Vol.4.Issue.2.2016 (April-June)

BULLETIN OF MATHEMATICS AND STATISTICS RESEARCH

A Peer Reviewed International Research Journal

http://www.bomsr.com

Email:editorbomsr@gmail.com

RESEARCH ARTICLE

ISSN:2348-0580

MERSENNE NUMBERS

JEAN V. LEYENDEKKERS[1], ANTHONY G. SHANNON[2]

[1]Faculty of Science, The University of Sydney, NSW 2006, Australia

[2]Emeritus Professor, University of Technology Sydney, NSW 2007, Australia

JEAN V. LEYENDEKKERS

ABSTRACT

In this paper the Integer Structure of the Mersenne system of numbers is summarized through their Fermat expansion. Prime and composite forms are compared through contrasting logarithmic plots which are strikingly distinct.

Keywords: Mersenne numbers, Fibonacci numbers, integer structure, modular rings.

AMS Classification Numbers: 11B39, 11B50.

1. INTRODUCTION

An analysis of Mersenne numbers, $M_n = (2^n - 1)$, in relation to primality problems and Fibonacci numbers was provided in a series of papers to demonstrate their integer structure [2,3,8]. The production of composites rather than primes is explained by a two parameter equation, the difference of squares and Fermat's Little Theorem. In particular, if *n* is composite, then so is $2^n - 1$ since $(2^x - 1) | (2^{xy} - 1)$. The Fermat expansion form

$$M_p = pK_M + 1 \tag{1.1}$$

was examined up to *p* = 257. The use of modular rings (Table 1) showed that these numbers are restricted to class $\bar{3}_4 \subset Z_4$ and they are composed of single class "nests" of integers [9].

Table 1: Classes and rows for Z_4

Row r_i ↓	Class i →	$\bar{0}_4$	$\bar{1}_4$	$\bar{2}_4$	$\bar{3}_4$	Comments
0		0	1	2	3	• $N = 4r_i + i$
1		4	5	6	7	• even $\bar{0}_4, \bar{2}_4$
2		8	9	10	11	• $(N^n, N^{2n}) \in \bar{0}_4$
3		12	13	14	15	• odd $\bar{1}_4, \bar{3}_4$; $N^{2n} \in \bar{1}_4$

The purpose of this paper is to analyse relationships of $K_M = \frac{1}{p}\left(M_p - 1\right)$ which distinguish composites and primes. Some M_p values relevant to this discussion are displayed in Table 2.

Table 2: Some prime subscripted Mersenne numbers

p	**Class of p**		**M_p**	**K_M**	**Integer type**	
	$\bar{1}_4$	$\bar{3}_4$			**prime**	**Comp.**
3		X	7	2	X	
5	X		31	6	X	
7		X	127	18	X	
11		X	2,047	186		X
13	X		8,191	630	X	
17	X		131,071	7,710	X	
19		X	524,287	27,594	X	
23		X	8,388,607	364,722		X
29	X		536,870,911	18,512,790		X
31		X	2,147,483,647	69,273,666	X	
37	X		137,438,953,471	3,714,566,310		X
41	X		2,199,023,255,551	53,634,713,550		X
43		X	8,796,093,022,207	204,560,302,842		X
47		X	140,737,488,355,327	2,994,414,645,858		X

2. FIBONACCI AND MERSENNE PRIMES – A LINK

Somewhat analogously to (1.1) prime Fibonacci numbers with prime subscripts satisfy [4-7]:

$$F_p = pK_F \pm 1 \tag{2.1}$$

in which K_F is an even integer. When F_p is composite, the factors, f, satisfy

$$f = kp \pm 1 \tag{2.2}$$

where k is an even integer. Often k = 2 for one of the factors. This has helped to test for primality of Fibonacci numbers [*cf*. 12]. These equations are satisfied for generalized Fibonacci numbers too [5-7].

The analogies with the Mersenne primes can be seen in Tables 3 and 4 in which $K_M = 4r_2 + 2$ or $r_2 = \left\lfloor \frac{1}{4}\left(K_M - 2\right)\right\rfloor$ and $9 \mid K_M$ except for p = 17.

Table 3: Some prime subscripted Mersenne primes

p	***Class of p***		**K_M**	**r_2**	**r_2***
	$\bar{1}_4$	$\bar{3}_4$			
3		X	2	0	0
5	X		6	1	1
7		X	18	4	4
13	X		630	157	7
17	X		7,710	1,927	7
19		X	27,594	6,898	8
31		X	69,273,666	17,318,416	6

It can be seen in Table 2 that the right-end-digit (RED) of r_2 is $r_2^* \in \begin{cases} \{1,7\}, & \left(p \in \bar{1}_4\right), \\ \{0,4,6,8\}, & \left(p \in \bar{3}_4\right). \end{cases}$

Table 4: Some prime subscripted Mersennecomposites

p	Class of p $\bar{1}_4$	Class of p $\bar{3}_4$	K_M	r_2	Factors	k
11		X	186	46	23;89	$2p+1;8p+1$
23		X	364,722	91,180	47;178481	$2p+1;7760p+1$
29	X		18,512,790	4,628,197	233;1103;2089	$8p+1;38p+1;72p+1$
37	X		3,714,566,310	928,641,577	223;6166318177	$6p+1;16657248p+1$
41	X		53,634,713,550	13408678387	13367;164511353	$326p+1;4012472p+1$
43		X	204,560,302,842	51,140,075,710	431;9719;2099863	$10p+1;226p+1;48834p+1$
47		X	2,994,414,645,858	748,603,661,464	2351;4513;13,264,529	$50p+1;96p+1;28224p+1$

Similarly it can be seen in Table 4 that

- $r_2 = \left\lfloor \frac{1}{4}(K_M - 2) \right\rfloor$ again,
- 9 does not divide K_M except when $p \in \{37,43\}$,and
- $r_2^* \in \begin{cases} \{7\}, & (p \in \bar{1}_4), \\ \{0,4,6\}, & (p \in \bar{3}_4). \end{cases}$

3. K_M FUNCTIONS

From (1.1) and

$$M_p = (2^p - 1) \tag{3.1}$$

we can obtain

$$K_M = \tfrac{2}{p}\left(2^{\frac{1}{2}(p-1)} - 1\right)\left(2^{\frac{1}{2}(p-1)} + 1\right). \tag{3.2}$$

We then plot ln K_M against n(the positional number for primes) to obtain the smooth exponential curve in Figure 2below which is in stark contrast to Figure 1, the corresponding logarithmic form of *ln*K_M against m (the positional number for composites). For composites,

$$K_M = k_1 k_2 p + (k_1 + k_2) \tag{3.3}$$

for two factors, and in general

$$K_M = A_1 p^n + A_2 p^{n-1} + \ldots + A_n p + 1, \tag{3.4}$$

which partly explains the contrasting curves.

4. STRUCTURE

In Table 5 we briefly summarise the main structures from above and [2,8].

Table 5: Classes of various components of the Mersenne sequence

$M_p \subset$	$K_M \subset$	$p \in$	$k \in$	factors $\in$
$\bar{3}_4$	$\bar{2}_4$	$\{\bar{1}_4, \bar{3}_4\}$	$\{\bar{0}_4, \bar{2}_4\}$	$\{\bar{1}_4, \bar{3}_4\}$

Some class combinations may be invalid, for example, $\bar{3}_4\bar{3}_4 \in \bar{1}_4$. This information can help when identifying factors (Table 6). The REDs for M_p are limited to 1 and 7 because $(2^n)^*$ is 2 or 8 when n is odd. When $p \in \bar{1}_4, M_p^* = 1$, but when $p \in \bar{3}_4, M_p^* = 7$.

Table 6: Factor class combinations for composite M_p

p	Factor classes	$M_p$class
11	$\bar{3}_4\ \bar{1}_4$	$\bar{3}_4$
23	$\bar{3}_4\ \bar{1}_4$	$\bar{3}_4$
29	$\bar{1}_4\ \bar{3}_4\ \bar{1}_4$	$\bar{3}_4$
37	$\bar{3}_4\ \bar{1}_4$	$\bar{3}_4$
41	$\bar{3}_4\ \bar{1}_4$	$\bar{3}_4$
43	$\bar{3}_4\ \bar{3}_4\ \bar{3}_4$	$\bar{3}_4$
47	$\bar{3}_4\ \bar{1}_4\ \bar{1}_4$	$\bar{3}_4$

5. CONCLUDING COMMENTS

There is no obvious reason to believe that the curve for the primes in Figure 1 should not be valid for all *n* [10]. The work of Dr Curtis Cooper and his colleagues in the Great Internet Mersenne Prime Search (GIMPS) would seem to lend credence to this assumption.Other curious properties of Mersenne numbers which are worthy of further elaboration include

- theMersenne numbers consist of all 1s in base 2 (binary repunits); for example. $2^3-1=7=111_2;\ 2^4-1=15=1111_2;\ 2^5-1=31=11111_2$, and so on [1];
- everyMersenne prime is associated with an even perfect number; that is, M_n is prime if, and only if, $\left(2^n-1\right)M_n$ is perfect [11].

Figures 1 and 2 are approximate only graphs to compare and contrast the overall behaviour of the primes and composites which is more obvious with the logarithms of these very large values for K_M.

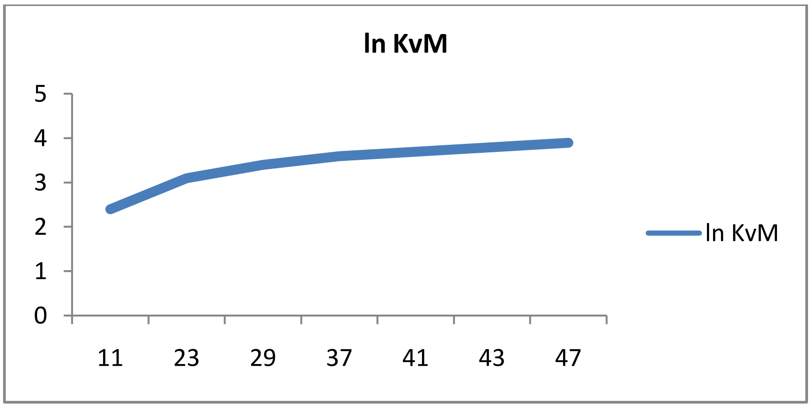

Figure 1: *p* vs ln $K_M$for composites

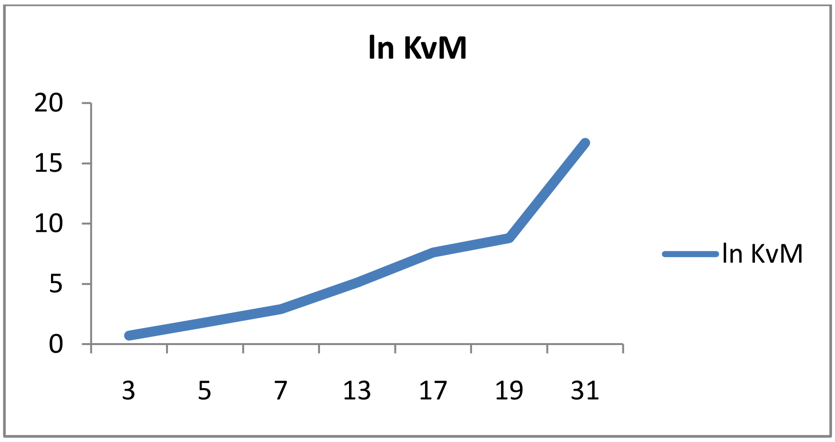

Figure 2: *p* vs $K_M$for primes

REFERENCES

[1] Gillies, Donald B. 1964. Three new Mersenne primes and a statistical theory. *Mathematics of Computation*. 18: 93-97.

[2] Leyendekkers, J.V., A.G Shannon. 1999. Fermat and Mersenne numbers and some related factors. *International Journal of Mathematical Education in Science and Technology*. 30 (4): 627-629.

[3] Leyendekkers, J.V., A.G Shannon. 2005. Mersenne and Fermat numbers. *Notes on Number Theory and Discrete Mathematics.* 11 (4): 17-25.

[4] Leyendekkers, J.V., A.G. Shannon. 2014. Fibonacci primes. *Notes on Number Theory and Discrete Mathematics*. 20 (2): 6-9.

[5] Leyendekkers, J.V., A.G. Shannon. 2015. An infinite primality conjecture for prime-subscripted Fibonacci numbers. *Notes on Number Theory and Discrete Mathematics*. 21 (1): 51-55.

[6] Leyendekkers, J.V., A.G. Shannon. 2015. Primes within Fibonacci sequences. *Notes on Number Theory and Discrete Mathematics*. 21 (4): 56-63.

[7] Leyendekkers, J.V., A.G. Shannon. 2016. Some Golden Ratio generalized Fibonacci and Lucas sequences. *Notes on Number Theory and Discrete Mathematics.* 22 (1): 33-41.

[8] Leyendekkers. J.V., J.M. Rybak, A.G. Shannon. 1999. An analysis of Mersenne-Fibonacci and Mersenne-Lucas primes. *Notes on Number Theory and Discrete Mathematics*. 5 (1): 1-26.

[9] Leyendekkers, J.V., A.G. Shannon, J.M. Rybak. 2007. *Pattern Recognition: Modular Rings and Integer Structure*. North Sydney: Raffles KvB Monograph No.9.

[10] Ribenboim, Paulo. 1997. Are there functions that generate prime numbers? *The College Mathematics Journal.* 28 (5): 352-359.

[11] Wagstaff, Samuel S., Jr. 1983. Divisors of Mersenne numbers. *Mathematics of Computation* 40 (161): 385-397.

[12] Williams, H.C. 1982. A note on the Fibonacci quotient $F_{p-\varepsilon}/F_p$. *Canadian Mathematical Bulletin*. 25 (3): 366-370.