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A STUDY ON BIPOLAR VALUED Q-FUZZY SUBGROUPS OF A GROUP

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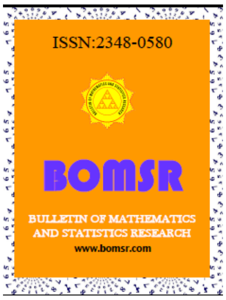

ABSTRACT

In this paper, we study some of the properties of bipolar valued Q-fuzzy subgroup and prove some results on these.

KEY WORDS: Bipolar-valued Q-fuzzy set, bipolar valued Q-fuzzy subgroup, bipolar valued Q-fuzzy normal subgroup, product, bipolar valued Q-fuzzy coset.

INTRODUCTION

In 1965, Zadeh [12] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since then it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval-valued fuzzy sets, vague sets, soft sets etc [5]. Lee [7] introduced the notion of bipolar-valued fuzzy sets. Bipolar-valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [–1, 1]. In a bipolar-valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [–1, 0) indicates that elements somewhat satisfy the implicit counter property. Bipolar-valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [7, 8]. We introduce the concept of bipolar valued Q-fuzzy subgroup and established some results.

1.PRELIMINARIES

1.1 Definition: A bipolar-valued Q-fuzzy set (BVQFS) A in X is defined as an object of the form A = { < (x, q), $A^+(x, q)$, $A^-(x, q)$ > / x in X and q in Q},

where $A^+ : X\times Q \to [0, 1]$ and $A^- : X\times Q \to [-1, 0]$. The positive membership degree $A^+(x, q)$ denotes the satisfaction degree of an element (x, q) to the property corresponding to a bipolar-valued Q-fuzzy set A and the negative membership degree $A^-(x, q)$ denotes the satisfaction degree of an element (x, q) to some implicit counter-property corresponding to a bipolar-valued Q-fuzzy set A. If $A^+(x, q) \neq 0$ and $A^-(x, q) = 0$, it is the situation that (x, q) is regarded as having only positive satisfaction for A and if $A^+(x, q) = 0$ and $A^-(x, q) \neq 0$, it is the situation that (x, q) does not satisfy the property of A, but somewhat satisfies the counter property of A. It is possible for an element (x, q) to be such that $A^+(x, q) \neq 0$ and $A^-(x, q) \neq 0$ when the membership function of the property overlaps that of its counter property over some portion of X.

1.1 Example: A = { < (a, q), 0.7, –0.4 >, < (b, q), 0.6, –0.7 >, < (c, q), 0.5, –0.8 >} is a bipolar-valued Q-fuzzy subset of X= {a, b, c }, where Q = {q }.

1.2 Definition: Let G be a group and Q be a non-empty set. A bipolar-valued Q-fuzzy subset A of G is said to be a bipolar-valued Q-fuzzy subgroup of G (BVQFSG) if the following conditions are satisfied,

(i) $A^+(xy, q) \geq \min\{ A^+(x, q), A^+(y, q) \}$,

(ii) $A^+(x^{-1}, q) \geq A^+(x, q)$,

(iii) $A^-(xy, q) \leq \max\{ A^-(x, q), A^-(y, q) \}$,

(iv) $A^-(x^{-1}, q) \leq A^-(x, q)$, for all x and y in G and q in Q.

1.2 Example: Let G = {1, –1, i, –i } be a group with respect to the ordinary multiplication and Q = {q }. Then A = { < (1, q), 0.5, – 0.6 >, < (–1, q), 0.4, – 0.5 >,< (i, q), 0.2, –0.4 >, < (–i, q), 0.2, –0.4 > } is a bipolar-valued Q-fuzzy subgroup of G.

1.3 Definition: Let $A = \langle A^+, A^- \rangle$ and $B = \langle B^+, B^- \rangle$ be any two bipolar-valued Q-fuzzy subsets of sets G and H, respectively. The product of A and B, denoted by A×B, is defined as $A\times B = \{\langle ((x, y), q), (A\times B)^+((x, y), q), (A\times B)^-((x, y), q) \rangle$ / for all x in G and y in H and q in Q}, where $(A\times B)^+((x, y), q) = \min\{ A^+(x, q), B^+(y, q) \}$ and $(A\times B)^-((x, y), q)= \max\{ A^-(x, q), B^-(y, q) \}$, for all x in G and y in H and q in Q.

1.4 Definition: Let G be a group. A bipolar valued Q-fuzzy subgroup A of G is said to be a bipolar valued Q-fuzzy normal subgroup of G if $A^+(xy, q) = A^+(yx, q)$ and $A^-(xy, q) = A^-(yx, q)$, for all x, y in G and q in Q.

1.5 Definition: Let A be a bipolar valued Q-fuzzy subgroup of a group G. For any a in G, aA defined by $(aA^+)(x, q) = A^+(a^{-1}x, q)$ and $(aA^-)(x, q) = A^-(a^{-1}x, q)$, for every x in G and q in Q is called the bipolar valued Q-fuzzy coset of the group G.

1.6 Definition: Let A be a bipolar valued Q-fuzzy subgroup of a group G and $H = \{x\in G/ A^+(x, q)=A^+(e, q)$ and $A^-(x, q)=A^-(e, q)\}$, then o(A), order of A is defined as o(A)= o(H).

1.7 Definition: Let A and B be two bipolar valued Q-fuzzy subgroups of a group G. Then A and B are said to be conjugate bipolar valued Q-fuzzy subgroup of G if for some g in G and q in Q, $A^+(x, q)= B^+(g^{-1}xg, q)$ and $A^-(x, q)= B^-(g^{-1}xg, q)$, for every x in G and q in Q.

1.8 Definition: Let A be a bipolar valued Q-fuzzy subgroup of a group G. Then for any a and b in G, a bipolar valued Q-fuzzy middle coset aAb of G is defined by $(aA^+b)(x, q)= A^+(a^{-1}xb^{-1}, q)$ and $(aA^-b)(x, q)= A^-(a^{-1}x b^{-1}, q)$, for every x in G and q in Q.

2. PROPERTIES

2.1 Theorem: Let $A = \langle A^+, A^- \rangle$ be a bipolar valued Q-fuzzy subgroup of a group G. If $A^+(x, q) < A^+(y, q)$ and $A^-(x, q) > A^-(y, q)$, for some x, y in G and q in Q, then $A^+(xy, q) = A^+(x, q) = A^+(yx, q)$ and $A^-(xy, q) = A^-(x, q) = A^-(yx, q)$.

Proof: Let $A = \langle A^+, A^-\rangle$ be a bipolar valued Q-fuzzy subgroup of a group G. Let $A^+(x, q) < A^+(y, q)$ and $A^-(x, q) > A^-(y, q)$, for some x, y in G and q in Q. Now, $A^+(xy, q) \geq \min\{A^+(x, q), A^+(y, q) \}= A^+(x, q)$; and $A^+(x, q) = A^+(xyy^{-1}, q) \geq \min\{ A^+(xy, q), A^+(y, q)\} = A^+(xy, q)$. Also, $A^+(yx, q) \geq \min\{ A^+(y, q), A^+(x, q)\} = A^+(x, q)$; and $A^+(x, q) = A^+(y^{-1}yx, q) \geq \min\{ A^+(y, q), A^+(yx, q)\} = A^+(yx, q)$. Therefore $A^+(xy, q) = A^+(x, q) = A^+(yx, q)$. Also $A^-(xy, q) \leq \max\{ A^-(x, q), A^-(y, q) \}= A^-(x, q)$; and $A^-(x, q) = A^-(xyy^{-1}, q) \leq \max\{A^-(xy, q),$

$A^-(y, q)$ }= $A^-(xy, q)$. And, $A^-(yx, q) \leq \max\{ A^-(y, q), A^-(x, q) \} = A^-(x, q)$; and $A^-(x, q) = A^-(y^{-1}yx, q) \leq \max\{ A^-(y, q), A^-(yx, q) \} = A^-(yx, q)$. Therefore $A^-(xy, q) = A^-(x, q) = A^-(yx, q)$.

2.2 Theorem: Let A= $\langle A^+, A^- \rangle$ be a bipolar valued Q-fuzzy subgroup of a group G. If $A^+(x, q) < A^+(y, q)$ and $A^-(x, q) < A^-(y, q)$, for some x, y in G and q in Q, then $A^+(xy, q) = A^+(x, q) = A^+(yx, q)$ and $A^-(xy, q) = A^-(y, q) = A^-(yx, q)$.

Proof: Let A= $\langle A^+, A^- \rangle$ be a bipolar valued Q-fuzzy subgroup of a group G. Let $A^+(x, q) < A^+(y, q)$ and $A^-(x, q) < A^-(y, q)$, for some x, y in G and q in Q. Now, $A^+(xy, q) \geq \min\{A^+(x, q), A^+(y, q)\} = A^+(x, q)$; and $A^+(x, q) = A^+(xyy^{-1}, q) \geq \min\{ A^+(xy, q), A^+(y, q)\} = A^+(xy, q)$. And $A^+(yx, q) \geq \min\{ A^+(y, q), A^+(x, q) \} = A^+(x, q)$; and $A^+(x, q) = A^+(y^{-1}yx, q) \geq \min\{ A^+(y, q), A^+(yx, q) \} = A^+(yx, q)$. Therefore $A^+(xy, q) = A^+(x, q) = A^+(yx, q)$. Now, $A^-(xy, q) \leq \max\{ A^-(x, q), A^-(y, q) \} = A^-(y, q)$; and $A^-(y, q) = A^-(x^{-1}xy, q) \leq \max\{ A^-(x, q), A^-(xy, q) \} = A^-(xy, q)$. And, $A^-(yx, q) \leq \max\{ A^-(y, q), A^-(x, q) \} = A^-(y, q)$; and $A^-(y, q) = A^-(yxx^{-1}, q) \leq \max\{ A^-(yx, q), A^-(x, q) \} = A^-(yx, q)$. Therefore $A^-(xy, q) = A^-(y, q) = A^-(yx, q)$.

2.3 Theorem: Let A = $\langle A^+, A^- \rangle$ be a bipolar valued Q-fuzzy subgroup of a group G. If $A^+(x, q) > A^+(y, q)$ and $A^-(x, q) > A^-(y, q)$, for some x, y in G and q in Q, then $A^+(xy, q) = A^+(y, q) = A^+(yx, q)$ and $A^-(xy, q) = A^-(x, q) = A^-(yx, q)$.

Proof: It is trivial.

2.4 Theorem: Let A = $\langle A^+, A^- \rangle$ be a bipolar valued Q-fuzzy subgroup of a group G. If $A^+(x, q) > A^+(y, q)$ and $A^-(x, q) < A^-(y, q)$, for some x, y in G and q in Q, then $A^+(xy, q) = A^+(y, q) = A^+(yx, q)$ and $A^-(xy, q) = A^-(y, q) = A^-(yx, q)$.

Proof: It is trivial.

2.5 Theorem: Let A = $\langle A^+, A^- \rangle$ be a bipolar valued Q-fuzzy subgroup of a finite group G, then o(A) divides o(G).

Proof: Let A be a bipolar valued Q-fuzzy subgroup of a finite group G with e as its identity element. Clearly H = $\{ x \in G \,/\, A^+(x, q) = A^+(e, q)$ and $A^-(x, q) = A^-(e, q) \}$ is a subgroup of the group G. By Lagranges theorem o(H) | o(G). Hence by the definition of the order of the bipolar valued Q-fuzzy subgroup of the group G, we have o(A) | o(G).

2.6 Theorem: Let A= $\langle A^+, A^- \rangle$ and B= $\langle B^+, B^- \rangle$ be two bipolar-valued Q-fuzzy subsets of an abelian group G. Then A and B are conjugate bipolar-valued Q-fuzzy subsets of the group G if and only if A = B.

Proof: Let A and B be conjugate bipolar-valued Q-fuzzy subsets of group G, then for some y in G and q in Q, we have $A^+(x, q) = B^+(y^{-1}xy, q) = B^+(y^{-1}yx, q) = B^+(ex, q) = B^+(x, q)$. Therefore $A^+(x, q) = B^+(x, q)$. And, $A^-(x, q) = B^-(y^{-1}xy, q) = B^-(y^{-1}yx, q) = B^-(ex, q) = B^-(x, q)$. Therefore $A^-(x, q) = B^-(x, q)$. Hence A = B. Conversely if A = B then for the identity element e of the group G, we have $A^+(x, q) = B^+(e^{-1}xe, q)$ and $A^-(x, q) = B^-(e^{-1}xe, q)$ for every x in G. Hence A and B are conjugate bipolar-valued Q-fuzzy subsets of the group G.

2.7 Theorem: If A= $\langle A^+, A^- \rangle$ and B= $\langle B^+, B^- \rangle$ are conjugate bipolar valued Q-fuzzy subgroups of the group G, then o(A) = o(B).

Proof: Let A and B are conjugate bipolar valued Q-fuzzy subgroups of the group G. Now, o(A) = order of $\{ x \in G \,/\, A^+(x, q) = A^+(e, q)$ and $A^-(x, q) = A^-(e, q) \}$ = order of $\{x \in G \,/\, B^+(y^{-1}xy, q) = B^+(y^{-1}ey, q)$ and $B^-(y^{-1}xy, q) = B^-(y^{-1}ey, q) \}$ = order of $\{ x \in G \,/\, B^+(x, q) = B^+(e, q)$ and $B^-(x, q) = B^-(e, q) \}$ = o(B). Hence o(A) = o(B).

2.8 Theorem: Let A= $\langle A^+, A^- \rangle$ be a bipolar valued Q-fuzzy normal subgroup of a group G. Then for any y in G and q in Q, we have $A^+(yxy^{-1}, q) = A^+(y^{-1}xy, q)$ and $A^-(yxy^{-1}, q) = A^-(y^{-1}xy, q)$, for every x in G and q in Q.

Proof: Let A be a bipolar valued Q-fuzzy normal subgroup of a group G. For any y in G and q in Q. Then we have, $A^+(yxy^{-1}, q) = A^+(x, q) = A^+(xyy^{-1}, q) = A^+(y^{-1}xy, q)$. Therefore $A^+(yxy^{-1}, q) = A^+(y^{-1}xy, q)$. And, $A^-(yxy^{-1}, q) = A^-(x, q) = A^-(xyy^{-1}, q) = A^-(y^{-1}xy, q)$. Therefore $A^-(yxy^{-1}, q) = A^-(y^{-1}xy, q)$.

2.9 Theorem: A bipolar valued Q-fuzzy subgroup $A = \langle A^+, A^- \rangle$ of a group G is normalized if and only if $A^+(e, q) = 1$ and $A^-(e, q) = 0$, where e is the identity element of the group G.

Proof: If A is normalized then there exists x in G and q in Q such that $A^+(x, q) = 1$ and $A^-(x, q) = 0$, but by properties of a bipolar valued Q-fuzzy subgroup A of the group G, $A^+(x, q) \leq A^+(e, q)$ and $A^-(x, q) \geq A^-(e, q)$ for every x in G and q in Q. Since $A^+(x, q) = 1$ and $A^-(x, q) = 0$ and $A^+(x, q) \leq A^+(e, q)$ and $A^-(x, q) \geq A^-(e, q)$. Therefore $1 \leq A^+(e, q)$ and $0 \geq A^-(e, q)$. But $1 \geq A^+(e, q)$ and $0 \leq A^-(e, q)$. Hence $A^+(e, q) = 1$ and $A^-(e, q) = 0$. Conversely if $A^+(e, q) = 1$ and $A^-(e, q) = 0$, then by the definition of normalized bipolar valued Q-fuzzy subset A is normalized.

2.10 Theorem: If $A = \langle A^+, A^- \rangle$ is a bipolar valued Q-fuzzy subgroup of a group G, then for any a in G the bipolar valued Q-fuzzy middle coset aAa^{-1} of G is also a bipolar valued Q-fuzzy subgroup of a group G.

Proof: Let A is a bipolar valued Q-fuzzy subgroup of a group G and a in G. To prove $aAa^{-1} = ((x, q), aA^+a^{-1}, aA^-a^{-1})$ is a bipolar valued Q-fuzzy subgroup of G. Let x and y in G and q in Q. Then $(a A^+a^{-1})(xy^{-1}, q) = A^+(a^{-1}xy^{-1}a, q) = A^+(a^{-1}xaa^{-1}y^{-1}a, q) = A^+(a^{-1}xa(a^{-1}ya)^{-1}, q) \geq \min\{A^+(a^{-1}xa, q), A^+(a^{-1}ya, q)\} = \min\{(aA^+a^{-1})(x, q), (aA^+a^{-1})(y, q)\}$.

Therefore $(aA^+a^{-1})(xy^{-1}, q) \geq \min \{ (aA^+a^{-1})(x, q), (aA^+a^{-1})(y, q) \}$. And $(aA^-a^{-1})(xy^{-1}, q) = A^-(a^{-1}xy^{-1}a, q) = A^-(a^{-1}xaa^{-1}y^{-1}a, q) = A^-(a^{-1}xa(a^{-1}ya)^{-1}, q) \leq \max\{A^-(a^{-1}xa, q), A^-(a^{-1}ya, q)\} = \max \{ (aA^-a^{-1})(x, q), (aA^-a^{-1})(y, q)\}$. Therefore $(aA^-a^{-1})(xy^{-1}) \leq \max \{(aA^-a^{-1})(x, q), (aA^-a^{-1})(y, q)\}$. Hence aAa^{-1} is a bipolar valued Q-fuzzy subgroup of a group G.

2.11 Theorem: Let $A = \langle A^+, A^- \rangle$ be a bipolar valued Q-fuzzy subgroup of a group G and aAa^{-1} be a bipolar valued Q-fuzzy middle coset of the group G, then $o(aAa^{-1}) = o(A)$, for any a in G.

Proof: Let A be a bipolar valued Q-fuzzy subgroup of a group G and a in G. By Theorem 2.10, the bipolar valued Q-fuzzy middle coset aAa^{-1} is a bipolar valued Q-fuzzy subgroup of a group G. Further by the definition of a bipolar valued Q-fuzzy middle coset of the group G we have $(aA^+a^{-1})(x, q) = A^+(a^{-1}xa, q)$ and $(aA^-a^{-1})(x, q) = A^-(a^{-1}xa, q)$, for every x in G and q in Q. Hence for any a in G, A and aAa^{-1} are conjugate bipolar valued Q-fuzzy subgroup of the group G as there exists a in G and q in Q such that $(aA^+a^{-1})(x, q) = A^+(a^{-1}xa, q)$ and $(aA^-a^{-1})(x, q) = A^-(a^{-1}xa, q)$ for every x in G and q in Q. By Theorem 2.6, $o(aAa^{-1}) = o(A)$ for any a in G.

2.12 Theorem: Let $A = \langle A^+, A^- \rangle$ be a bipolar valued Q-fuzzy subgroup of a group G and $B = \langle B^+, B^- \rangle$ be a bipolar-valued Q-fuzzy subset of a group G. If A and B are conjugate bipolar-valued Q-fuzzy subsets of the group G then B is a bipolar valued Q-fuzzy subgroup of a group G.

Proof: Let A be a bipolar valued Q-fuzzy subgroup of a group G and B be a bipolar valued Q-fuzzy subset of a group G. And let A and B are conjugate bipolar-valued Q-fuzzy subsets of the group G. To prove B is a bipolar valued Q-fuzzy subgroup of the group G. Let x and y in G and q in Q. Then xy^{-1} in G. Now, $B^+(xy^{-1}, q) = A^+(g^{-1}xy^{-1}g, q) = A^+(g^{-1}xgg^{-1}y^{-1}g, q) = A^+(g^{-1}xg(g^{-1}yg)^{-1}, q) \geq \min \{A^+(g^{-1}xg, q), A^+(g^{-1}yg, q)\} = \min \{B^+(x, q), B^+(y, q) \}$. Therefore $B^+(xy^{-1}, q) \geq \min \{B^+(x, q), B^+(y, q) \}$. And $B^-(xy^{-1}, q) = A^-(g^{-1}xy^{-1}g, q) = A^-(g^{-1}xgg^{-1}y^{-1}g, q) = A^-(g^{-1}xg(g^{-1}ygq)^{-1}, q) \leq \max \{A^-(g^{-1}xg, q), A^-(g^{-1}yg, q)\} = \max\{B^-(x, q), B^-(y, q)\}$. Therefore $B^-(xy^{-1}, q) \leq \max \{B^-(x, q), B^-(y, q)\}$.

Hence B is a bipolar valued Q-fuzzy subgroup of the group G.

2.13 Theorem: Let a bipolar valued Q-fuzzy subgroup $A = \langle A^+, A^- \rangle$ of a group G be conjugate to a bipolar valued Q-fuzzy subgroup $M = \langle M^+, M^- \rangle$ of G and a bipolar valued Q-fuzzy subgroup $B = \langle B^+, B^- \rangle$ of a group H be conjugate to a bipolar valued Q-fuzzy subgroup $N = \langle N^+, N^- \rangle$ of H. Then a bipolar valued Q-fuzzy subgroup $A\times B = \langle (A\times B)^{+}, (A\times B)^{-} \rangle$ of a group $G\times H$ is conjugate to a bipolar valued Q-fuzzy subgroup $M\times N = \langle (M\times N)^{+}, (M\times N)^{-} \rangle$ of $G\times H$.

Proof: Let A and B be bipolar valued Q-fuzzy subgroups of the groups G and H. Let x, x^{-1} and f be in G and y, y^{-1} and g be in H. Then (x, y), (x^{-1}, y^{-1}) and (f, g) are in $G\times H$ and q in Q. Now $(A\times B)^{+}((f, g), q)= \min\{A^{+}(f, q), B^{+}(g, q)\}= \min\{ M^{+}(xfx^{-1}, q), N^{+}(ygy^{-1}, q)\} = (M\times N)^{+}((xfx^{-1}, ygy^{-1}), q) = (M\times N)^{+}[(x, y)(f, g)(x^{-1}, y^{-1}), q] = (M\times N)^{+}[(x, y)(f, g)(x, y)^{-1}, q]$. Therefore, $(A\times B)^{+}((f, g), q) = (M\times N)^{+}[(x, y)(f, g)(x, y)^{-1}, q]$. And, $(A\times B)^{-}((f, g), q) = \max \{A^{-}(f, q), B^{-}(g, q)\}= \max\{M^{-}(xfx^{-1}, q), N^{-}(ygy^{-1}, q)\}= (M\times N)^{-}((xfx^{-1}, ygy^{-1}), q) = (M\times N)^{-}[(x, y)(f, g)(x^{-1}, y^{-1}), q] = (M\times N)^{-}[(x, y)(f, g)(x, y)^{-1}, q]$. Therefore, $(A\times B)^{-}((f, g), q) = (M\times N)^{-}[(x, y)(f, g)(x, y)^{-1}, q]$. Hence a bipolar valued Q-fuzzy subgroup $A\times B$ of a group $G\times H$ is conjugate to a bipolar valued Q-fuzzy subgroup $M\times N$ of $G\times H$.

REFERENCES

[1]. Anthony.J.M. and Sherwood.H, Fuzzy groups Redefined, Journal of mathematical analysis and applications, 69,124 -130 (1979).

[2]. Arsham Borumand Saeid, bipolar-valued fuzzy BCK/BCI-algebras, World Applied Sciences Journal 7 (11): 1404-1411(2009).

[3]. Azriel Rosenfeld, Fuzzy groups, Journal of mathematical analysis and applications 35, 512-517 (1971).

[4]. Choudhury.F.P. and Chakraborty.A.B. and Khare.S.S., A note on fuzzy subgroups and fuzzy homomorphism, Journal of mathematical analysis and applications, 131, 537 -553 (1988).

[5]. Gau, W.L. and D.J. Buehrer, Vague sets, IEEE Transactons on Systems, Man and Cybernetics, 23: 610-614(1993).

[6]. Kyoung Ja Lee, bipolar fuzzy subalgebras and bipolar fuzzy ideals of BCK/BCI-algebras, Bull. Malays.Math. Sci. Soc. (2) 32(3), 361–373 (2009).

[7]. Lee, K.M., Bipolar-valued fuzzy sets and their operations. Proc. Int. Conf. on Intelligent Technologies, Bangkok, Thailand, pp: 307-312(2000).

[8]. Lee, K.M., Comparison of interval-valued fuzzy sets, intuitionistic fuzzy sets and bipolarvalued fuzzy sets. J. Fuzzy Logic Intelligent Systems, 14 (2): 125-129(2004).

[9]. Mustafa Akgul, some properties of fuzzy groups, Journal of mathematical analysis and applications, 133, 93 -100 (1988).

[10]. Samit Kumar Majumder, Bipolar Valued Fuzzy Sets in Γ-Semigroups, Mathematica Aeterna, Vol. 2, no. 3, 203 – 213(2012).

[11]. Young Bae Jun and Seok Zun Song, Subalgebras and closed ideals of BCH-algebras based on bipolar-valued fuzzy sets, Scientiae Mathematicae Japonicae Online, 427-437 (2008).

[12]. Zadeh, L.A., Fuzzy sets, Inform. And Control, 8: 338-353(1965).