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RESEARCH ARTICLE

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A MANPOWER MODEL WHEN COMPLETE LENGTH OF SERVICE FOLLOWS RIGHT TRUNCATED EXPONENTIAL DISTRIBUTION

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ABSTRACT

Many researchers studied manpower models with the assumption that Complete Length of Service (CLS) of an employee in the organization follows an infinite range distribution.This assumption is meaningful only when the CLS of an employee is sufficiently large. However, in many situations due to the push and pull factors , the CLS may not very large. Thus, this paper argues that, if the period of stay of an employee in the organization is not sufficiently large, then approximating CLS with infinite range distributions does not provide accurate results and hence there is a need to analyze manpower models with CLS with truncated distributions. In this paper, a manpower model developed and analyzed, by assuming CLS follows the Right Truanted Exponential Distribution (RTED).The various lifetime characteristics of RTED are obtained and analyzed through numerical illustration and graphs.This paper found that there is a significance difference in right truncated distributions as compared with non-truncated distribution.

Keywords: Manpower Models, Manpower planning, Truncated distributions, Life Time Distributions

1. INTRODUCTION

In any organization, manpower planning is required to know the various manpower characteristics like recruitment, promotion and leaving also called wastage etc.A manpower model is a statistical description of how a change of employees takes place in the organization. Due to the unpredictable and the uncertainties of the events in the organization, the manpower system requires manpower modelling through stochastic modelling. The stochastic modelling provides the basic framework for developing the manpower models.In manpower planning the Completed

Length of Service (CLS) is an important variable. Many researchers studied the manpower models by approximating the CLS with a probability distribution having infinite range.However, this assumption is meaningful , only when the CLS of an employee of the organization is sufficiently large. However, in many situations due to the push and pull factors the CLS of an organization may not be very large. Thus to develop suitable manpower strategies with optimum operating feasibilities, it is reasonable to approximate the CLS distribution with the truncated distributions. In this paper, a manpower model with the Right Truncated Exponential Distribution (RTED) developed and analyzed various properties through numerical illustrations and graphs.

2. Literature Review

Vajda (1947) is one of the pioneers in the development of the manpower models.Bartholomew (1959) developed the labour turnover process and defined survival function as complementary of the distribution function. Lane and Andrew (1955) express that, the probability of an employee leaving as a function of his length of service. Bartholomew (1959) has given an approximate solution to the integral equation for the renewal density, it's intended for the use when the CLS distribution is extremely skewed. Bartholomew (1959) showed the application of the renewal theory to the study of manpower systems and discussed possible applications in the manpower planning. Bartholomew (1971) explained the role of a statistical approach to the manpower planning and studied on mixed Exponential and Gamma distributions extensively with infinite range distributions. Marshall(1970) suggested that an employee’s probability of leaving may often be successfully related to his length of service by means of several well-known distributions. Bartholomew(1973) made a renewal theory, application to manpower planning. This work was theoretical in emphasis and used for continuing time. Robinson (1974) made a renewal theory approach to the study of manpower systems. Gary (1975) developed a model for manpower management by classifying employees as two categories namely good and poor performers. In an organization, wastage occurs due to the employee leaving the organization for various reasons. In particular, the leaving is frequent and common in a private organization. Therefore, there is a need to study on wastage or leaving the process of the organizations in order to take precautionary measures. Bartholomew and Forbes (1979) studied CLS & wastage and mentioned that in any organization, the required staff strength needed to maintain through new recruitment. Stochastic analysis of manpower levels affecting business with varying recruitment rates using stochastic analysis (Kumar et al ,2014).

The literature review shows that different researchers have approximated the Complete Length of Service (CLS) with infinite range distributions such as Exponential, Gama, Log-Normal etc. due to mathematical convenience. However,if a period of stay of an employee in an organization is not sufficiently large, then approximating the CLS with infinite range distributions does not give exact results, which may play a dominant role in developing optimal operating manpower strategies. To tackle this sort of situations, it is reasonable to approximate the CLS distribution with the distributions having a finite range or right-truncated at a finite point. These truncated distributions are different from their non-truncated distributions and give the original non-truncated distributions if the truncating parameter tends to infinity.

The truncated distributions can be used in many practical situations in a variety of ways (Khasawneh, Bowling, Kaewkuekool, and Cho 2004, 2005; Faris 2008). Technologies such as MathCAD, MathLAB and R-language helped researchers to solve truncated distributions. Saralees, et al, (2006) developed various models using different truncated distribution, i.e., whether it is a doubled, right and left using ‘R-language’, an open source statistical software. These programs accept any value for the truncation points or any truncated distribution function. In lifetime distribution, one can study various lifetime functions such as the probability density function, the cumulative distribution function , survival function, hazard rate function,the cumulative hazard rate and the mean residual life function (Rinne, 2009 & 2010).

3. Manpower planning model with Right Truncated Exponential Distribution(RTED)

In this paper, an attempt was made to fill the gap in the research area by using right truncated distributions for CLS.In this paper, the manpower model is approximating the CLS distribution as a Right Truncated Exponential Distribution (RTED).The functions such as a probability density function, the cumulative distribution function survival function, Renewal equation are derived and analyzed through numerical illustrations and graphs.

3.1 Assumptions

a) 'T', the Complete Length of Service (CLS) is a continuous non-negative random variable.
b) Individuals leaving the organization are considered as wastage and it is identically independent random variables.
c) Complete Length of Service (CLS) distribution is same for all periods of time.
d) In an organization, the Complete Length of Service (CLS) of an employee follows a Right Truncated Exponential Distribution (RTED).

3.2 Notations

f(t) : Probability Density Function (PDF).

F(t) : Cumulative Distribution Function (CDF).

S(t) : Survival function or reliability function, it is also called the complementary cumulative distribution function, i.e 1-F(t)

R(t) : Renewal equation or Renewal density

E(t) : Mean time to failure

V(t): Variance of time to failure

A: Truncated value of ' T', T will be truncated on the right at 'A', Where 'A' is known constant.

3.3 Manpower planning models having Right Truncated Exponential Distribution as CLS

Using assumptions and notations mentioned in Section 3.1 and 3.2, various functions of RTED are derived and their behavior studied through numerical illustrations and graphs , which are given in Table 1 and Figures 1 to 4.

$$f(t) = \frac{\lambda.e^{-\lambda.t}}{1-e^{-\lambda.A}} \text{ , Where } \lambda>0,\ 0<t\leq A \text{ , and A is a known constant} \tag{1}$$

$$F(t) = \frac{1-e^{-\lambda.t}}{1-e^{-\lambda.A}} \tag{2}$$

$$E(t) = \frac{1}{\lambda} - \frac{A.e^{-\lambda A}}{1-e^{-\lambda.A}} \tag{3}$$

$$V(t) = \frac{(1-e^{-\lambda.A}).(e^{-\lambda,A}+(\lambda.A)^2-2)}{\lambda^2.(1-e^{-\lambda.A})^2} \tag{4}$$

$$S(t) = \frac{e^{-\lambda.t}-e^{-\lambda.A}}{1-e^{-\lambda.A}} \tag{5}$$

Bartholomew and Forbes (1979) used the concept of renewal equation in manpower studies, the approximate solution to the renewal function of this model is given by

$$R(t) = f(t) + \frac{F^2(t)}{\int_0^t S(t).dt} \tag{6}$$

The Probability Density Function (PDF), Cumulative Distribution Ffunction (CDF) and survival function of the RTED given in equations 1,2 and 5 respectively, and by solving equation no. 6, the Renewal equation or Renewal function R(t) of the Manpower model having CLS follow a Right Truncated Exponential Distribution model is

$$R(t) = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}-e^{-\lambda A}}\left[e^{-\lambda t} + \frac{1-e^{-\lambda t}}{1-e^{-\lambda t}-\lambda.t.e^{-\lambda A}}\right] \quad (7)$$

4. Manpower planning model having Changing CLS follows Right Truncated Exponential Distributions

In Section 3, the paper discussed in the manpower model having CLS as RTED. In this model , it was assumed that the Complete Length of Service (CLS) distribution is same for all periods of time. But in some organizations, it was observed that the CLS is changing over time due to different situations prevailing in and around the organisation.The CLS will have a change after some period of time .Therefore, in this case, the CLS distributions of the organizations before and after the change are assumed as right truncated exponential distributions with parameters λ_1, A and λ_1, A respectively. The behaviour of the model was analyzed through graphs given in Figures 5 and 6.

The important functions of manpower planning models with RTED having changing CLS are:

a) The density function of the CLS distribution of the members recruited before the change is

$$f_1\ (t) = \frac{\lambda_1.e^{-\lambda_1 t}}{1-e^{-\lambda_1 A}} \text{ , Where } \lambda_1 > 0\ , 0 < T \leq A \quad (8)$$

b) The density function of the CLS distribution of the members recruited after the change is

$$f_2\ (t) = \frac{\lambda_2.e^{-\lambda_2 t}}{1-e^{-\lambda_2 A}} \text{ , Where } \lambda_2 > 0\ , 0 < T \leq A \quad (9)$$

c) The distribution functions before and after the change are defined as

$$F_1\ (t) = \frac{1-e^{-\lambda_1 t}}{1-e^{-\lambda_1 A}}\ , \qquad F_2\ (t) = \frac{1-e^{-\lambda_2 t}}{1-e^{-\lambda_2 A}} \quad (10)$$

d)The Survival function before and after change are defined as

$$S_1\ (t) = \frac{e^{-\lambda_1 t}-e^{-\lambda_1 A}}{1-e^{-\lambda_1 A}}, \quad S_2\ (t) = \frac{e^{-\lambda_2 t}-e^{-\lambda_2 A}}{1-e^{-\lambda_2 A}} \quad (11)$$

e)The Renewal density or equation of the RTED having changing CLS model is given by

$$R(t) = \frac{1}{r}\left[r_1 + \frac{r_2}{r_3}\right] \quad (12)$$

Where ,

$$r = 1- e^{-\lambda_1 A}\ (1+\lambda_1 A)\ ,\ r_1 = \lambda_1[e^{-\lambda_1 t} - e^{-\lambda_1 A}],$$

$$r_2 = \lambda_2[1 - e^{-\lambda_1 t} - t.\lambda_1 e^{-\lambda_1 A}]\ [1-e^{-\lambda_2 t}]$$

$$r_3 = [1-e^{-\lambda_2 t}] - t.\lambda_2 e^{-\lambda_2 A}$$

5. Results and Discussions

The numerical illustrations of the mean and variance of the manpower planning models having CLS follow Right Truncated Exponential Distribution (RTED) for various values of λ and 'A',is given in Table 1, it was observed from the numerical results , when 'λ' increases, the mean is decreasing and variance is increasing, whereas, when 'A' increases both mean and variance are increasing.The behaviour of different shapes of frequency curves of manpower planning models having CLS follows Right Truncated Exponential Distribution(RTED) for fixed λ and for different values is given Figure 1 and 2.

λ	Mean	Variance	A	Mean	Variance
0.3	1.278	-14.669	1	0.475	-50.123
1	0.843	7.419	5	1.897	6.767
3	0.333	8.779	10	2.809	82.435
10	0.1	8.98	15	3.165	205.181

Table 1: Mean and Variance of (RTED) for various values of λ and A

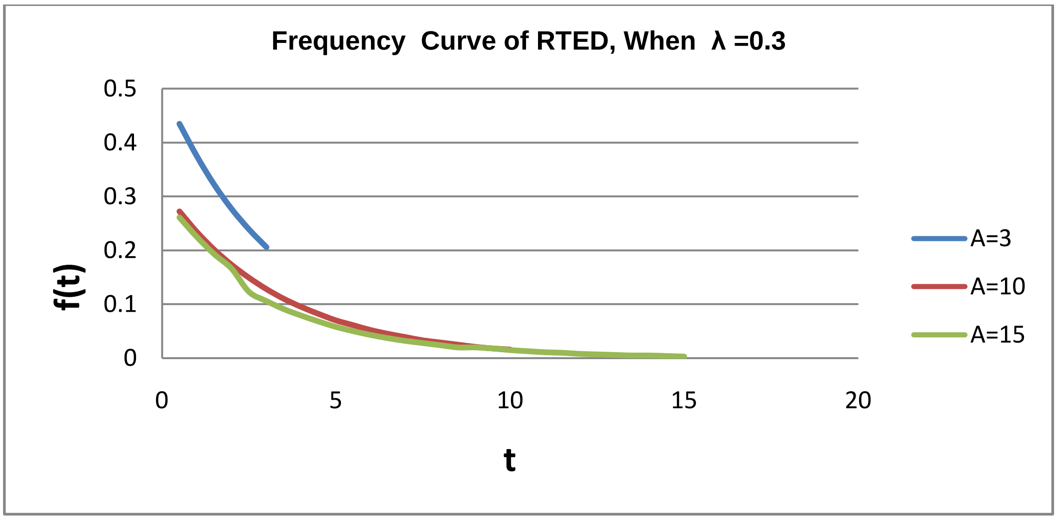

Figure 1: Frequency curve of Right Truncated Exponential Distribution ,When A=3,10 and 15 and λ=0.3

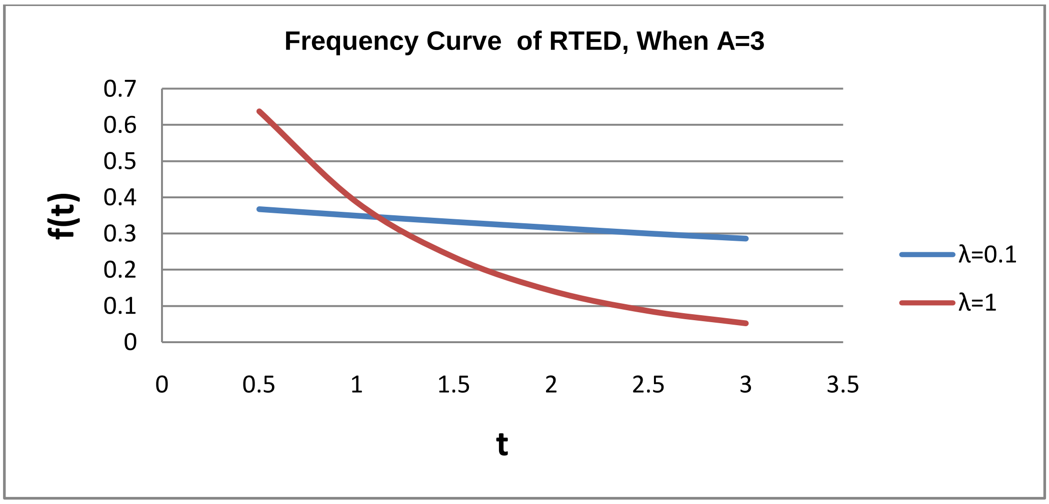

Figure 2: Frequency curve of Right Truncated Exponential Distribution,When A=3 and λ=0.1, 1

The right truncated exponential frequency curves are all right skewed. As λ increases the slopes of the frequency curves are increasing and as 'A' increases, the slope of the curve is also increasing. The graph of renewal equation R (t) of the model for different values of λ and for a fixed value of 'A' is shown in Figure 3, for a fixed value of λ and for different values of A is shown Figure 4.

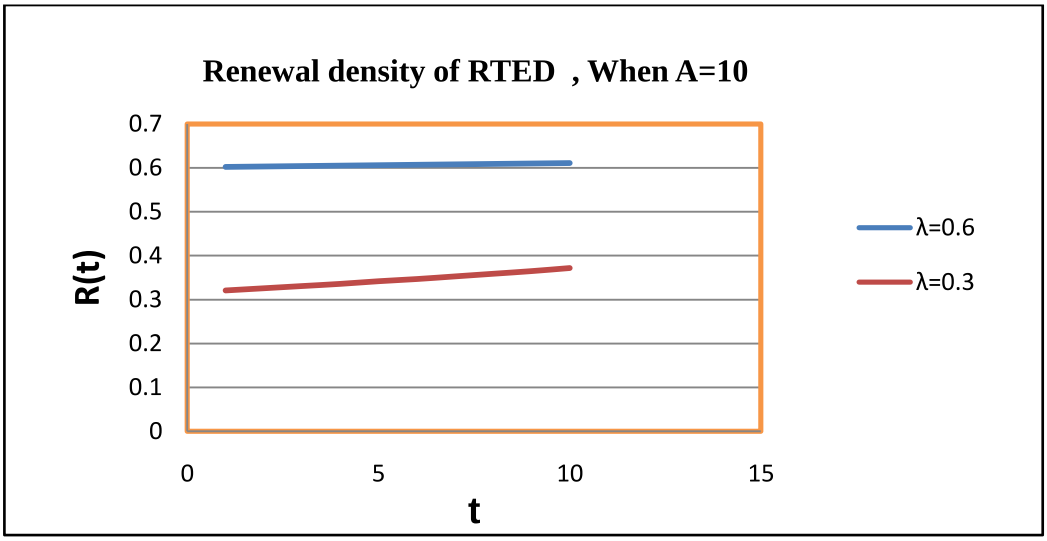

Figure 3: A graph of Renewal Density R (t) when A=10 and λ=0. 3 and 0.6

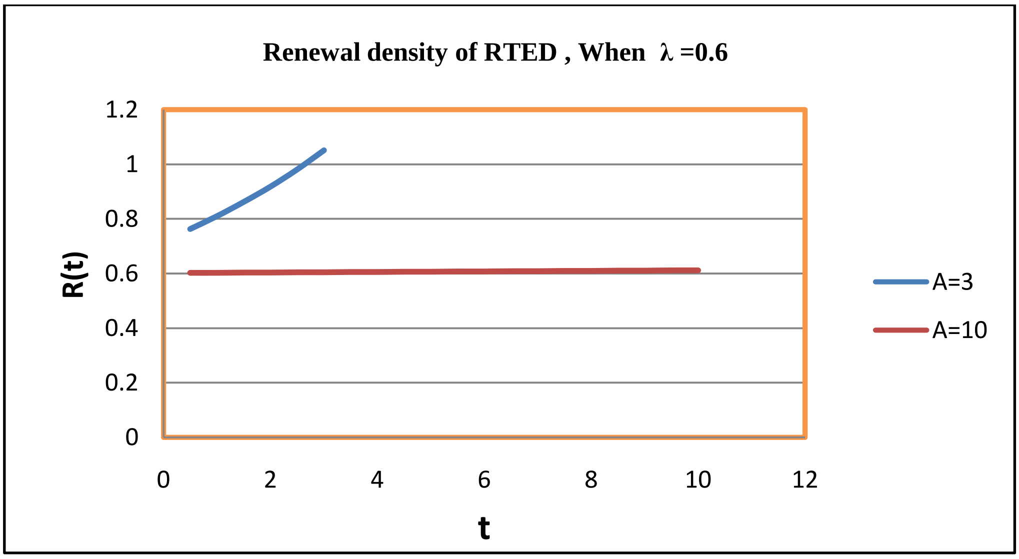

Figure 4: A graph of Renewal Density, when A=3 and 10 and λ= 0.6

It was observed from the Figures 3 and 4, when 'A' is 3, the renewal density is an increasing function of t, that means, for a small value of 'A' the expected recruitment level is increasing over time. As 'A' increases over time, the recruitment level is decreasing. If A tends to infinity (for larger values of 'A'), R (t) = λ.

The graph of the renewal density of the manpower planning model with the Right Truncated Exponential Distribution (RTED) having changing CLS distribution, for various values of λ_1 ,λ_2 and 'A' are given in figures 5 and 6.

It was observed from the figures 5 and 6, the behaviour of that the renewal density R (t) is a decreasing function of t for fixed values of λ_1 and λ_2 and A. The R (t) is decreasing the function of A, for fixed values of λ_1 and λ_2. As λ_2 decreases R (t) decreases, when other parameters are fixed. If $\lambda_1 = \lambda_2$, then, the renewal density of this model is same as of the non-truncated models. Further, it was observed that the slope of R (t) is increasing as λ_2 increases. As 'A' is increasing, R (t) is also increasing. In RTED having a changing CLS distribution model, the renewal density will start increasing after some value of it and this value is approximately equal to A/2, it is one of the significant change in truncated model.

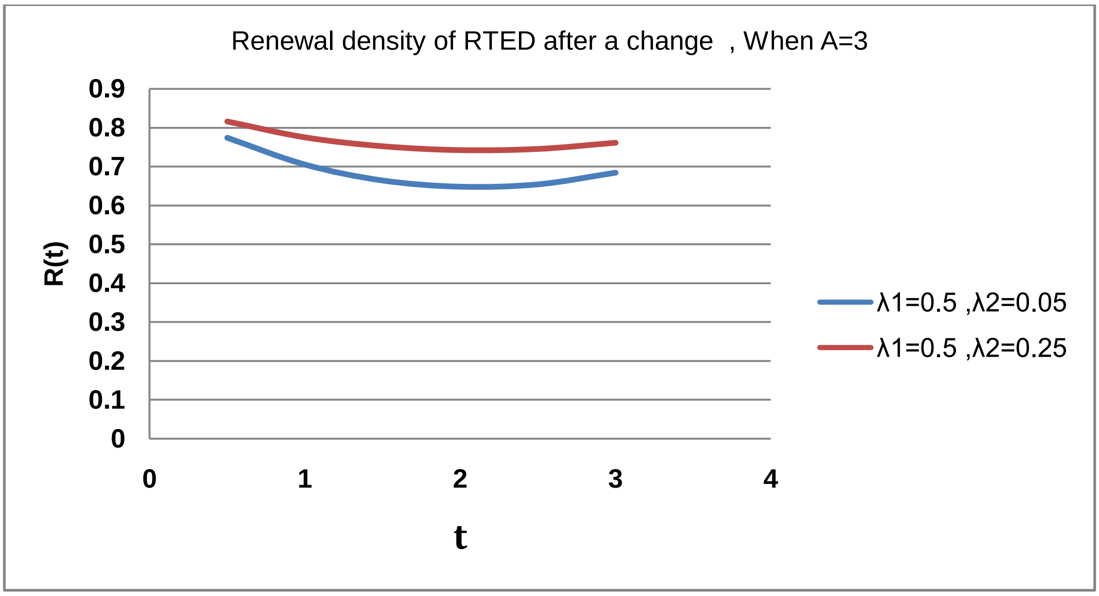

Figure 5: A graph of Renewal Density of RTED after change, when A=3

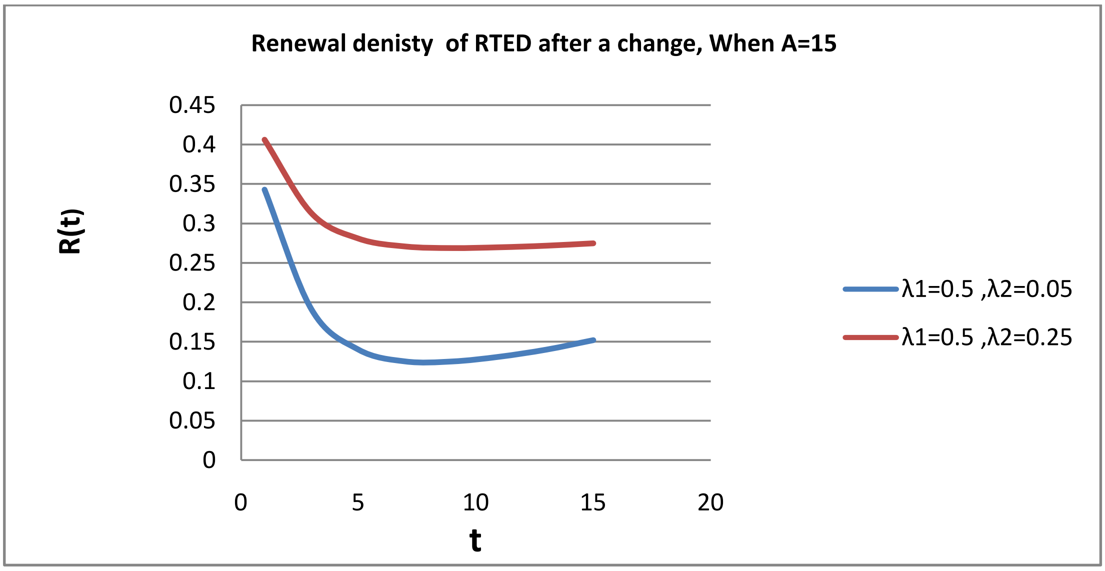

Figure 6: A graph of Renewal Density of RTED after change, when A=15

The results show that there is a significant change in characteristics of manpower model having CLS as Right Truncated Exponential Distribution (RTED), for an example, larger values of 'A' , the renewal equation gives λ.Therefore, this paper concluded that there is a significant difference in truncated distributions as compared to the non-truncated distributions, particularly for smaller values of 'A' .This type of situation is very common in private organizations, where the leaving of employees is common as compared to the government or public sector organizations.

6. Conclusions

In this paper, studied the Right Truncated Exponential Distribution (RTED) with the assumption that the Completed Length of Service (CLS) of an employee is very small, and developed and analyzed various lifetime functions. The numerical results and figures of the model show that the period of stay of an employee in an organization plays a vital role in this manpower model. The readers should note that this model includes some of the earlier models as particular cases for specific or limiting values of parameters. It was found that there is a significant difference in the truncated distributions as compared to the non-truncated distributions, particularly for smaller values. The proposed model is particularly useful in the private sector, where the employees leaving the organisation are more as compared to the public sector organizations.

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