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RESEARCH ARTICLE

THE EDGE DEGREE AND THE EDGE REGULAR PROPERTIES OF TRUNCATIONS OF FUZZY GRAPHS

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ABSTRACT

In this paper, degree of an edge in truncations of fuzzy graphs is obtained and edge regular properties of truncations of fuzzy graphs are studied. Truncations of fuzzy graph of an edge regular fuzzy graph need not be edge regular. Conditions under which it is edge regular are provided.

KEYWORDS: Strong fuzzy graph, complete fuzzy graph, edge regular fuzzy graph, totally edge regular fuzzy graph, truncations of fuzzy graph.

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1. INTRODUCTION

Fuzzy graph theory was introduced by Azriel Rosenfeld in 1975 [10]. Though it is very young, it has been growing fast and has numerous applications in various fields. During the same time Yeh and Bang have also introduced various connectedness concepts in fuzzy graphs [11]. A. Nagoorgani and K. Radha discussed the concepts of lower and upper truncations of a fuzzy graph [7]. K.Radha and N.Kumaravel (2014) introduced the concept of edge regular fuzzy graphs [8]. In this paper, we study about edge regular property of truncations of fuzzy graphs.

First we go through some basic definitions in the next section from [1] – [11].

2. BASIC CONCEPTS

Let V be a non-empty finite set and $E \subseteq V \times V$. A fuzzy graph $G:(\sigma, \mu)$ is a pair of functions $\sigma : V \to [0, 1]$ and $\mu : E \to [0, 1]$ such that $\mu(x, y) \le \sigma(x) \wedge \sigma(y)$ for all $x, y \in V$ [4]. The underlying crisp graph is denoted by $G^* : (V, E)$ .The order and size of a fuzzy graph $G:(\sigma, \mu)$

are defined by $O(G)=\sum_{x\in V}\sigma(x)$ and $S(G)=\sum_{xy\in E}\mu(xy)$ [3]. A fuzzy Graph $G:(\sigma,\mu)$ is strong, if $\mu(xy)=\sigma(x)\wedge\sigma(y)$ for all $xy\in E$ [3]. A fuzzy Graph $G:(\sigma,\mu)$ is complete, if $\mu(xy)=\sigma(x)\wedge\sigma(y)$ for all $x,y\in V$ [4].

The degree of a vertex x is $d_G(x)=\sum_{x\neq y}\mu(xy)$ [4]. If each vertex in G has same degree k, then G is said to be a regular fuzzy graph or $k-$ regular fuzzy graph [4]. The degree of an edge $e=uv\in E$ in G^* is defined by $d_{G^*}(uv)=d_{G^*}(u)+d_{G^*}(v)-2$ [1]. If each edge in G^* has same degree, then G^* is said to be edge regular. The degree of an edge $xy\in E$ in G is $d_G(xy)=\sum_{x\neq z}\mu(xz)+\sum_{z\neq y}\mu(zy)-2\mu(xy)$ [9]. If each edge in G has same degree k, then G is said to be an edge regular fuzzy graph or $k-$ edge regular fuzzy graph [8].

The adjacency sequence of an edge e in a fuzzy graph G is defined as a sequence of membership values of edges adjacent to e arranged in increasing order. It is denoted by $as(e)$ [5].

For $t>0$, $\sigma^t=\{u\in V/\sigma(u)\geq t\}$. The lower truncation $\sigma_{(t)}$ and upper truncation $\sigma^{(t)}$ of σ at level $'t'$, $0<t\leq 1$ are the fuzzy subsets,

$$\sigma_{(t)}(u)=\begin{cases}\sigma(u), & \text{if } u\in\sigma^t\\ 0, & \text{if } u\notin\sigma^t\end{cases} \text{ and } \sigma^{(t)}(u)=\begin{cases}t, & \text{if } u\in\sigma^t\\ \sigma(u), & \text{if } u\notin\sigma^t\end{cases}.$$

Taking $V_{(t)}=\sigma^t, E_{(t)}=\mu^t, G_{(t)}:(\sigma_{(t)},\mu_{(t)})$ is a fuzzy graph with underlying crisp graph $G_{(t)}{}^*:(V_{(t)},E_{(t)})$. This is called the lower truncation of the fuzzy graph G at level t.

Taking $V^{(t)}=V, E^{(t)}=E, G^{(t)}:(\sigma^{(t)},\mu^{(t)})$ is a fuzzy graph with underlying crisp graph $G^{(t)*}:(V^{(t)},E^{(t)})$. This is called the upper truncation of the fuzzy graph G at level t [7].

2.1. Theorem [8]: Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$. If μ is a constant function, then G is edge regular if and only if G^* is edge regular.

2.2. Theorem [9]: The size of a $k-$ edge regular fuzzy graph $G:(\sigma,\mu)$ on a k_1- edge regular graph $G^*:(V,E)$ is $\dfrac{qk}{k_1}$, where $q=|E|$.

3. DEGREE OF AN EDGE IN TRUNCATIONS OF FUZZY GRAPH

3.1. Degree of an edge in lower truncation of fuzzy graph:

$$d_{G_{(t)}}(uv)=\sum_{uw\in E_{(t)},w\neq v}\mu_{(t)}(uw)+\sum_{wv\in E_{(t)},w\neq u}\mu_{(t)}(wv),\forall uv\in E_{(t)}.$$

$$=\sum_{uw\in E,w\neq v}\mu(uw)-\sum_{\substack{uw\in E,w\neq v\\ \mu(uw)<t}}\mu(uw)+\sum_{wv\in E,w\neq u}\mu(wv)-\sum_{\substack{wv\in E,w\neq u\\ \mu(vw)<t}}\mu(wv),\forall uv\in E_{(t)}.$$

$$=d_G(uv)-\sum_{\substack{uw\in E,w\neq v\\ \mu(uw)<t}}\mu(uw)-\sum_{\substack{wv\in E,w\neq u\\ \mu(vw)<t}}\mu(wv),\forall uv\in E_{(t)}. \quad \text{.....................................(3.1)}$$

3.2. Degree of an edge in upper truncation of fuzzy graph:

$$d_{G^{(t)}}(uv) = \sum_{uw\in E^{(t)}, w\neq v} \mu^{(t)}(uw) + \sum_{wv\in E^{(t)}, w\neq u} \mu^{(t)}(wv), \forall uv \in E^{(t)}.$$

$$= \sum_{uw\in E, w\neq v} \mu(uw) - \sum_{\substack{uw\in E, w\neq v\\ \mu(uw)\geq t}} \left(\mu(uw)-t\right) + \sum_{wv\in E, w\neq u} \mu(wv) - \sum_{\substack{wv\in E, w\neq u\\ \mu(vw)\geq t}} \left(\mu(wv)-t\right), \forall uv\in E.$$

$$= d_G(uv) - \sum_{\substack{uw\in E, w\neq v\\ \mu(uw)>t}} \left(\mu(uw)-t\right) - \sum_{\substack{wv\in E, w\neq u\\ \mu(vw)>t}} \left(\mu(wv)-t\right), \forall uv\in E. \quad \text{......................(3.2)}$$

3.3. Theorem: Let $G:(\sigma,\mu)$ be a fuzzy graph such that $\mu(uv)\geq t, \forall uv\in E$, where $0<t\leq 1$. Then for any $uv\in E_{(t)}$, $d_{G_{(t)}}(uv) = d_G(uv)$.

Proof: From (3.1), for any $uv\in E_{(t)}$, $d_{G_{(t)}}(uv) = d_G(uv) - \sum_{\substack{uw\in E, w\neq v\\ \mu(uw)<t}} \mu(uw) - \sum_{\substack{wv\in E, w\neq u\\ \mu(vw)<t}} \mu(wv)$.

$\Rightarrow d_{G_{(t)}}(uv) = d_G(uv)$.

3.4. Theorem: Let $G:(\sigma,\mu)$ be a fuzzy graph such that $\mu(uv)=c, \forall uv\in E$, where c is a constant. Then for any $uv\in E^{(t)}$, $d_{G^{(t)}}(uv) = \begin{cases} d_G(uv), & \text{if } c<t \\ d_G(uv)-(c-t)d_{G^*}(uv), & \text{if } c\geq t \end{cases}$.

Proof: Let $t>c$. Then from (3.2) and from the definition of $\mu^{(t)}$,

$\Rightarrow d_{G^{(t)}}(uv) = d_G(uv) - \sum_{\substack{uw\in E, w\neq v\\ c>t}} \left(c-t\right) - \sum_{\substack{wv\in E, w\neq u\\ c>t}} \left(c-t\right)$.

Hence $d_{G_{(t)}}(uv) = d_G(uv)$.

Similarly, when $t\leq c$, $d_{G^{(t)}}(uv) = d_G(uv) - \sum_{\substack{uw\in E, w\neq v\\ c>t}} \left(c-t\right) - \sum_{\substack{wv\in E, w\neq u\\ c>t}} \left(c-t\right)$.

$$= d_G(uv) - \left(c-t\right)\left(d_{G^*}(u)-1\right) - \left(c-t\right)\left(d_{G^*}(v)-1\right).$$

$$= d_G(uv) - \left(c-t\right)\left(d_{G^*}(u)+d_{G^*}(v)-2\right).$$

Hence $d_{G^{(t)}}(uv) = d_G(uv) - \left(c-t\right)d_{G^*}(uv)$.

4. EDGE REGULAR PROPERTY OF TRUNCATIONS OF FUZZY GRAPH

4.1. Remark: If $G:(\sigma,\mu)$ is an edge regular fuzzy graph, then $G_{(t)}:(\sigma_{(t)},\mu_{(t)})$ and $G^{(t)}:(\sigma^{(t)},\mu^{(t)})$ need not be edge regular fuzzy graphs. For example, in figure 4.1 $G:(\sigma,\mu)$ is 1.6 – edge regular fuzzy graph, but $G_{(0.5)}:(\sigma_{(0.5)},\mu_{(0.5)})$ and $G^{(0.5)}:(\sigma^{(0.5)},\mu^{(0.5)})$ are not an edge regular fuzzy graphs.

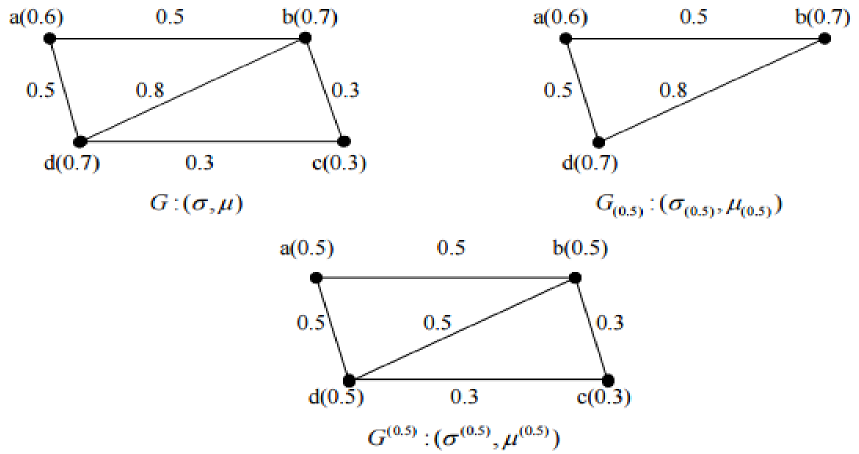

Fig.4.1.

4.2. Remark: If $G_{(t)}:(\sigma_{(t)},\mu_{(t)})$ and $G^{(t)}:(\sigma^{(t)},\mu^{(t)})$ are edge regular fuzzy graphs, then $G:(\sigma,\mu)$ need not be an edge regular fuzzy graph. For example, in figure 4.2 $G_{(0.5)}:(\sigma_{(0.5)},\mu_{(0.5)})$ is 0.6 – edge regular and $G^{(0.4)}:(\sigma^{(0.4)},\mu^{(0.4)})$ is 0.8 – edge regular. But $G:(\sigma,\mu)$ is not an edge regular fuzzy graph.

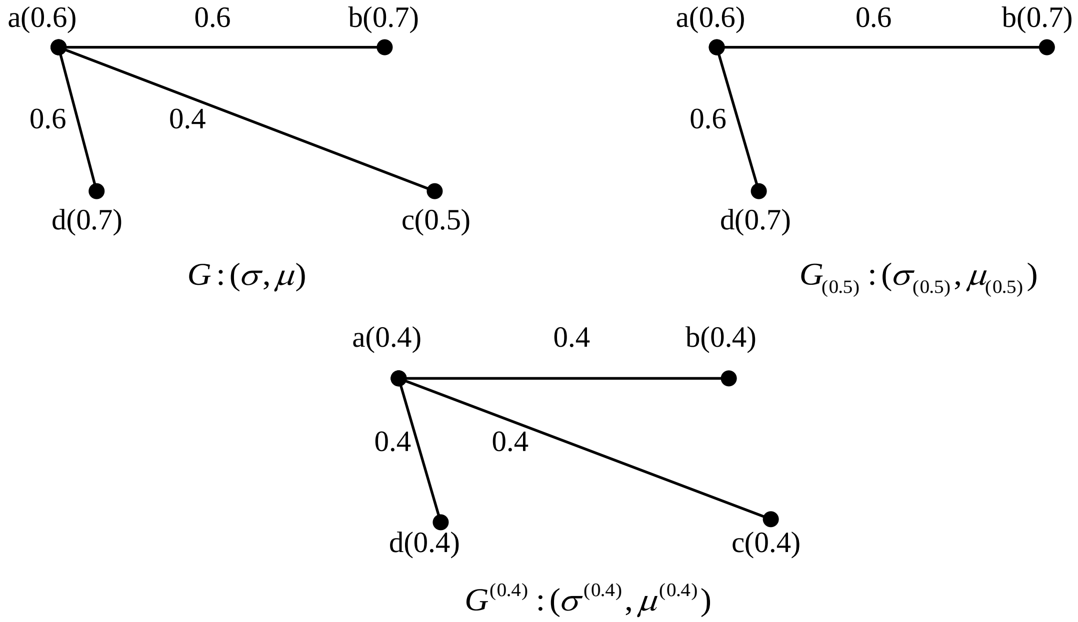

Fig.4.2.

In the following theorems, we obtain some conditions under which $G_{(t)}$ and $G_{(t)}$ are edge regular.

4.3. Theorem: Let $G:(\sigma,\mu)$ be a fuzzy graph such that μ is a constant function with constant value *c*. For every $0<t\leq c$, $G_{(t)}$ is edge regular if and only if G is edge regular.

Proof: If $0<t\leq c$, then $V_{(t)}=V$, because $\sigma(v)\geq t,\forall v\in V$ and $E_{(t)}=E$, because $\mu(e)\geq t,\forall e\in E$.

$\therefore$ From theorem 3.3, $d_G(e)=d_{G_{(t)}}(e)$, for every $e\in E$.

Hence *G* is edge regular if and only if $G_{(t)}$ is edge regular.

4.4. Remark: If $c < t$, then $G_{(t)}$ is an empty graph.

4.5. Theorem: Let $G:(\sigma,\mu)$ be a fuzzy graph such that μ is a constant function. Then $G:(\sigma,\mu)$ is an edge regular fuzzy graph if and only if $G^{(t)}$ is an edge regular fuzzy graph, where $0 < t \le 1$.

Proof: Assume that $G:(\sigma,\mu)$ is an m – edge regular fuzzy graph.

Since μ is a constant function, by theorem 2.1, $G^*:(V,E)$ is an edge regular graph.

Let G^* be k – edge regular.

When $t > c$, by theorem 3.4, $d_{G^{(t)}}(uv) = d_G(uv) = m$, for every $uv \in E^{(t)}$.

Therefore $G^{(t)}$ is m – edge regular.

When $t \le c$, by theorem 3.4, $d_{G^{(t)}}(uv) = m-(c-t)k$, for every $uv \in E^{(t)}$.

Therefore $G^{(t)}$ is $\left(m-(c-t)k\right)$ – edge regular.

Conversely, assume that $G^{(t)}$ is edge regular, for every $0 < t \le 1$.

When $c < t$, by theorem 3.4, $d_G(uv) = d_{G^{(t)}}(uv)$, $\forall\ uv \in E^{(t)}$ implies that G is also edge regular.

Let $c \ge t$. Then $\mu^{(t)}$ is a constant function of constant value t. Therefore by theorem 2.1, $G^{(t)*}$ is edge regular. Since underlying crisp graphs of G and $G^{(t)}$ are same, G^* is edge regular.

From theorem 3.4, $d_G(uv) = d_{G^{(t)}}(uv) + (c-t)d_{G^*}(uv)$, for every $uv \in E$. Hence G is an edge regular fuzzy graph.

4.6. Theorem: Let $G:(\sigma,\mu)$ be a fuzzy graph on an odd cycle $G^*:(V,E)$. Then G is edge regular if and only if μ is a constant function.

Proof: Let G be a k – edge regular fuzzy graph on an odd cycle $v_1v_2,\ldots,v_nv_1$.

Let $\mu(v_1v_2) = s$.

Edge degree of an edge in a cycle is the sum of membership values of the two edges adjacent to it.

$$\therefore\ d(v_2v_3) = \mu(v_1v_2) + \mu(v_3v_4) \Rightarrow k = s + \mu(v_3v_4) \Rightarrow \mu(v_3v_4) = k - s.$$

Similarly, $\mu(v_5v_6) = s$, $\mu(v_7v_8) = k-s$ and so on.

Proceeding like this, we get

$$\mu(v_nv_1) = \begin{cases} s, & if\ \ n-1 \equiv 0 \pmod 4 \\ k-s, & if\ \ n-1 \not\equiv 0 \pmod 4 \end{cases}$$

Case 1: $\mu(v_nv_1) = s$.

Proceeding as above, we get

$$\mu(v_2v_3) = k-s,\ \mu(v_4v_5) = s,\ \mu(v_6v_7) = k-s \ldots\ldots$$

Since $n-1 \equiv 0 \pmod 4$, $\mu(v_{n-1}v_n) = s$.

Now, $$d(v_nv_1) = k \Rightarrow \mu(v_{n-1}v_n) + \mu(v_1v_2) = k \Rightarrow s+s = k \Rightarrow s = \frac{k}{2}.$$

$$\therefore\ k - s = k - \frac{k}{2} = \frac{k}{2}.$$

$$\therefore\ \mu(v_iv_{i+1}) = \frac{k}{2}, \forall\, i = 1,2,\ldots,n, \text{ where } v_{n+1} = v_1.$$

Case 2: $\mu(v_n v_1) = k - s$.

Proceeding as above, we get

$\mu(v_2 v_3) = s$, $\mu(v_4 v_5) = k - s$, $\mu(v_6 v_7) = s$.......

Since $n-1 \not\equiv 0$ (mod4), $\mu(v_{n-1} v_n) = s$.

Now, proceeding as above,

$$d(v_n v_1) = k \Rightarrow s = \frac{k}{2}.$$

$$\therefore\ k - s = k - \frac{k}{2} = \frac{k}{2}.$$

$$\therefore\ \mu(v_i v_{i+1}) = \frac{k}{2}, \forall i = 1, 2, ..., n, \text{ where } v_{n+1} = v_1.$$

Hence μ is a constant function.

Conversely, assume that μ is a constant function with constant value c.Then G is $2c-$edge regular.

4.7. Theorem: Let $G:(\sigma,\mu)$ be a fuzzy graph on an even cycle $G^*:(V,E)$ with n vertices and let $n \not\equiv 0 \pmod 4$. Then G is an edge regular fuzzy graph if and only if μ is a constant function.

Proof: Let G be a fuzzy graph on an even cycle $v_1 v_2, ..., v_n v_1$, where $n \not\equiv 0 \pmod 4$.

If μ is a constant function with constant value c, then G is $2c-$edge regular.

Conversely, let G be k $-$edge regular.

Since n is even and $n \not\equiv 0 \pmod 4$, we have $n - 2 \equiv 0 \pmod 4$.

Therefore $\frac{n-2}{2}$ is an even number. Hence the number of edges that lie alternatively from $v_1 v_2$ is $\frac{n}{2} - 1$, an even number.

Let $\mu(v_1 v_2) = s$. Then proceeding as in the previous theorem $\mu(v_3 v_4) = k - s$, $\mu(v_5 v_6) = s$, ..., $\mu(v_{n-3} v_{n-2}) = k - s$ and $\mu(v_{n-1} v_n) = s$, which is the $\left(\frac{n-2}{2}\right)^{th}$ term(even term) of the sequence.

$$\therefore\ d(v_n v_1) = \mu(v_{n-1} v_n) + \mu(v_1 v_2) \Rightarrow k = s + s \Rightarrow s = \frac{k}{2}.$$

$$\therefore\ \ k - s = \frac{k}{2}.$$

Therefore $\mu(v_1 v_2) = \mu(v_3 v_4) = ... = \mu(v_{n-1} v_n) = \frac{k}{2}$.

Similarly, if $\mu(v_2 v_3) = r$, then proceeding as above, $\mu(v_n v_1) = r$ and $d(v_1 v_2) = k$ gives $r = \frac{k}{2}$.

Therefore μ is a constant function.

4.8. Theorem: Let $G:(\sigma,\mu)$ be a fuzzy graph on an even cycle $G^*:(V,E)$ with n vertices and let $n \equiv 0 \pmod 4$. Then G is a $k-$edge regular fuzzy graph if and only if μ assumes exactly four values r, s, t and l such that consecutive adjacent edges receives these values in cyclic order with $r + t = k$ and $s + l = k$.

Proof: Let G be a $k-$edge regular fuzzy graph on an even cycle $v_1v_2,...,v_nv_1$, where $n \equiv 0 \pmod 4$.

Let $\mu(v_1v_2) = r$.

Since $n \equiv 0 \pmod 4$, $n-2$ is even and $n-2 \not\equiv 0 \pmod 4$. Therefore $\frac{n-2}{2}$ is an odd number.

$\therefore$ The number of edges that lie alternatively from v_1v_2 is $\frac{n}{2}-1$, an odd number.

Now $\mu(v_1v_2) = r$ and $\mu(v_2v_3) = k$ gives $\mu(v_3v_4) = k-r$.

Similarly, $\mu(v_5v_6) = r$ … $\mu(v_{n-1}v_n) = k-r$.

If $\mu(v_2v_3) = s$, then proceeding as above, $\mu(v_4v_5) = k-s$, $\mu(v_6v_7) = s$ … $\mu(v_nv_1) = k-s$.

$\therefore$ The consecutive adjacent edges of the cycle receives the four values $r, s, k-r$ and $k-s$ in cyclic order.

Conversely, by our assumption, $d(e) = r+t$ or $s+l, \forall e \in E$.

$$= k, \ \forall e \in E.$$

Therefore G is a $k-$edge regular fuzzy graph.

4.9. Remark: If the minimum of the membership values of the edges is not unique, then the fuzzy graph on a cycle is a fuzzy cycle (not a fuzzy tree). Hence all edge regular fuzzy graphs on cycles are fuzzy cycles except for *n* = 4. When *n* = 4 and *r* ≠ *s*, the minimum of the membership values of the four edges is unique, in which case, the edge regular fuzzy graph on cycle is a fuzzy tree.

4.10. Theorem: Let $G:(\sigma,\mu)$ be a fuzzy graph on a cycle G^* with $n \not\equiv 0 (\mathrm{mod}\, 4)$, where $|V| = n$. Then $G:(\sigma,\mu)$ is an edge regular fuzzy graph if and only if $G^{(t)}$ are edge regular fuzzy graphs, where $0 < t \le 1$.

Proof: Given G is a fuzzy graph on a cycle G^* with $n \not\equiv 0 (\mathrm{mod}\, 4)$, by theorems 4.6 and 4.7, μ is a constant function, the result follows from theorem 4.5.

4.11. Theorem: If all the edges of G have the same adjacency sequence, then all the edges of $G_{(t)}$ have the same adjacency sequence.

Proof: Suppose that all the edges of G have the same adjacency sequence, say $(k_1, k_2, ..., k_n)$.

If $t > k_n$, then there is no edge in $G_{(t)}$.

If $t \le k_1$, then $as(e) = (k_1, k_2, ..., k_n)$ for each $e \in E_{(t)}$.

If $k_{i-1} < t \le k_i$, for some i, then $as(e) = (k_i, k_{i+1}, ..., k_n)$ for each $e \in E_{(t)}$.

Hence the theorem follows.

4.12. Remark: Converse of theorem 4.11 need not be true. For example, all the edges in $G_{(0.5)}$ have the same adjacency sequence $(0.6, 0.7)$. But in $G:(\sigma,\mu)$, $as(p) = (0.4, 0.6, 0.7)$ $\ne (0.6, 0.6, 0.7, 0.7) = as(t)$.

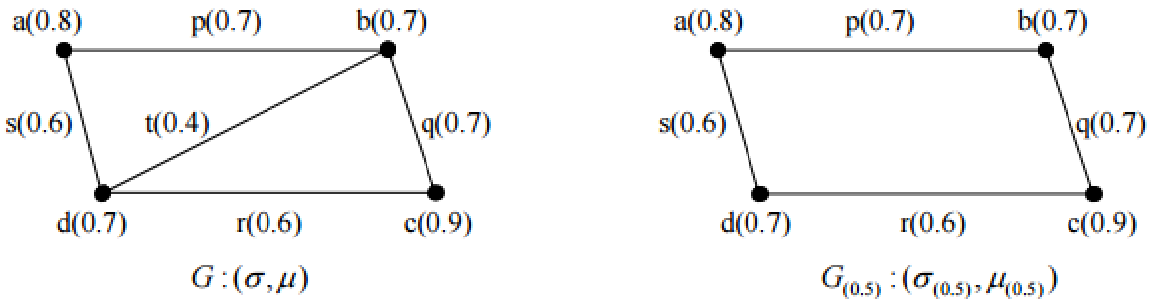

Fig.4.3.

4.13. Theorem: If all the edges of G have the same adjacency sequence, then all the edges of $G^{(t)}$ have the same adjacency sequence.

Proof: Let $(k_1,k_2,...,k_n)$ be the adjacency sequence of each edge in G.

If $t>k_n$, then $as(e)=(k_1,k_2,...,k_n)$ for each $e\in E^{(t)}$. .... (4.1)

If $t\le k_1$, then $as(e)=(t,t,...,t)$ for each $e\in E^{(t)}$. .... (4.2)

If $k_{i-1}<t\le k_i$, then $as(e)=(k_1,k_2,...,k_{i-1},t,t,...,t)$ for each $e\in E^{(t)}$. .... (4.3)

Hence the theorem follows.

4.14. Remark: Converse of theorem 4.13 need not be true. For example, all the edges in $G^{(0.4)}$ have the same adjacency sequence $(0.2,0.4,0.4)$.

But in $G:(\sigma,\mu)$, $as(p)=(0.2,0.5,0.7)\ne(0.2,0.6,0.7)=as(q)$.

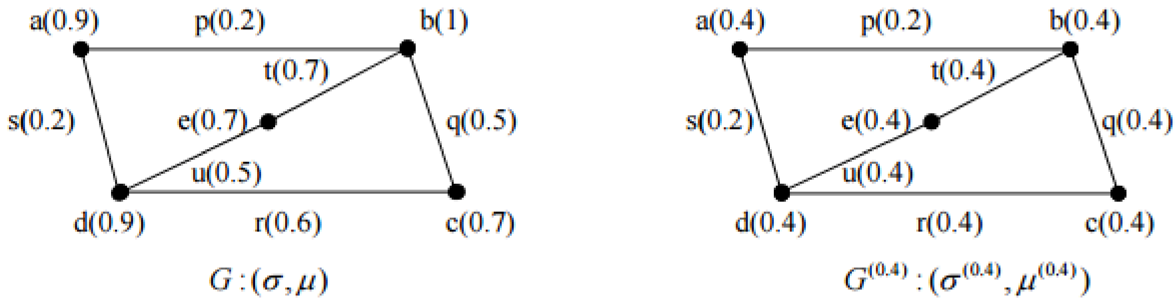

Fig.4.4.

4.15. Theorem: If all the edges of G have the same adjacency sequence, then $G_{(t)}$ and $G^{(t)}$ are edge regular fuzzy graphs.

Proof: When all the edges of G have the same adjacency sequence, the same holds for $G_{(t)}$ and $G^{(t)}$ also. Since the sum of all the elements of the adjacency sequence of an edge is its degree, $G_{(t)}$ and $G^{(t)}$ are edge regular fuzzy graphs.

4.16. Remark: Converse of theorem 4.15 need not be true. For example, in the figure 4.2, $G_{(t)}$ and $G^{(t)}$ are edge regular fuzzy graphs, but $G:(\sigma,\mu)$ do not have same adjacency sequence, that is, $as(ab)=(0.4,0.6)\ne(0.6,0.6)=as(ac)$.

5. PROPERTIES OF TRUNCATIONS OF FUZZY GRAPHS

5.1. Theorem: Let $G:(\sigma,\mu)$ be a fuzzy graph on $G^*:(V,E)$ such that $\mu(e)=c,\forall e\in E$, where c is a constant. Then the size of $G^{(t)}$ is either tq or cq, where $|E|=q$.

Proof: When $c \geq t$. Then $\sigma^{(t)}(v) = t, \forall v \in V$ & $\mu^{(t)}(e) = t, \forall e \in E$ in $G^{(t)}$ .... (5.1)

When $c < t$. Then $\mu^{(t)}(e) = \mu(e) = c, \forall e \in E$ in $G^{(t)}$ .... (5.2)

Therefore the result follows from the equations (5.1) and (5.2).

5.2. Theorem: Let $G:(\sigma, \mu)$ be a fuzzy graph on $G^*:(V, E)$ such that $\mu(e) = c, \forall e \in E$, where c is a constant. Then the size of $G_{(t)}$ is either cq or 0, where $|E| = q$.

Proof: When $0 < t \leq c$. Then $V_{(t)} = V$, because $\sigma(v) \geq t, \forall v \in V$ and $E_{(t)} = E$, because $\mu(e) \geq t, \forall e \in E$.

$\therefore$ $\sigma_{(t)}(v) = \sigma(v), \forall v \in V$ & $\mu_{(t)}(e) = \mu(e) = c, \forall e \in E$ in $G_{(t)}$. Therefore the size of $G_{(t)}$ is cq.

When $t > c$, $G_{(t)}$ has no edge. Therefore the size of $G_{(t)}$ is 0.

5.3. Theorem: If all the edges of G have the same adjacency sequence $(k_1, k_2, ..., k_n)$ with $|E| = q$, then the size of $G^{(t)}$ is either $S(G)$ or qt or $\frac{qk}{n}$,

where $k = k_1 + k_2 + ... + k_{i-1} + (n - i + 1)t, k_{i-1} < t \leq k_i$.

Proof: If $t > k_n$, then $as(e) = (k_1, k_2, ..., k_n)$ for each $e \in E^{(t)}$. So the size of $G^{(t)}$ is $S(G)$.

If $t \leq k_1$, then $as(e) = (t, t, ..., t)$ for each $e \in E^{(t)}$. So the size of $G^{(t)}$ is qt.

If $k_{i-1} < t \leq k_i$, then $as(e) = (k_1, k_2, ..., k_{i-1}, t, t, ..., t)$ for each $e \in E^{(t)}$.

If $k = k_1 + k_2 + ... + k_{i-1} + (n - i + 1)t$, then $G^{(t)}$ is $k-$ edge regular. Also $G^{(t)^*}$ is n − edge regular. Hence by theorem 2.2, the size of $G^{(t)}$ is $\frac{qk}{n}$.

5.4. Theorem: If all the edges of G have the same adjacency sequence $(k_1, k_2, ..., k_n)$ with $|E| = q$, then the size of $G_{(t)}$ is either 0 or $S(G)$ or $\frac{qk}{n-i+1}$, $k = k_i + k_{i+1} + ... + k_n$, $k_{i-1} < t \leq k_i$.

Proof: If $t > k_n$, then there is no edge in $G_{(t)}$. So the size of $G_{(t)}$ is 0.

If $t \leq k_1$, then $as(e) = (k_1, k_2, ..., k_n)$ for each $e \in E_{(t)}$. So the size of $G_{(t)}$ is $S(G)$.

If $k_{i-1} < t \leq k_i$, then $as(e) = (k_i, k_{i+1}, ..., k_n)$ for each $e \in E_{(t)}$.

If $k = k_i + k_{i+1} + ... + k_n$, then $G_{(t)}$ is k − edge regular.

Also $G_{(t)}^*$ is $n - i + 1$ − edge regular. Hence by theorem 2.2, the size of $G_{(t)}$ is $\frac{qk}{n-i+1}$.

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