x_1), \nu_{[B]}(y_1)\}, \text{rmax}\{ \nu_{[A]}(x_2), \nu_{[B]}(y_2)\}\} = \text{rmax}\{ \nu_{[A]\times[B]}(x_1, y_1), \nu_{[A]\times[B]}(x_2, y_2)\}$. Therefore, $\nu_{[A]\times[B]}[(x_1, y_1)(x_2, y_2)] \leq \text{rmax}\{ \nu_{[A]\times[B]}(x_1, y_1), \nu_{[A]\times[B]}(x_2, y_2)\}$. Hence [A]×[B] is an interval valued intuitionistic fuzzy subsemiring of $R_1 \times R_2$.

2.4 Theorem: Let [A] be an interval valued intuitionistic fuzzy subset of a semiring R and [V] be the strongest interval valued intuitionistic fuzzy relation of R. If [A] is an interval valued intuitionistic fuzzy subsemiring of R, then [V] is an interval valued intuitionistic fuzzy subsemiring of R×R.

Proof: Suppose that [A] is an interval valued intuitionistic fuzzy subsemiring of a semiring R. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ in R×R, we have $\mu_{[V]}(x+y) = \mu_{[V]}[(x_1, x_2)+(y_1, y_2)] = \mu_{[V]}(x_1+y_1, x_2+y_2) = \text{rmin}\{ \mu_{[A]}(x_1+y_1), \mu_{[A]}(x_2+y_2) \} \geq \text{rmin}\{ \text{rmin}\{\mu_{[A]}(x_1), \mu_{[A]}(y_1) \}, \text{rmin}\{ \mu_{[A]}(x_2), \mu_{[A]}(y_2)\}\} = \text{rmin}\{ \text{rmin}\{ \mu_{[A]}(x_1), \mu_{[A]}(x_2) \}, \text{rmin}\{ \mu_{[A]}(y_1), \mu_{[A]}(y_2)\}\} = \text{rmin}\{ \mu_{[V]}(x_1, x_2), \mu_{[V]}(y_1, y_2) \} = \text{rmin}\{ \mu_{[V]}(x), \mu_{[V]}(y) \}$. Therefore $\mu_{[V]}(x+y) \geq \text{rmin}\{ \mu_{[V]}(x), \mu_{[V]}(y)\}$ for all x and y in R×R. And $\mu_{[V]}(xy) = \mu_{[V]}[(x_1, x_2)(y_1, y_2)] = \mu_{[V]}(x_1y_1, x_2y_2) = \text{rmin}\{ \mu_{[A]}(x_1y_1), \mu_{[A]}(x_2y_2) \} \geq \text{rmin}\{ \text{rmin}\{ \mu_{[A]}(x_1), \mu_{[A]}(y_1) \}, \text{rmin}\{ \mu_{[A]}(x_2), \mu_{[A]}(y_2) \}\} = \text{rmin}\{ \text{rmin}\{ \mu_{[A]}(x_1), \mu_{[A]}(x_2) \}, \text{rmin}\{ \mu_{[A]}(y_1), \mu_{[A]}(y_2)\}\} = \text{rmin}\{ \mu_{[V]}(x_1, x_2), \mu_{[V]}(y_1, y_2) \} = \text{rmin}\{ \mu_{[V]}(x), \mu_{[V]}(y) \}$. Therefore $\mu_{[V]}(xy) \geq \text{rmin}\{ \mu_{[V]}(x), \mu_{[V]}(y) \}$ for all x and y in R×R. We have $\nu_{[V]}(x+y) = \nu_{[V]}[(x_1, x_2)+(y_1, y_2)] = \nu_{[V]}(x_1+y_1, x_2+y_2) = \text{rmax}\{ \nu_{[A]}(x_1+y_1), \nu_{[A]}(x_2+y_2) \} \leq \text{rmax}\{ \text{rmax}\{\nu_{[A]}(x_1), \nu_{[A]}(y_1) \}, \text{rmax}\{\nu_{[A]}(x_2), \nu_{[A]}(y_2)\}\} = \text{rmax}\{ \text{rmax}\{ \nu_{[A]}(x_1), \nu_{[A]}(x_2)\}, \text{rmax}\{\nu_{[A]}(y_1), \nu_{[A]}(y_2)\}\} = \text{rmax}\{ \nu_{[V]}(x_1, x_2), \nu_{[V]}(y_1, y_2) \} = \text{rmax}\{\nu_{[V]}(x), \nu_{[V]}(y) \}$. Therefore $\nu_{[V]}(x+y) \leq \text{rmax}\{ \nu_{[V]}(x), \nu_{[V]}(y) \}$ for all x and y in R×R. And $\nu_{[V]}(xy) = \nu_{[V]}[(x_1, x_2)(y_1, y_2)] = \nu_{[V]}(x_1y_1, x_2y_2) = \text{rmax}\{ \nu_{[A]}(x_1y_1), \nu_{[A]}(x_2y_2) \} \leq \text{rmax}\{ \text{rmax}\{ \nu_{[A]}(x_1), \nu_{[A]}(y_1) \}, \text{rmax}\{\nu_{[A]}(x_2), \nu_{[A]}(y_2) \}\} = \text{rmax}\{ \text{rmax}\{ \nu_{[A]}(x_1), \nu_{[A]}(x_2) \}, \text{rmax}\{ \nu_{[A]}(y_1), \nu_{[A]}(y_2) \}\} = \text{rmax}\{ \nu_{[V]}(x_1, x_2), \nu_{[V]}(y_1, y_2) \} = \text{rmax}\{ \nu_{[V]}(x), \nu_{[V]}(y) \}$. Therefore $\nu_{[V]}(xy) \leq \text{rmax}\{ \nu_{[V]}(x), \nu_{[V]}(y) \}$ for all x and y in R×R. Hence [V] is an interval valued intuitionistic fuzzy subsemiring of R×R.

2.5 Theorem: If [A] is an interval valued intuitionistic fuzzy subsemiring of a semiring (R, +, ·), then $H = \{ x / x \in R: \mu_{[A]}(x) = [1], \nu_{[A]}(x) = [0] \}$ is either empty or a subsemiring of R.

Proof: If no element satisfies this condition, then H is empty. If x, y in H, then $\mu_{[A]}(x+y) \geq$ rmin { $\mu_{[A]}(x)$, $\mu_{[A]}(y)$ } = rmin { [1], [1] } = [1]. Therefore $\mu_{[A]}(x+y)$ = [1]. And $\mu_{[A]}(xy) \geq$ rmin { $\mu_{[A]}(x)$, $\mu_{[A]}(y)$ } = rmin { [1], [1] } = [1]. Therefore $\mu_{[A]}(xy)$ = [1]. Now $\nu_{[A]}(x+y) \leq$ rmax { $\nu_{[A]}(x)$, $\nu_{[A]}(y)$ } = rmax { [0], [0] } = [0]. Therefore $\nu_{[A]}(x+y)$ = [0]. And $\nu_{[A]}(xy) \leq$ rmax { $\nu_{[A]}(x)$, $\nu_{[A]}(y)$ }= rmax { [0], [0] } = [0]. Therefore $\nu_{[A]}(xy)$ = [0]. We get x+y, xy in H. Therefore H is a subsemiring of R. Hence H is either empty or a subsemiring of R.

2.6 Theorem: Let [A] be an interval valued intuitionistic fuzzy subsemiring of a semiring (R, +, ·). (i) If $\mu_{[A]}(x+y)$ = [0], then either $\mu_{[A]}(x)$ = [0] or $\mu_{[A]}(y)$ = [0] for all x, y in R. (ii) If $\nu_{[A]}(x+y)$ = [1], then either $\nu_{[A]}(x)$ = [1] or $\nu_{[A]}(y)$ = [1] for all x, y in R. (iii) If $\mu_{[A]}(xy)$ = [0], then either $\mu_{[A]}(x)$ = [0] or $\mu_{[A]}(y)$ = [0] for all x, y in R. (iv) If $\nu_{[A]}(xy)$ = [1], then either $\nu_{[A]}(x)$ = [1] or $\nu_{[A]}(y)$ = [1] for all x, y in R.

Proof: Let x and y in R. (i) By the definition $\mu_{[A]}(x+y) \geq$ rmin { $\mu_{[A]}(x)$, $\mu_{[A]}(y)$ } which implies that [0] $\geq$ rmin { $\mu_{[A]}(x)$, $\mu_{[A]}(y)$ }. Therefore either $\mu_{[A]}(x)$ = [0] or $\mu_{[A]}(y)$ = [0]. (ii) By the definition $\nu_{[A]}(x+y) \leq$ rmax { $\nu_{[A]}(x)$, $\nu_{[A]}(y)$ } which implies that [1] $\leq$ rmax {$\nu_{[A]}(x)$, $\nu_{[A]}(y)$ }. Therefore either $\nu_{[A]}(x)$ = [1] or $\nu_{[A]}(y)$ = [1]. (iii) By the definition $\mu_{[A]}(xy) \geq$ rmin { $\mu_{[A]}(x)$, $\mu_{[A]}(y)$ } which implies that [0] $\geq$ rmin { $\mu_{[A]}(x)$, $\mu_{[A]}(y)$ }. Therefore either $\mu_{[A]}(x)$ = [0] or $\mu_{[A]}(y)$ = [0]. (iv) By the definition $\nu_{[A]}(xy) \leq$ rmax {$\nu_{[A]}(x)$, $\nu_{[A]}(y)$ } which implies that [1] $\leq$ rmax { $\nu_{[A]}(x)$, $\nu_{[A]}(y)$ }. Therefore either $\nu_{[A]}(x)$ = [1] or $\nu_{[A]}(y)$ = [1].

2.7 Theorem: If [A] is an interval valued intuitionistic fuzzy subsemiring of a semiring (R, +, ·), then $\square$[A] is an interval valued intuitionistic fuzzy subsemiring of R.

Proof: Let [A] be an interval valued intuitionistic fuzzy subsemiring of a semiring R. Consider [A] = { 〈 x, $\mu_{[A]}(x)$, $\nu_{[A]}(x)$ 〉 } for all x in R, we take $\square$[A] = [B] = { 〈 x, $\mu_{[B]}(x)$, $\nu_{[B]}(x)$ 〉 }, where $\mu_{[B]}(x) = \mu_{[A]}(x)$, $\nu_{[B]}(x)$ = [1]– $\mu_{[A]}(x)$. Clearly $\mu_{[B]}(x+y) \geq$ rmin {$\mu_{[B]}(x)$, $\mu_{[B]}(y)$ } for all x, y in R and $\mu_{[B]}(xy) \geq$ rmin { $\mu_{[B]}(x)$, $\mu_{[B]}(y)$ } for all x, y in R. Since [A] is an interval valued intuitionistic fuzzy subsemiring of R, we have $\mu_{[A]}(x+y) \geq$ rmin { $\mu_{[A]}(x)$, $\mu_{[A]}(y)$ } for all x, y in R which implies that [1]– $\nu_{[B]}(x+y) \geq$ rmin { [1]– $\nu_{[B]}(x)$, [1]– $\nu_{[B]}(y)$ } which implies that $\nu_{[B]}(x+y) \leq$ [1]– rmin { [1]– $\nu_{[B]}(x)$, [1]– $\nu_{[B]}(y)$ } = rmax {$\nu_{[B]}(x)$, $\nu_{[B]}(y)$ }. Therefore $\nu_{[B]}(x+y) \leq$ rmax { $\nu_{[B]}(x)$, $\nu_{[B]}(y)$ } for all x, y in R. And $\mu_{[A]}(xy) \geq$ rmin { $\mu_{[A]}(x)$, $\mu_{[A]}(y)$ } for all x, y in R which implies that [1]– $\nu_{[B]}(xy) \geq$ rmin { [1]– $\nu_{[B]}(x)$, [1]– $\nu_{[B]}(y)$ } which implies that $\nu_{[B]}(xy) \leq$ [1]– rmin { [1]– $\nu_{[B]}(x)$, [1]– $\nu_{[B]}(y)$ } = rmax { $\nu_{[B]}(x)$, $\nu_{[B]}(y)$ }. Therefore $\nu_{[B]}(xy) \leq$ rmax { $\nu_{[B]}(x)$, $\nu_{[B]}(y)$ } for all x, y in R. Hence [B] = $\square$[A] is an interval valued intuitionistic fuzzy subsemiring of R.

2.8 Theorem: If [A] is an interval valued intuitionistic fuzzy subsemiring of a semiring (R, +, ·), then $\Diamond$[A] is an interval valued intuitionistic fuzzy subsemiring of R.

Proof: Let [A] be an interval valued intuitionistic fuzzy subsemiring of R. That is [A]= { 〈 x, $\mu_{[A]}(x)$, $\nu_{[A]}(x)$ 〉 } for all x in R. Let $\Diamond$[A] = [B] = { 〈x, $\mu_{[B]}(x)$, $\nu_{[B]}(x)$ 〉 }, where $\mu_{[B]}(x)$ = [1]–$\nu_{[A]}(x)$, $\nu_{[B]}(x) = \nu_{[A]}(x)$. Clearly $\nu_{[B]}(x+y) \leq$ rmax{ $\nu_{[B]}(x)$, $\nu_{[B]}(y)$ } for all x, y in R and $\nu_{[B]}(xy) \leq$ rmax{ $\nu_{[B]}(x)$, $\nu_{[B]}(y)$ } for all x, y in R. Since [A] is an interval valued intuitionistic fuzzy subsemiring of R, we have $\nu_{[A]}(x+y) \leq$ rmax{ $\nu_{[A]}(x)$, $\nu_{[A]}(y)$ }, for all x, y in R which implies that [1]–$\mu_{[B]}(x+y) \leq$ rmax { [1]– $\mu_{[B]}(x)$, [1]– $\mu_{[B]}(y)$ } which implies that $\mu_{[B]}(x+y) \geq$ [1]– rmax { [1]– $\mu_{[B]}(x)$, [1]– $\mu_{[B]}(y)$} = rmin { $\mu_{[B]}(x)$, $\mu_{[B]}(y)$ }. Therefore $\mu_{[B]}(x+y) \geq$ rmin{ $\mu_{[B]}(x)$, $\mu_{[B]}(y)$ } for all x, y in R. And $\nu_{[A]}(xy) \leq$ rmax {$\nu_{[A]}(x)$, $\nu_{[A]}(y)$ } for all x, y in R which implies that [1]–$\mu_{[B]}(xy) \leq$ rmax { [1]–$\mu_{[B]}(x)$, [1]–$\mu_{[B]}(y)$ } which implies that $\mu_{[B]}(xy) \geq$ [1]– rmax { [1]–$\mu_{[B]}(x)$, [1]–$\mu_{[B]}(y)$ } = rmin {$\mu_{[B]}(x)$, $\mu_{[B]}(y)$ }. Therefore $\mu_{[B]}(xy) \geq$ rmin { $\mu_{[B]}(x)$, $\mu_{[B]}(y)$ } for all x, y in R. Hence [B] = $\Diamond$[A] is an interval valued intuitionistic fuzzy subsemiring of R.

2.9 Theorem: Let [A] be an interval valued intuitionistic fuzzy subsemiring of a semiring (R, +, .), then the pseudo interval valued intuitionistic fuzzy coset $(a[A])^p$ is an interval valued intuitionistic fuzzy subsemiring of R, for every a in R.

Proof: Let [A] be an interval valued intuitionistic fuzzy subsemiring of R. For every x, y in R, we have $((a\mu_{[A]})^p)(x+y) = p(a)\mu_{[A]}(x+y) \geq p(a)$ rmin $\{\mu_{[A]}(x), \mu_{[A]}(y)\}$ = rmin $\{p(a)\mu_{[A]}(x), p(a)\mu_{[A]}(y)\}$ = rmin $\{((a\mu_{[A]})^p)(x), ((a\mu_{[A]})^p)(y)\}$. Therefore $((a\mu_{[A]})^p)(x+y) \geq$ rmin $\{((a\mu_{[A]})^p)(x), ((a\mu_{[A]})^p)(y)\}$ for all x, y in R. Now $((a\mu_{[A]})^p)(xy) = p(a)\mu_{[A]}(xy) \geq p(a)$ rmin$\{\mu_{[A]}(x), \mu_{[A]}(y)\}$ = rmin $\{p(a)\mu_{[A]}(x), p(a)\mu_{[A]}(y)\}$ = rmin $\{((a\mu_{[A]})^p)(x), ((a\mu_{[A]})^p)(y)\}$. Therefore $((a\mu_{[A]})^p)(xy) \geq$ rmin $\{((a\mu_{[A]})^p)(x), ((a\mu_{[A]})^p)(y)\}$ for all x, y in R. For every x, y in R, we have $((a\nu_{[A]})^p)(x+y) = p(a)\nu_{[A]}(x+y) \leq p(a)$ rmax $\{\nu_{[A]}(x), \nu_{[A]}(y)\}$ = rmax $\{p(a)\nu_{[A]}(x), p(a)\nu_{[A]}(y)\}$ = rmax $\{((a\nu_{[A]})^p)(x), ((a\nu_{[A]})^p)(y)\}$. Therefore $((a\nu_{[A]})^p)(x+y) \leq$ rmax $\{((a\nu_{[A]})^p)(x), ((a\nu_{[A]})^p)(y)\}$ for all x, y in R. Now $((a\nu_{[A]})^p)(xy) = p(a)\nu_{[A]}(xy) \leq p(a)$ rmax $\{\nu_{[A]}(x), \nu_{[A]}(y)\}$ = rmax $\{p(a)\nu_{[A]}(x), p(a)\nu_{[A]}(y)\}$ = rmax $\{((a\nu_{[A]})^p)(x), ((a\nu_{[A]})^p)(y)\}$. Therefore $((a\nu_{[A]})^p)(xy) \leq$ rmax $\{((a\nu_{[A]})^p)(x), ((a\nu_{[A]})^p)(y)\}$ for all x, y in R. Hence $(a[A])^p$ is an interval valued intuitionistic fuzzy subsemiring of R.

2.10 Theorem: If *[A]* is an interval valued intuitionistic fuzzy subsemiring of a semiring R, then ?(*[A]*) is an interval valued intuitionistic fuzzy subsemiring of R.

Proof: For every x, y in R, we have $\mu_{?([A])}(x+y)$ = rmin { [½, ½], $\mu_{[A]}(x+y)$ } ≥ rmin { [½, ½], rmin { $\mu_{[A]}(x)$, $\mu_{[A]}(y)$}}= rmin { rmin {[½, ½], $\mu_{[A]}(x)$ }, rmin {[½, ½], $\mu_{[A]}(y)$}}= rmin{$\mu_{?([A])}(x)$, $\mu_{?([A])}(y)$}. Therefore $\mu_{?([A])}(x+y)$ ≥ rmin{$\mu_{?([A])}(x)$, $\mu_{?([A])}(y)$}, for all x, y in R. And $\mu_{?([A])}(xy)$ = rmin { [½, ½], $\mu_{[A]}(xy)$ } ≥ rmin {[½, ½], rmin {$\mu_{[A]}(x)$, $\mu_{[A]}(y)$ } } = rmin { rmin { [½, ½], $\mu_{[A]}(x)$ }, rmin { [½, ½], $\mu_{[A]}(y)$ }} = rmin {$\mu_{?([A])}(x)$, $\mu_{?([A])}(y)$ }. Therefore $\mu_{?([A])}(xy)$ ≥ rmin { $\mu_{?([A])}(x)$, $\mu_{?([A])}(y)$ } for all x, y in R. Also $\nu_{?([A])}(x+y)$ = rmax {[½, ½], $\nu_{[A]}(x+y)$} ≤ rmax {[½, ½], rmax {$\nu_{[A]}(x)$, $\nu_{[A]}(y)$}} = rmax { rmax {[½, ½], $\nu_{[A]}(x)$ }, rmax {[½, ½], $\nu_{[A]}(y)$}} = rmax{ $\nu_{?([A])}(x)$, $\nu_{?([A])}(y)$ }. Therefore $\nu_{?([A])}(x+y)$ ≤ rmax { $\nu_{?([A])}(x)$, $\nu_{?([A])}(y)$ } for all x, y in R. And $\nu_{?([A])}(xy)$ = rmax{[½, ½], $\nu_{[A]}(xy)$ } ≤ rmax{[½, ½], rmax{$\nu_{[A]}(x)$, $\nu_{[A]}(y)$}} = rmax{ rmax{ [½, ½], $\nu_{[A]}(x)$ }, rmax { [½, ½], $\nu_{[A]}(y)$ }} = rmax { $\nu_{?([A])}(x)$, $\nu_{?([A])}(y)$ }. Therefore $\nu_{?([A])}(xy)$ ≤ rmax{ $\nu_{?([A])}(x)$, $\nu_{?([A])}(y)$ } for all x, y in R. Hence ?(*[A]*) is an interval valued intuitionistic fuzzy subsemiring of R.

2.11 Theorem: If *[A]* is an interval valued intuitionistic fuzzy subsemiring of a semiring R, then !(*[A]*) is an interval valued intuitionistic fuzzy subsemiring of R.

Proof: For every x, y in R, we have $\mu_{!([A])}(x+y)$ = rmax { [½, ½], $\mu_{[A]}(x+y)$ }≥ rmax {[½, ½], rmin { $\mu_{[A]}(x)$, $\mu_{[A]}(y)$ } }= rmin { rmax { [½, ½], $\mu_{[A]}(x)$ }, rmax { [½, ½], $\mu_{[A]}(y)$ } } = rmin { $\mu_{!([A])}(x)$, $\mu_{!([A])}(y)$ }. Therefore $\mu_{!([A])}(x+y)$ ≥ rmin { $\mu_{!([A])}(x)$, $\mu_{!([A])}(y)$ } for all x, y in R. And $\mu_{!([A])}(xy)$ = rmax { [½, ½], $\mu_{[A]}(xy)$ } ≥ rmax {[½, ½], rmin { $\mu_{[A]}(x)$, $\mu_{[A]}(y)$ } } = rmin { rmax { [½, ½], $\mu_{[A]}(x)$ }, rmax { [½, ½], $\mu_{[A]}(y)$ }} = rmin { $\mu_{!([A])}(x)$, $\mu_{!([A])}(y)$ }. Therefore $\mu_{!([A])}(xy)$ ≥ rmin { $\mu_{!([A])}(x)$, $\mu_{!([A])}(y)$ } for all x, y in R. Also $\nu_{!([A])}(x+y)$ = rmin { [½, ½], $\nu_{[A]}(x+y)$ } ≤ rmin { [½, ½], rmax { $\nu_{[A]}(x)$, $\nu_{[A]}(y)$ } } = rmax { rmin { [½, ½], $\nu_{[A]}(x)$ }, rmin { [½, ½], $\nu_{[A]}(y)$ } } = rmax {$\nu_{!([A])}(x)$, $\nu_{!([A])}(y)$ }. Therefore $\nu_{!([A])}(x+y)$ ≤ rmax { $\nu_{!([A])}(x)$, $\nu_{!([A])}(y)$ } for all x, y in R. And $\nu_{!([A])}(xy)$ = rmin { [½, ½], $\nu_{[A]}(xy)$ } ≤ rmin { [½, ½], rmax { $\nu_{[A]}(x)$, $\nu_{[A]}(y)$ }}= rmax { rmin{[½, ½], $\nu_{[A]}(x)$ }, rmin { [½, ½], $\nu_{[A]}(y)$ }} = rmax { $\nu_{!([A])}(x)$, $\nu_{!([A])}(y)$ }.

Therefore $\nu_{!([A])}(xy)$ ≤ rmax { $\nu_{!([A])}(x)$, $\nu_{!([A])}(y)$ } for all x, y in R. Hence !(*[A]*) is an interval valued intuitionistic fuzzy subsemiring of R.

2.12 Theorem: If *[A]* is an interval valued intuitionistic fuzzy subsemiring of a semiring R, then $Q_{\alpha,\beta}$ (*[A]*) is an interval valued intuitionistic fuzzy subsemiring of R.

Proof: For every x, y in R, for α, β ∈ D[0,1] and α+β ≤ [1], we have $\mu_{Q_{\alpha,\beta}([A])}(x+y)$ = rmin { α, $\mu_{[A]}(x+y)$ }≥ rmin { α, rmin { $\mu_{[A]}(x)$, $\mu_{[A]}(y)$ }} = rmin { rmin { α, $\mu_{[A]}(x)$ }, rmin {α, $\mu_{[A]}(y)$ } } = rmin { $\mu_{Q_{\alpha,\beta}([A])}(x)$, $\mu_{Q_{\alpha,\beta}([A])}(y)$ }. Therefore $\mu_{Q_{\alpha,\beta}([A])}(x+y)$ ≥ rmin {$\mu_{Q_{\alpha,\beta}([A])}(x)$, $\mu_{Q_{\alpha,\beta}([A])}(y)$ } for all x, y in R. And $\mu_{Q_{\alpha,\beta}([A])}(xy)$ = rmin { α, $\mu_{[A]}(xy)$ }≥ rmin { α, rmin { $\mu_{[A]}(x)$, $\mu_{[A]}(y)$ } } = rmin { rmin { α, $\mu_{[A]}(x)$ }, rmin { α,

$\mu_{[A]}$(y) }}= rmin { $\mu_{Q_{\alpha,\beta}([A])}$(x), $\mu_{Q_{\alpha,\beta}([A])}$(y) }. Therefore $\mu_{Q_{\alpha,\beta}([A])}$(xy) ≥ rmin { $\mu_{Q_{\alpha,\beta}([A])}$(x), $\mu_{Q_{\alpha,\beta}([A])}$(y) } for all x, y in R. Also $\nu_{Q_{\alpha,\beta}([A])}$(x+y) = rmax { β, $\nu_{[A]}$(x+y) } ≤ rmax { β, rmax { $\nu_{[A]}$(x), $\nu_{[A]}$(y) }} = rmax { rmax { β, $\nu_{[A]}$(x) }, rmax { β, $\nu_{[A]}$(y) }} = rmax { $\nu_{Q_{\alpha,\beta}([A])}$(x), $\nu_{Q_{\alpha,\beta}([A])}$(y) }. Therefore $\nu_{Q_{\alpha,\beta}([A])}$(x+y) ≤ rmax { $\nu_{Q_{\alpha,\beta}([A])}$(x), $\nu_{Q_{\alpha,\beta}([A])}$(y) } for all x, y in R. And $\nu_{Q_{\alpha,\beta}([A])}$(xy) = rmax { β, $\nu_{[A]}$(xy) } ≤ rmax { β, rmax {$\nu_{[A]}$(x), $\nu_{[A]}$(y) }} = rmax { rmax { β, $\nu_{[A]}$(x) }, rmax { β, $\nu_{[A]}$(y) }} = rmax {$\nu_{Q_{\alpha,\beta}([A])}$(x), $\nu_{Q_{\alpha,\beta}([A])}$(y) }. Therefore $\nu_{Q_{\alpha,\beta}([A])}$(xy) ≤ rmax { $\nu_{Q_{\alpha,\beta}([A])}$(x), $\nu_{Q_{\alpha,\beta}([A])}$(y) } for all x, y in R. Hence $Q_{\alpha,\beta}$ (*[A]*) is an interval valued intuitionistic fuzzy subsemiring of R.

2.13 Theorem: If *[A]* is an interval valued intuitionistic fuzzy subsemiring of a semiring R, then $P_{\alpha,\beta}$ (*[A]*) is an interval valued intuitionistic fuzzy subsemiring of R.

Proof: For every x, y in R, for α, β ∈D[0,1] and α +β ≤ [1], we have $\mu_{P_{\alpha,\beta}([A])}$(x+y) = rmax { α, $\mu_{[A]}$(x+y) } ≥ rmax { α, rmin { $\mu_{[A]}$(x), $\mu_{[A]}$(y) }} = rmin { rmax { α, $\mu_{[A]}$(x) }, rmax {α, $\mu_{[A]}$(y) }} = rmin { $\mu_{P_{\alpha,\beta}([A])}$(x), $\mu_{P_{\alpha,\beta}([A])}$(y) }. Therefore $\mu_{P_{\alpha,\beta}([A])}$(x+y) ≥ rmin {$\mu_{P_{\alpha,\beta}([A])}$(x), $\mu_{P_{\alpha,\beta}([A])}$(y) } for all x, y in R. And $\mu_{P_{\alpha,\beta}([A])}$(xy) = rmax { α, $\mu_{[A]}$(xy)} ≥ rmax { α, rmin { $\mu_{[A]}$(x), $\mu_{[A]}$(y) } } = rmin { rmax { α, $\mu_{[A]}$(x) }, rmax { α, $\mu_{[A]}$(y) }}= rmin { $\mu_{P_{\alpha,\beta}([A])}$(x), $\mu_{P_{\alpha,\beta}([A])}$(y) }. Therefore $\mu_{P_{\alpha,\beta}([A])}$(xy) ≥ rmin { $\mu_{P_{\alpha,\beta}([A])}$(x), $\mu_{P_{\alpha,\beta}([A])}$(y) } for all x, y in R. Also $\nu_{P_{\alpha,\beta}([A])}$(x+y) = rmin { β, $\nu_{[A]}$(x+y) } ≤ rmin { β, rmax { $\nu_{[A]}$(x), $\nu_{[A]}$(y) } } = rmax { rmin { β, $\nu_{[A]}$(x) }, rmin { β, $\nu_{[A]}$(y) }} = rmax {$\nu_{P_{\alpha,\beta}([A])}$(x), $\nu_{P_{\alpha,\beta}([A])}$(y) }. Therefore $\nu_{P_{\alpha,\beta}([A])}$(x+y) ≤ rmax { $\nu_{P_{\alpha,\beta}([A])}$(x), $\nu_{P_{\alpha,\beta}([A])}$(y) }, for all x, y in R. And $\nu_{P_{\alpha,\beta}([A])}$(xy) = rmin {β, $\nu_{[A]}$(xy)} ≤ rmin { β, rmax {$\nu_{[A]}$(x), $\nu_{[A]}$(y)}} = rmax { rmin { β, $\nu_{[A]}$(x) }, rmin { β, $\nu_{[A]}$(y) } } = rmax { $\nu_{P_{\alpha,\beta}([A])}$(x), $\nu_{P_{\alpha,\beta}([A])}$(y) }. Therefore $\nu_{P_{\alpha,\beta}([A])}$(xy) ≤ rmax{$\nu_{P_{\alpha,\beta}([A])}$(x), $\nu_{P_{\alpha,\beta}([A])}$(y) } for all x, y in R. Hence $P_{\alpha,\beta}$ (*[A]*) is an interval valued intuitionistic fuzzy subsemiring of R.

2.14 Theorem: If *[A]* is an interval valued intuitionistic fuzzy subsemiring of a semiring R, then $G_{\alpha,\beta}$ (*[A]*) is an interval valued intuitionistic fuzzy subsemiring of R.

Proof: For every x, y in R, for α, β ∈D[0,1] and α+β ≤ [1], we have $\mu_{G_{\alpha,\beta}([A])}$(x+y) = α $\mu_{[A]}$(x+y) ≥ α (rmin { $\mu_{[A]}$(x), $\mu_{[A]}$(y) }) = rmin { α$\mu_{[A]}$(x), α $\mu_{[A]}$(y)}= rmin {$\mu_{G_{\alpha,\beta}([A])}$(x), $\mu_{G_{\alpha,\beta}([A])}$(y) }. Therefore $\mu_{G_{\alpha,\beta}([A])}$(x+y) ≥ rmin { $\mu_{G_{\alpha,\beta}([A])}$(x), $\mu_{G_{\alpha,\beta}([A])}$(y) } for all x, y in R. And $\mu_{G_{\alpha,\beta}([A])}$(xy) = α $\mu_{[A]}$(xy) ≥ α (rmin { $\mu_{[A]}$(x), $\mu_{[A]}$(y) }) = rmin { α$\mu_{[A]}$(x), α$\mu_{[A]}$(y) } = rmin { $\mu_{G_{\alpha,\beta}([A])}$(x), $\mu_{G_{\alpha,\beta}([A])}$(y) }. Therefore $\mu_{G_{\alpha,\beta}([A])}$(xy) ≥ rmin {$\mu_{G_{\alpha,\beta}([A])}$(x), $\mu_{G_{\alpha,\beta}([A])}$(y) } for all x, y in R. Also $\nu_{G_{\alpha,\beta}([A])}$(x+y) = β $\nu_{[A]}$(x+y) ≤ β (rmax { $\nu_{[A]}$(x), $\nu_{[A]}$(y) }) = rmax { β $\nu_{[A]}$(x), β $\nu_{[A]}$(y) } = rmax {$\nu_{G_{\alpha,\beta}([A])}$(x), $\nu_{G_{\alpha,\beta}([A])}$(y) }. Therefore $\nu_{G_{\alpha,\beta}([A])}$(x+y) ≤ rmax { $\nu_{G_{\alpha,\beta}([A])}$(x), $\nu_{G_{\alpha,\beta}([A])}$(y) } for all x, y in R. And $\nu_{G_{\alpha,\beta}([A])}$(xy) = β $\nu_{[A]}$(xy) ≤ β rmax { $\nu_{[A]}$(x), $\nu_{[A]}$(y) }) = rmax { β $\nu_{[A]}$(x), β $\nu_{[A]}$(y) }= rmax { $\nu_{G_{\alpha,\beta}([A])}$(x), $\nu_{G_{\alpha,\beta}([A])}$(y) }. Therefore $\nu_{G_{\alpha,\beta}([A])}$(xy) ≤ rmax { $\nu_{G_{\alpha,\beta}([A])}$(x), $\nu_{G_{\alpha,\beta}([A])}$(y) } for all x, y in R. Hence $G_{\alpha,\beta}$ (*[A]*) is an interval valued intuitionistic fuzzy subsemiring of R.

REFERENCE

[1]. Akram.M and Dar.K.H, On fuzzy d-algebras, Punjab university journal of mathematics, 37(2005), 61-76.

[2]. Asok Kumer Ray, On product of fuzzy subgroups, Fuzzy sets and systems, 105, 181-183 (1999).

[3]. Azriel Rosenfeld, Fuzzy Groups, Journal of mathematical analysis and applications, 35, 512-517 (1971).

[4]. Biswas.R, Fuzzy subgroups and Anti-fuzzy subgroups, Fuzzy sets and systems, 35,121-124 (1990).

[5]. Grattan-Guiness, Fuzzy membership mapped onto interval and many valued quantities, Z.Math.Logik. Grundladen Math. 22 (1975), 149-160.

[6]. Indira.R, Arjunan.K and Palaniappan.N, Notes on interval valued fuzzy rw-Closed, interval valued fuzzy rw-Open sets in interval valued fuzzy topological space, International Journal of Fuzzy Mathematics and Systems, Vol. 3, Num.1, pp 23-38, 2013.

[7]. Jahn.K.U., interval wertige mengen, Math Nach.68, 1975, 115-132.

[8]. Jun.Y.B and Kin.K.H, interval valued fuzzy R-subgroups of nearrings, Indian Journal of Pure and Applied Mathematics, 33(1) (2002), 71-80.

[9]. Palaniappan. N & K. Arjunan, 2007. Operation on fuzzy and anti fuzzy ideals, Antartica J. Math., 4(1): 59-64.

[10]. Solairaju.A and Nagarajan.R, Charactarization of interval valued Anti fuzzy Left h-ideals over Hemirings, Advances in fuzzy Mathematics, Vol.4, Num. 2 (2009), 129-136.

[11]. Somasundara moorthy.M.G. & Arjunan.K., Interval valued fuzzy subring of a ring under homomorphism, International journal of scientific Research, Vol.3, Iss. 4, (2014), 292-296.

[12]. Zadeh.L.A, Fuzzy sets, Information and control, Vol.8, 338-353 (1965).

[13]. Zadeh.L.A, The concept of a linguistic variable and its application to approximation reasoning-1, Inform. Sci. 8(1975), 199-249.